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Sparse Warcasting

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Bilateral Assistance
& Capacity Building
for Central Banks

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Abstract

Forecasting economic activity during an invasion is a nontrivial exercise. The lack of timely statistical data and the expected nonlinear effect of military action challenge the use of established nowcasting and short-term forecasting methodologies. In a recent study (Constantinescu (2023b)), I explore the use of Partial Least Squares (PLS) augmented with an additional variable selection step to nowcast quarterly Ukrainian GDP using Google search data. Model outputs are benchmarked against both static and Dynamic Factor Models. Preliminary results outline the usefulness of PLS in capturing the effects of large shocks in a setting rich in data, but poor in statistics.

Keywords: Nowcasting, quarterly GDP, Google Trends, Machine Learning, Partial Least Squares, Sparsity, Markov Blanket

JEL: C38, C53, 55, E32, E37

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The views expressed in this paper are solely those of the author and do not necessarily reflect those of the National Bank of Ukraine.

1 Introduction

The 2022 Russian invasion is a shock of varying regional and temporal intensity with highly heterogeneous impact on the Ukrainian economy. Whereas the initial stages of the assault increased uncertainty and affected Ukrainian economic activity as a macro shock, subsequent internal migration from east to west and south-west gave way to a much more nuanced pattern. Deep and long lasting contractions in regions experiencing intense military aggression and constant population outflows were contrasted by shorter and shallower cycles in regions acting as emigration gateways to the European Union. The launch of the "Black Sea Grain Initiative" further revived southern oblasts¹.

An immediate effect of the invasion was the introduction of Martial Law on February 24th. As a result, data gathering and processing by Ukrainian state statistical agencies were virtually suspended in the first quarters of 2022. The lack of up-to-date economic data was widespread affecting both the functioning of the national statistical agency as well as that of private surveying companies.

The current context differs to the Covid period. Among both developing and developed economies, data gathering witnessed a robust extension in both scope and coverage, with many high-frequency alternative data sources used to complement official hard and soft data. Full-scale war virtually freezes all sampling and processing of information related to economic activity of both public and private entities. For security reasons, access to novel data used during the Covid period on mobility, mobile phone and internet access, electricity and fuel consumption, had also been suspended.

Central banking nowcasting models, developed around regular statistical releases, become severally constrained. These models may still be employed albeit with little flexibility beyond scenario analysis and comparative exercises using past conflicts as empirical counterparts. Without any readings on consumption, investment and production aggregates or their survey-based micro estimates, assessing the scale and speed of changes in the Ukrainian economy mutated from a well-tuned multistage process into a creative exploration.

The first necessary step in this context is replacing hard and soft data with alternative measures expected to correlate well with the variables of interest, in our case quarterly Gross

¹<https://www.un.org/en/black-sea-grain-initiative>

Domestic Product. This implies moving further back on the data creation value chain and finding appropriate alternative inputs and models, the output of which correlate well with GDP. In [Constantinescu, Kappner and Szumilo \(2022, 2023\)](#) we present preliminary results of forecasting annual regional GDP using a multitude of alternative data sources. Nightlights, social media activity and Google search volumes may become valuable substitutes when no official data are present.

2 Literature Review

Standard nowcasting models, as those put forward by [Giannone, Reichlin and Small \(2008\)](#); [Stock and Watson \(2002, 2012\)](#), build on the basis of an extended list of variables, measured both at quarterly and monthly or higher frequency. These models have been developed to distill a large number of potentially useful time-series in a much smaller number of driving factors. These factors, generally described by a multivariate state-space VAR, become input in subsequent estimation exercises, such as bridge equations or state-space models. [Bańbura et al. \(2013\)](#); [Bok et al. \(2018\)](#) are widely employed specifications. More recent literature leverages high frequency data to extract signals of lower frequency targets. Bayesian alternatives open up new modeling avenues as in [Cimadomo et al. \(2022\)](#).

The current context, rich in alternative data but poor in official statistics, sets up the task of nowcasting quarterly aggregate GDP in terms similar to the above literature, yet with a notable exception. Many hard and soft variables included in the estimation exercises referenced above, are gathered following statistical guidelines and have, to varying degrees and via theoretical justifications, a connection to the target variable. Extensions with high-frequency social media or web-search activity build on top of an already existing skeleton of hard and soft variables.

In the current study, no hard or soft data are used in the aggregate nowcasting exercise as none was available in the first quarters of 2022 when the first steps of the current exercise were undertaken.

Empirical univariate and multivariate exercises cement the usefulness of alternative data inputs as well as the need of a pre-selection stage when the list of possibly useful variables grows larger than the sample size. Various methodological alternatives are present in the

literature to deal with this issue, ranging from sparse principal components as in [Zou, Hastie and Tibshirani \(2006\)](#); [Pena, Smucler and Yohai \(2021\)](#) to sparse dynamic factors as the model put forward in [Mosley, Chan and Gibberd \(2023\)](#).

The regularization algorithm proposed for example in [Ferrara and Simoni \(2022\)](#), dubbed by the authors *Ridge after Model Selection*, requires the availability of both Google Trends and official variables. The second stage of their procedure employs a Ridge regularization of Google Trends, preselected in the first stage, together with official variables. They show Google Trends improve GDP predictions conditional on hard or soft data being available with forecasting gains depending on the stage of the economic cycle.

2.1 Partial Least Squares

At the core of most big-data nowcasting applications one may find algorithms estimating a small number of potential latent factors (with a dynamic structure) driving a large number of observed explanatory variables. In this respect, principal components (PC) are currently the most widely employed estimation strategy of nowcasting models. Some notable exceptions are the recent applications of PLS in macroeconomic forecasting in [Eickmeier and Ng \(2011\)](#); [Cubadda, Guardabascio and Hecq \(2013\)](#); [Groen and Kapetanios \(2016\)](#) and in finance in the study by [Preda and Saporta \(2005\)](#) who use PLS to forecast stock returns. Leveraging both cross-sectional and time-series information, [Kelly and Pruitt \(2015\)](#) propose a multi-stage algorithm which features PLS regressions as a special case.

[Wold, Ruhe, Wold and W. J. Dunn \(1984\)](#); [Wold, Martens and Wold \(2006\)](#) introduce the partial least square method as an alternative projection method to PCR. [Stoica and Söderström \(1998\)](#) explore conditions under which PCR and PLS produce equivalent results, deriving in the process, asymptotic formulas for bias and variance of the PLS estimator. [Helland \(1990\)](#) further illuminates the relationship between PLS and PCR presenting conditions under which the two methodologies yield similar results. [Cubadda and Hecq \(2010\)](#) build a bridge to PLS using economic terminology and widely employed VAR canonical models. They indicate the improved performance of PLS in estimating sources of autocorrelation commonality in a data-rich environment.

[Götz and Knetsch \(2019\)](#) use well-established bridge equation models augmented with Google search time-series and outline their utility in improving the estimation of both GDP,

GDP components and monthly activity indicators. The authors highlight that Google search data may become a possible alternative to surveys in the manufacturing sector. PLS is employed in this study but only as an intermediary step to extract relevant factors later used in a PCR estimation. More recent publications leverage new machine-learning models in conjunction with large volumes and varieties of online search data to nowcast GDP growth rates as in [Dauphin et al. \(2022\)](#).

3 Data Description

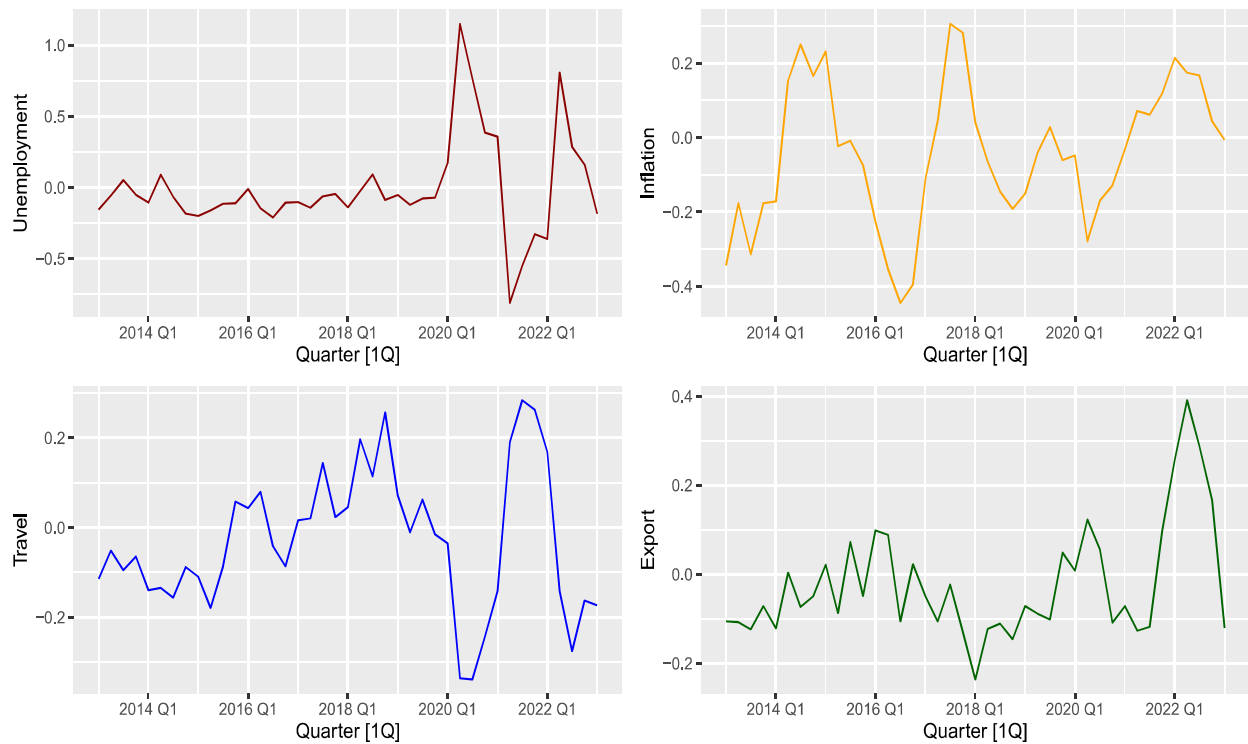
The target variable is deflated quarterly changes in aggregate GDP. The initial estimation timeframe is 2012 to 2021, reduced to 2013 to 2021 once appropriate lags are accounted for. The explanatory variables are a large number of Google Trends selected based on prior studies and observed local preferences². The raw daily data is aggregated at quarterly level. Quarterly changes are then computed based on the quarterly means and medians of monthly values.

Google Topics are available as time series ranging from 1 to 100, for a selected period in time and geographical area, representing the relative ranking of keywords (car) or categories of keywords (car, automobile, etc.) possibly in different languages present in a given geographical area. Low search volumes are set to 0 to preserve anonymity of searches. A value of 100 represents the most searched for term in the selected (time frame, geography) tuple. The widespread coverage of topics and categories allows for a possible matching of Trends to many official statistical series. The empirical relevance has already been established for a wide number of Google Trends, see for the earliest studies [Ettredge et al. \(2005\)](#), [Choi and Varian \(2009\)](#) or [Mclaren and Shanbhogue \(2011\)](#). Labor market dynamics implied by Google Trends are presented in [Mclaren and Shanbhogue \(2011\)](#) who uses volume of online searches to track labor and housing markets in the United Kingdom. [Askitas and Zimmermann \(2009\)](#) validate the use of internet searches during the 2008 Great Recession to better pinpoint the contraction in the German labor market.

In [Figure 1](#), the quarterly changes in Trends tracking searches related to Unemployment,

²See for example the evolution of searches on "work.ua", Ukraine's largest online job portal; a close competitor is the job section of "olx.ua". Searches about these two sites correlate strongly with the topic "Labor".

Figure 1



Note: Evolution over time of a few Google Trends for Ukraine

Inflation, Travel and Export are shown³. For example, searches related to Inflation reflect the dynamics of large inflationary swings associated with the 2014 Revolution of Dignity and the subsequent profound reforms of the NBU. Note that overall, following the introduction of Inflation Targeting in 2016, changes in inflation searches are on average negative reflecting the decrease in relative interest in the topic. In retrospect, the post-Covid robust increase in search volume may have foreshadowed the above target inflationary pressures flaring already in 2021. The Covid and war spikes in Unemployment, post-Covid strong rebound in Travel are further preliminary indications of the ability of Google Trends to recompose the mosaic of economic information present in official statistics.

The nature of the available data brings about the need for a new methodological lens. Most models and applications referenced above use variables with a proven track record in nowcasting and forecasting, and are shown to load on the latent factors under a variety

³Data wrangling and exploratory data analysis are performed with the R packages *tidyverse* by Wickham et al. (2019), *fabletools* by O'Hara et. al and *tsibble* by Wang et al. (2020).

of macroeconomic contexts. In the current case one should expect a much larger number of variables to be irrelevant or only weakly related to the underlying latent factors. Furthermore, a "large p (number of variables) small n (sample size)" setting adds its own challenges. Although PLS has been successfully used in bioinformatics applications with $p \gg n$ ⁴, the risk of overfitting to samples and predictors has been indicated as a challenge and various regularization techniques have been developed and employed (see details below). Overfitting will result in non-generalizability (of both variable selection and factor loading estimates) and lead so to poor out-of-sample performance.

4 Methodology

Principal Components, a dimensionality reduction unsupervised algorithm, does not have a predefined target. PCR's goal is to identify a lower rank representation of a high dimensionality matrix X , where many of the vectors may be highly correlated⁵. In a second stage, PCs from the first stage are used as inputs in a linear or non-linear regression model.

The Partial Least Squares algorithm, a supervised algorithm, is designed to account for the possible presence of multi-collinearity among the vectors of X *and* also to reflect their correlation with a selected target (or targets)⁶. This is a fundamental difference as the underlying vector space representation and projections of X are computed to reflect their correlation with a chosen target variable. Different target variables will therefore lead to different projections and loadings, depending on the strength of correlation between X and different targets y .

The PLS model and the associated estimation algorithm are worth exploring in detail, in particular highlighting similarities and differences to the widely used PCR estimation algorithm and its underlying assumptions. A simple simulation is presented in the Appendix. It features a small sample and relatively many explanatory variables driven by two latent factors. It is used to showcase settings in which PCR fails to properly estimate the latent

⁴High dimensional genomic studies regularly employ a number of highly collinear explanatory variables which is several tens of times larger than the number of samples or observations

⁵The PCR algorithm is used for example to compress images, represented as large matrices of 1s and 0s, for transmission over the internet. Via PCR, redundant information is eliminated, reducing the size of the file. In the case of an image, nearby pixels are highly correlated (in terms of color, etc), the reduced form representation being of much smaller size, appearing almost indistinguishable to the human eye.

⁶In ML jargon, a variable supervises the outcome of the algorithm when the algorithm optimizes over inputs in an attempt to predict as closely as possible the supervising variable

factors, unlike PLS which, despite the small n , recovers their structure to a very good degree. Leave-one-out cross-validation is used to select the optimal number of components, with up to 5 potential factors considered.

4.1 The Latent Factor Model

In the PLS algorithm, the structural assumption is that a few latent factors K drive both the p vectors in X and univariate observations in y (Helland (1990); Stoica and Söderström (1998)), with $K \ll p$. The objective of the algorithm is to recover both the latent factors as well as the loadings of both X and y on the factors via repeated partial regressions of y on the vectors in X , with regressions ordered according to the strength of the covariance between y and x_j . The assumed latent structure is given in equations 1 to 3 below.

$$T = X \times W^*, \quad X \in \mathbb{R}^{n \times p}, T \in \mathbb{R}^{n \times K}, K < p \quad (1)$$

$$X = T \times P' + \epsilon, \quad P \in \mathbb{R}^{p \times K} \quad (2)$$

$$y = T \times C' + \xi, \quad C \in \mathbb{R}^{1 \times K} \quad (3)$$

Most of the time, for computation purposes, both y and X will be scaled and centered. The first equation represents the relation between the latent factor T and X . The scores computed in T are linear combinations of the vectors in X and their respective weights. They summarize the information in the original predictor variables that is most relevant to the response variable y . As will be shown below, the weights W^* are the directions in the predictor variables' space which maximize the covariance with the response variable.

The second equation indicates that X is approximated by the latent factors in T times the loading matrix P and a remainder random noise matrix ϵ .

The third structural equation shows y to depend on the same latent factors T and a random noise component.

Given predictions from the above model for both $\hat{X} = T * P'$ and $\hat{y} = T * C'$, one can cast the problem in terms of the traditional multivariate regression expressing y as a linear model of X :

$$\hat{y} = T \times C' = X \times W^* \times C' = X \times \hat{b}^{OLS} \quad (4)$$

The second equality follows by replacing T with its inflated equivalent in terms of X . The third equality should be read as an identity of the OLS regression parameters from the model $y = X * b + \epsilon$ as a function of the latent factor parameters in equations 1 to 3. Specifically, $\hat{b}^{OLS} = W^* \times C'$.

The graph of a simple model with two latent variables is shown in Figure 2 below. L_1 drives X_1, X_2 while X_3 only loads on L_2 . The target y is driven by both L_1 and L_2 .

The structure of the model should reflect, even if at first in a heuristic manner, the presumed link between X and y . In the context of traditional nowcasting and near forecasting exercises, many variables included in a prototypical factor model, can be presumed to both be driven by the underlying latent factors and themselves have idiosyncratic components driving y . This is sustained by both theoretical and empirical exercises if one considers labor market variables, data related to firm investment decision, surveys of import and export and so on. In this case, y will be affected by a selected x via its indirect loading on the latent factors and via direct shocks in x . This case would be reflected in the graph with additional arrows pointing from X_1, X_2, X_3 to y .

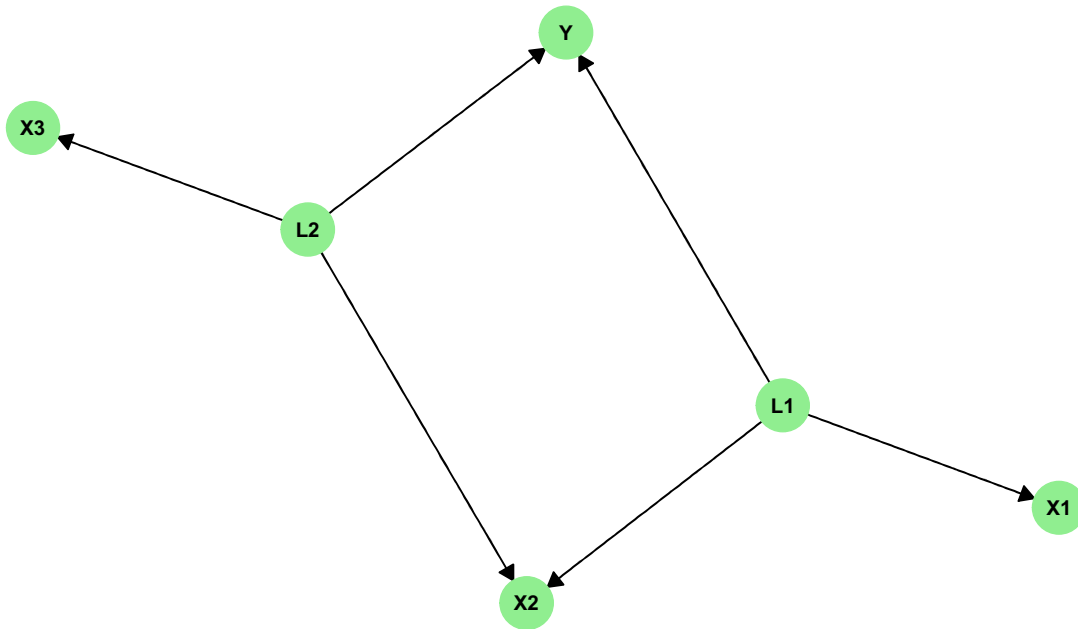


Figure 2 – Network Graph of a Simple Latent Factor Model

4.2 The PLS algorithm

The PLS algorithm is described in [Wold, Ruhe, Wold and W. J. Dunn \(1984\)](#); [Helland \(1990\)](#); [Bastien, Vinzi and Tenenhaus \(2005\)](#); [Wold, Martens and Wold \(2006\)](#) as the iterated use of OLS regressions defined such that the loadings reflect the covariance between the explanatory variables and the target variable. Given a vector of demeaned variables, the PLS regression model with K latent factors and p centered explanatory variables is given as

$$\mathbf{y} = \sum_{h=1}^K c_h \times t_h + \epsilon \quad (5)$$

$$t_h = \sum_{j=1}^p w_{hj}^* \mathbf{x}_j \quad (6)$$

with t_h extracted to be independent of each other. The first PLS factor, $\mathbf{t}_1 = \mathbf{X} \times \mathbf{w}_1^*$ is computed as

$$\mathbf{t}_1 = \frac{1}{\sqrt{\sum_{j=1}^p \text{cov}(\mathbf{y}, \mathbf{x}_j)^2}} \sum_{j=1}^p \text{cov}(\mathbf{y}, \mathbf{x}_j) \mathbf{x}_j. \quad (7)$$

The covariance between y and x is the regression coefficient in the simple OLS model of y and scaled x . Following the calculation of the first component, the algorithm proceeds to retrieve the second component with respect to the residuals from the first stage. The remaining variation in y and x_j , purged via t_1 , is then used to build the second factor t_2 . The algorithm is repeated either for a pre-imposed number of steps (to yield a determined number of factors) or is subject to cross-validation or bootstrap. In the current setting, bootstrap or CV is employed as no a-priori knowledge is available on the optimal number of factors nor on the amount of penalty. This is particularly important in the quarterly PLS model with regional Trends, where, depending on the specification, well over 2000 variables may appear in the list of possible explanatory variables (25 regions * 40 Google Trends per region + appropriate autocorrelation structures). For the yearly regional GDP model, the difference between the target sample size (9 yearly GDP changes) and the number of explanatory variables poses similar, though significantly smaller, challenges.

Note the difference to standard PCR regression in which only the covariance between explanatory variables is considered. Given the same p explanatory variables (centered and

scaled), M Principal Components Z_m are extracted as

$$Z_m = \sum_{j=1}^p \lambda_{jm} X_j \quad (8)$$

The λ_{jm} parameters, the principal component loadings, are selected so that the extracted components are orthogonal. In a second stage, an OLS regression is estimated with $M \ll p$ PCs.

$$y_i = \sum_{m=1}^M \xi_m z_{im} + \epsilon_i, \quad i = 1, \dots, n \quad (9)$$

Another useful methodological lens through which to consider the PLS and PCR algorithms is to view them as constrained optimization problems ([Hastie et al. \(2013\)](#)). The objective function reflects the supervised nature of PLS as opposed to the PCR, which only looks for lower dimensional representation of X .

The latent components stored in T are computed using the recursive solutions W^* of the following maximization problem:

$$\max_w \text{Corr}^2(y, X \times w) \text{Var}(X \times w) \quad (10)$$

$$\text{s.t. } w' \times w = 1, \quad w' \times \Sigma_X \times w_j, \quad \forall j = 1, \dots, K - 1 \quad (11)$$

Contrast the above with the PCR optimization problem in eq. 12 below:

$$\max_w \text{Var}(X \times w) \quad (12)$$

$$\text{s.t. } w' \times w = 1, \quad w' \times \Sigma_X \times w_j, \quad \forall j = 1, \dots, M - 1 \quad (13)$$

Alternative algorithms modifying standard PC have been proposed for example in [Bair, Hastie, Paul and Tibshirani \(2006\)](#), where a classification problem is tackled via supervised principal components. The proposed algorithm, similar in spirit to PLS, conducts feature selection as a preliminary step to the estimation of the latent factors. The authors highlight the superior performance of their proposed algorithm against a suite of alternatives, including PLS. Nevertheless the standard PLS, lacking a regularization step, is employed as benchmark. As indicated above, PLS will shrink parameters of variables with a weak connection to the latent factor but will not fully exclude them from the estimation exercise. In this respect, the standard PLS is more similar to performing a ridge regression rather than a lasso regression.

4.3 Sparse PLS and PCR

Given a mix of potentially useful and less useful predictors, a sparsity step is needed as the inclusion of more data is not guaranteed to improve the performance of a prediction model (Boivin and Ng (2006)). Regularization restrains estimation in the presence of a large number of variables, by screening potentially irrelevant or little relevant inputs via a penalty term, the Lasso being one such L_1 example (Tibshirani (1996)). This step is even more important at quarterly frequency given that both target and explanatory variables may have autocorrelation structures which apriori are not necessarily homogeneous with respect to the modelled latent factor. This leads to a several fold increase of the number of possible variables used in the estimation stage. In this setting, the risk of fitting noise looms large and threatens the validity of out-of-sample predictions.

Why is sparsity needed in a PLS regression? Chun and Keleş (2010) indicate challenges to asymptotic consistency of the PLS estimator in a "large p small n " context, with fixed p_1 relevant and increasing $p - p_1$ irrelevant variables. The intuition for the lack of asymptotic consistency comes from the ridge-like nature of the PLS algorithm. Given that PLS latent factors load on all variables available in X , a larger fraction of irrelevant variables weaken the ability of the algorithm to identify the true factor directions. Sparsity is achieved via variable selection in a multitude of ways, depending on the joint specificities of data sample and machine learning model. Tsamardinos and Aliferis (2003) highlight the close connection between variable selection, estimated model and metric used for prediction performance. The authors also present necessary properties of successful variable selection algorithms which account for the overall network of dependencies among variables. The Markov Blanket of the associated Bayesian network offers a new avenue to test dependence structures in a large set of variables when the overall goal is to identify causal mechanisms in large datasets.

Variable selection, also known as feature selection⁷, is a step in the process of building predictive machine learning models particularly useful for small samples or when model performance may be negatively impacted by noisy features. It offers a means to improve model accuracy, reduce complexity, and enhance interpretability. Selection models are usually classified into three categories: Filter methods, Wrapper methods, and Embedded methods

⁷"Features" is the term generally used in the ML literature to refer to prediction or explanatory variables

Tsamardinos and Aliferis (2003).

Filter methods rely on a selected statistical measure to assign a score to each variable. This score is then compared to a previously chosen threshold, hard or soft, and based on this comparison to the threshold, a variable is either included or excluded from a model. Typical score choices for PLS applications are the loading weights from the PLS algorithm or regression coefficients from the subsequent PLS regression. These methods are often univariate and consider the features independently of the subsequent model in which they are used, that is, no further model tuning is undertaken once variables have been selected. Although straightforward and computationally efficient, making them a good choice for large datasets, they may have limited accuracy in selecting the optimal sets with possible interactions. More importantly, filter methods do not consider overall model performance.

Wrapper methods evaluate different variables subsets based on their predictive performance in a specific machine learning model. The considered model is used to evaluate a combination of features iteratively. Different feature combinations are evaluated in terms of model accuracy, using for example cross-validated R2 or MSRE. Examples of such methods are recursive feature elimination, forward selection, and backward elimination in linear models. Computationally expensive, they are model dependent. The chosen features depend on the model, limiting the ability to generalize the selection across different models. The same feature selection wrapper will produce, for example, different optimal subsets for PCR as compared to PLS or Generalized Additive Models.

Regularization methods are the most common type of embedded methods. Examples of these methods are LASSO (Least Absolute Shrinkage and Selection Operator), Elastic Net, and Ridge Regression as in Tibshirani (1996). Embedded solutions, as the algorithm put forward in Chun and Keleş (2010), incorporate the variable selection step within the PLS algorithm. The identification of the optimal subset of variables is performed for each considered factor. Chun and Keleş (2010) propose an L_1 and L_2 penalty in the optimization problem recasting the algorithm in terms of the traditional PLS problem augmented with additional sparsity inducing penalties and associated constraints. In case of the univariate PLS regression, the problem is equivalent to imposing a Lasso penalty. In applications, both the amount of penalty and number of relevant number of factors are determined via cross-validation. Like wrapper methods, these are computationally fast as variable selection

and model training are done simultaneously. They may nevertheless overfit in sample and by design, are model dependent.

Given the current exercise, which method should one use? [Mehmood, Sæbø and Liland \(2020\)](#) provides some guidelines, benchmarking several variable selection methods in mainly bioinformatics and genomics PLS applications. Their Monte Carlo analysis points out that the choice depends on the amount of collinearity among explanatory variables and the overall number of predictive variables. The compromise faced across the different methods is one between good and stable variable selection accuracy and overall prediction ability. Given the time series exercise, emphasis is placed on Genetic Algorithms Wrappers and [Chun and Keleş \(2010\)](#)'s embedded method.

A further related issue is data sampling variability. Sample variability is a potential issue for topics with low level of searches, especially when the forecasting exercise is performed at high-frequencies (weekly or daily). As Google does not return the entire history of searches, but only a sample, repeated query for the same (topic, period, geographical area) can return quite different results. This variability is dependent on the underlying searches, and indirectly, on the population size in a given area. Establishing the relationship between population size and with-in month sampling variability is the goal of [Eichenauer, Indergand, Martinez and Sax \(2021\)](#). The authors focus on the necessary conditions to create a daily index, accounting for both sampling variance and the lack of consistency between daily and lower frequency searches. To tackle this issue, in line with their recommendations, repeated samples should be produced for the latest period.

4.4 Genetic Algorithms

GAs are part of a larger group of evolutionary algorithms that employ search heuristics inspired by the process of natural evolution ([Holland \(1992\)](#); [Goldberg and Deb \(1991\)](#)). A Genetic Algorithm starts with a population of candidate solutions, representing different subsets of potential features. These solutions, often referred to as 'chromosomes,' are usually represented as binary strings, where each bit corresponds to the presence (1) or absence (0) of a feature in the subset. Each initial solution's quality is assessed by a fitness function, in our case, the performance of the PLS model trained using the corresponding subset of features.

Following the evaluation of the first generation of feature subsets, chromosomes are chosen to form the basis of the next generation. The new generation is assembled via selection. Selection operates under the principle of "survival of the fittest," where fitter chromosomes, subsets of variables performing well in terms of insample PLS fit, have a higher chance of being selected. Techniques for selection include roulette wheel selection, tournament selection, and rank selection (Goldberg and Deb (1991)). Selected chromosomes undergo crossover, mimicking biological reproduction. Two parent chromosomes are combined to form one or more offspring, each containing features from both parents. To avoid getting trapped in local optima, mutation introduces small random modifications to offspring, enhancing the diversity of the solution space. In feature selection, mutation might involve randomly swapping a feature's presence or absence. This is repeated over multiple generations until a stopping criterion is satisfied. Genetic Algorithms can handle non-linearity and interactions between variables in an effective manner. They can be computationally demanding, as multiple solutions have to be evaluated over multiple generations and may converge prematurely in populations with little diversity of considered features combinations. R implementations are available, for example in the *GA* package⁸.

Genetic algorithms (GA) and their variants have been successfully employed in time series forecasting for example in Hansen et al. (1999), where neural networks with a GA-determined network structure show substantial predictive improvement over traditional ARIMA models and Messias et al. (2015) where GA are used to optimize load allocation in cloud computing. Hasegawa et al. (1997) and Leardi (2000) are early applications of GA as variable selection methods in PLS regressions.

5 Results

The Dynamic Factor Model is estimated and used a benchmark alongside with the considered PLS variants. The two stage DFM version ala Doz, Giannone and Reichlin (2011) as well as the quasi-MLE specification of Bańbura and Modugno (2012) are estimated and employed to produce one and two-quarters ahead forecasts. Bai and Ng (2002)'s IC_2 test plateaus between 4 and 8 components, followed by a further decrease above 10 PCs. A similar profile

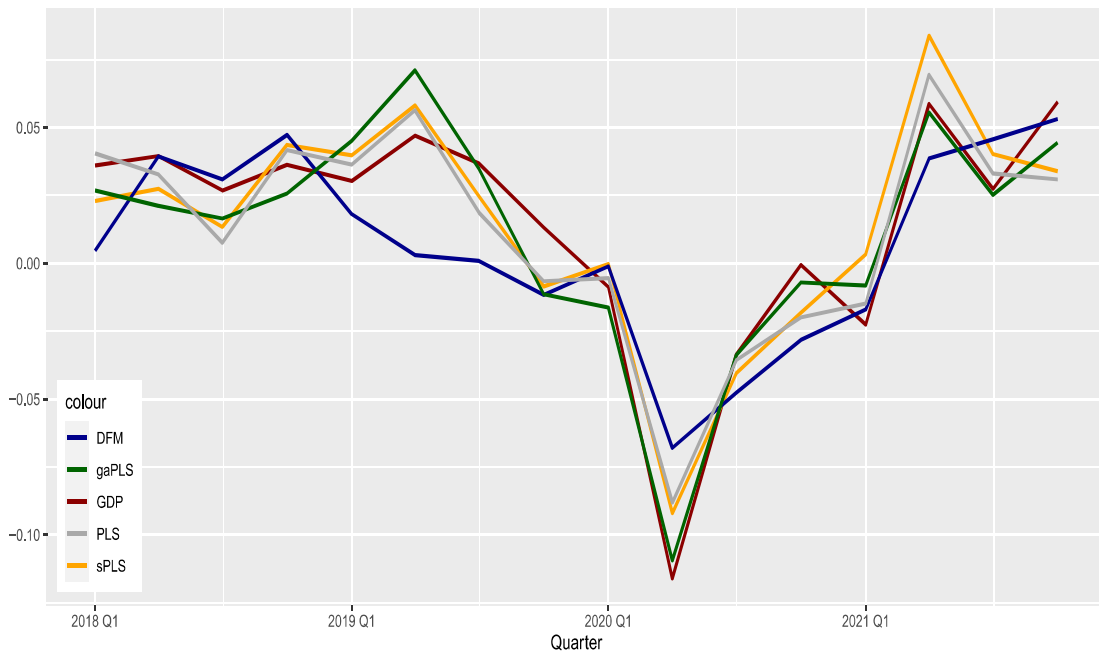
⁸<https://cran.r-project.org/web/packages/GA/vignettes/GA.html>

is returned when considering the IC_1 and IC_3 tests.

Results of Monte Carlo simulations in [Hastie et al. \(2013\)](#) show CV will select fewer PLS latent factors as compared to similar PCR exercises of the same data. On a preliminary basis, and considering the higher number of PCs used for example in [Giannone et al. \(2008\)](#), this provides initial support in favor of employing PLS estimations for small data samples.

Figure 3 shows the in-sample goodness of fit over the period 2018 Q1 - 2021 Q4, comparing the performance of the Dynamic Factor Model, the sparse PLS as in [Chun and Keleş \(2010\)](#) and the GA PLS. To facilitate comparison, a shorter time window is selected for illustration. In the appendix, the fit for the entire estimation period is shown in Figure 8.

Figure 3

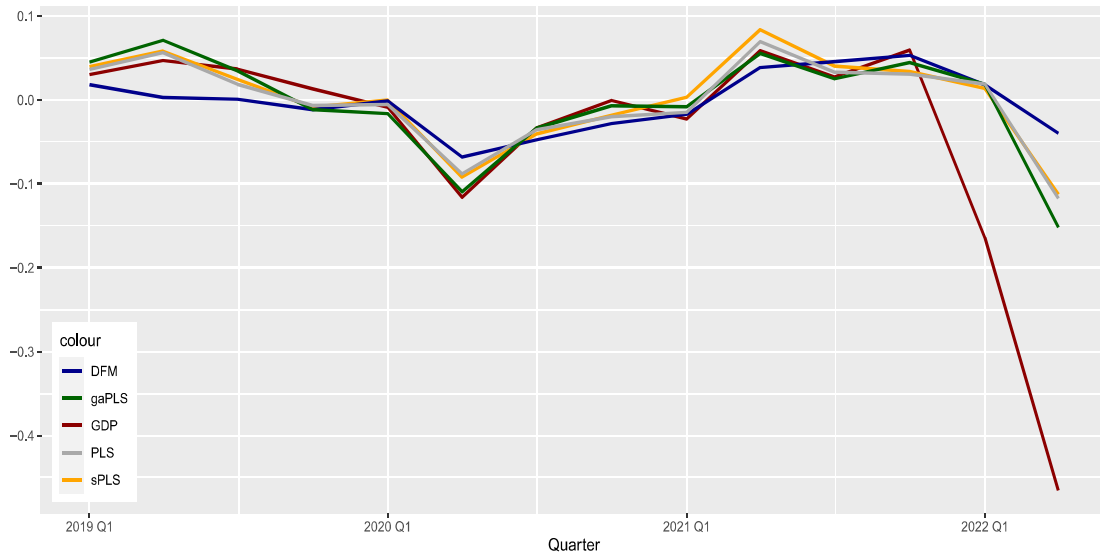


Note: In-sample Performance of Latent Factor Models

Several important observations follow. Overall, there are qualitative and quantitatively non-negligible differences among the DFM and PLS alternatives regarding insample predictions. The insample fit is much closer for PLS and its variants as compared to DFM estimates. This should not be surprising as the PLS algorithms specifically target the outcome variable. But insample fit is a nowcaster’s false friend. Relevant criteria are out-of-sample performance, ability in picking turning points and properly gauging surges and contractions as they occur.

DFM produce a similar directional nowcast in the first quarters of 2021 as sPLS and gaPLS. It does however not properly capture the Q3 slowdown in growth, most likely owing to the strong autocorrelation feature uniformly forced on all variables via the state-space AR(1) specification. The standard PLS and sPLS also indicate a slowdown in 2021 Q4 when none was observed. The gaPLS points in the same direction as actual GDP, although in a more muted fashion.

Figure 4



Note: Out-of-sample forecasts

The models trained using 2013-2021 data are then employed to produce nowcasts for Q1 and Q2 2022. The Q2 2022 value may be more properly considered a short-term forecast rather than a nowcast as in May 2022, Google Trends data was available yet no other macro-data had been released since the start of the invasion. Three sets of nowcasts are shown, using a comparable number of latent factors to facilitate performance comparison. In the tradition of ML estimation, this represents a test on previously unseen data. All considered models identify the contraction although of markedly different size.

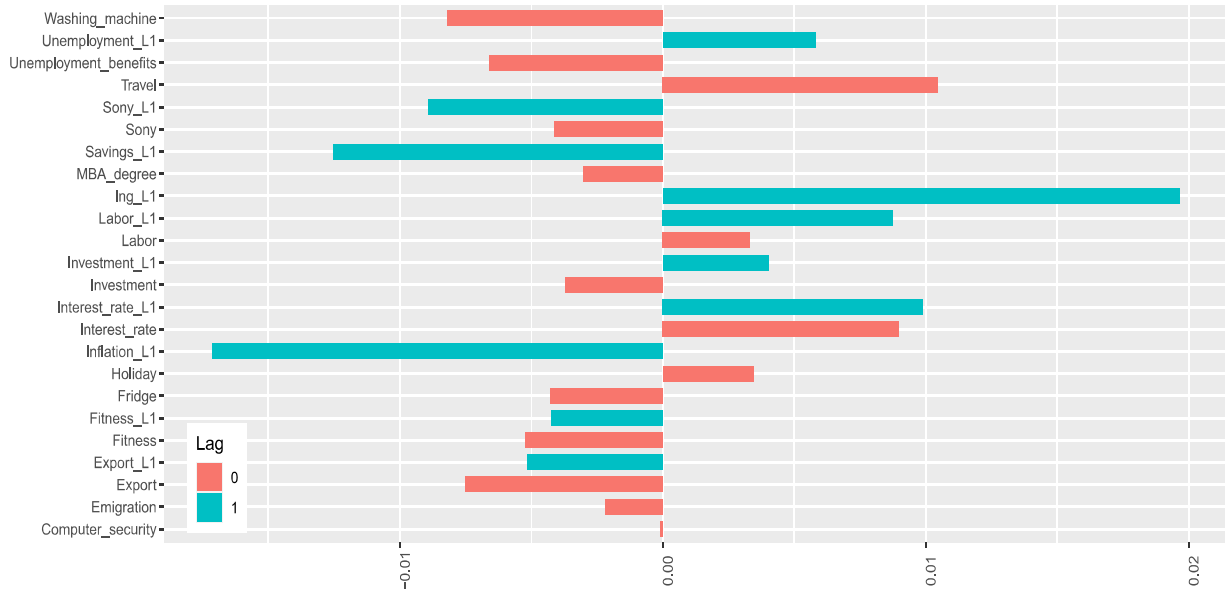
The regional model, using as input variables regional Google Trends, strengthens the case in favor of shrinkage. Most variables, related to Unemployment, Economy, Inflation and various consumer goods, are selected from economically large regions, such as Kyiv city, Kharkivska, Lvivska, and Odeska. Very few of the smaller oblasts' variables weigh on the

evolution of aggregate, according to the Sparse PLS model.

5.1 Variable Selection

Which variables are selected and what are their regression parameters? In Figure 5 one may find the result of the GA wrapper. All estimations are performed with standardized data. Note the high positive parameter associated with "lng_L1", the first lag of changes in GDP. The AR(1) component is autonomously picked by the algorithm, being part of the general pool of genes. Positive impact comes from contemporaneous changes in searches on "Holiday", "Travel" and "Interest Rate" and lags in "Unemployment", "Labor" and "Interest Rate". High past "Inflation" searches have a high and negative impact on current GDP, as do changes in "Savings" and "Sony". Past searches related to particular brands may act as consumption proxies. Although preliminary, the results support general economic intuition of the link between the macro variables possibly tracked by Google Trends and their effect on GDP.

Figure 5



Note: Variable Selection and Parameter Values

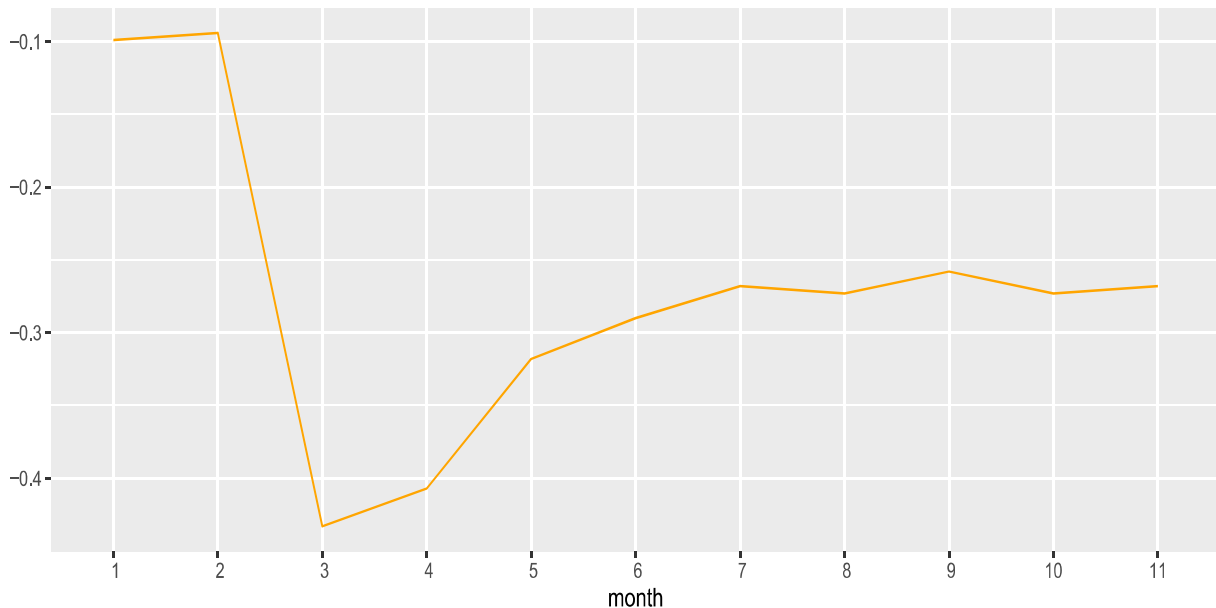
In the Appendix, the results of the sPLS algorithms are available in figure 9. Zeros are overlaid to offer a better view of the sparsity structure. Although there is some overlap in

terms of selected variables and lags, as well as in the signs of the parameters, this is far from uniform.

5.2 A Regional Factor Model

A regional factor model is estimated using annual regional GDP growth rates. PLS latent factors for each region are extracted from their respective cash-turnover growth rates. Payments data contain strong, timely signals about the direction of the economy, as shown in [Galbraith and Tkacz \(2018\)](#); [Aprigliano et al. \(2019\)](#). Recent studies leverage the ability of ML algorithms to handle large volumes of data to predict short-run macro dynamics, as for example the work of [Chapman and Desai \(2022\)](#). In particular for Ukraine, cash turnover may be more relevant to track due to the much higher cash usage.

Figure 6



Note: Aggregate Nowcasts - Regional PLS Model

The nowcast is computed using annual bridge equations estimated over the period 2013-2020. Regional nowcasts are then aggregated using 2020 national GDP shares. The model captures well the depth of the March contraction, the mid-year rebound as well as the ensuing dynamics for the year-end. The official y-o-y 2022 contraction is of -29.1 percent.

5.3 Bayesian Blankets and PLS

Bayesian networks, also known as directed acyclic graphical models, are a type of probabilistic graphical model that represent the conditional dependencies among a set of variables (Pearl (1986)). These models use a directed acyclic graph (DAG) where nodes represent variables and edges symbolize direct dependencies between the variables. The absence of an edge represents a specific conditional independence assertion in the joint distribution. Figure 2 is one such example.

In a Bayesian network graph, directed edges typically have associated probabilities. For each node, we attach a conditional probability that quantifies the influence of its parents on the node itself. The entire network then defines a unique joint probability distribution over the variables. Bayesian networks are employed in a multitude of applications such as prediction, anomaly detection, diagnostics, automated insight, reasoning under uncertainty, and providing compact representations of joint probability distributions in large datasets.

For a given node in a Bayesian network, the Markov Blanket consists of its parents, its children, and any other nodes that share a child with the node (Pearl (1988)). This set of nodes is essential because it shields the node from the rest of the network. Specifically, given the variables in its Markov Blanket, a node is conditionally independent of all other nodes in the network. When it comes to variable selection in a dataset, the Markov Blanket of a particular variable is the minimal subset of variables needed for optimal prediction of that variable. This can significantly simplify complex models by reducing the number of variables considered.

The PC Algorithm (Spirtes et al. (2000, 2012)), is a widely used method to learn the structure of the Bayesian Network and subsequently identify the Markov Blanket of a particular node. This constraint-based algorithm learns the structure of the network by systematically testing conditional independence among subsets of variables. The algorithm begins with a fully connected, undirected graph, and proceeds with two key steps: skeleton identification and v-structure orientation. In the first phase, the algorithm tests conditional independence for every pair of nodes, given a conditioning set. If a pair of nodes is conditionally independent, the edge connecting them is removed. This process is repeated with increasing conditioning set sizes until no further edges can be removed.

In the v-structure orientation phase, the algorithm assigns directions to the edges to form a directed acyclic graph (DAG). It identifies structures where two nodes have a common child but lack a direct edge between them, orienting the edges towards the common child. The result is a learned structure of the Bayesian network. To identify the Markov Blanket of a specific node, one needs to find the parents, children, and nodes sharing a child with that node. An extension of the standard PC, the temporal PC algorithm, which accounts for the time series structure of variables, is presented in [Petersen et al. \(2021\)](#).

The temporal PC Algorithm serves as a tool for discovering relationships among time-ordered variables viewed through the lens of Bayesian networks. The temporal partially directed DAG of the Google Trends and GDP data is presented in [Figure 7](#) below. Directed and undirected edges point towards current and past values of GDP indicating the complex nature of their joint distribution. The directed edge between GDP and its first temporal lag is identified along with a number of temporal direct and indirect effects. For example, as uncovered also by the Genetic Algorithm, changes in searches related to lagged "Inflation" and present "Export" impact present changes in GDP.

What is the PLS algorithm doing differently as compared to PCR? By allowing flexibility in the identification of factors, PLS may be better positioned to capture correlation of features with a target variable and heterogeneous lag structures. Factors are not constrained to load on current and past values of variables in a uniform AR fashion. In a standard DFM, the AR parameters of the state space equation impose a structure on the relationship between factor representations which may not be faithfully present in joint probability distribution of variables. Furthermore, sparsity identifies variables with strong predictive power across considered lag structures without the need to explicitly model their autoregressive feature. In the current study, PLS factors load on current and past values of considered variables differently across the identified components, with parameters linked to the strength of their predictive ability.

6 Conclusions

The current study provides evidence of the ability of Partial Least Squares in conjunction with Genetic Algorithm variable selection algorithms to estimate economic activity during

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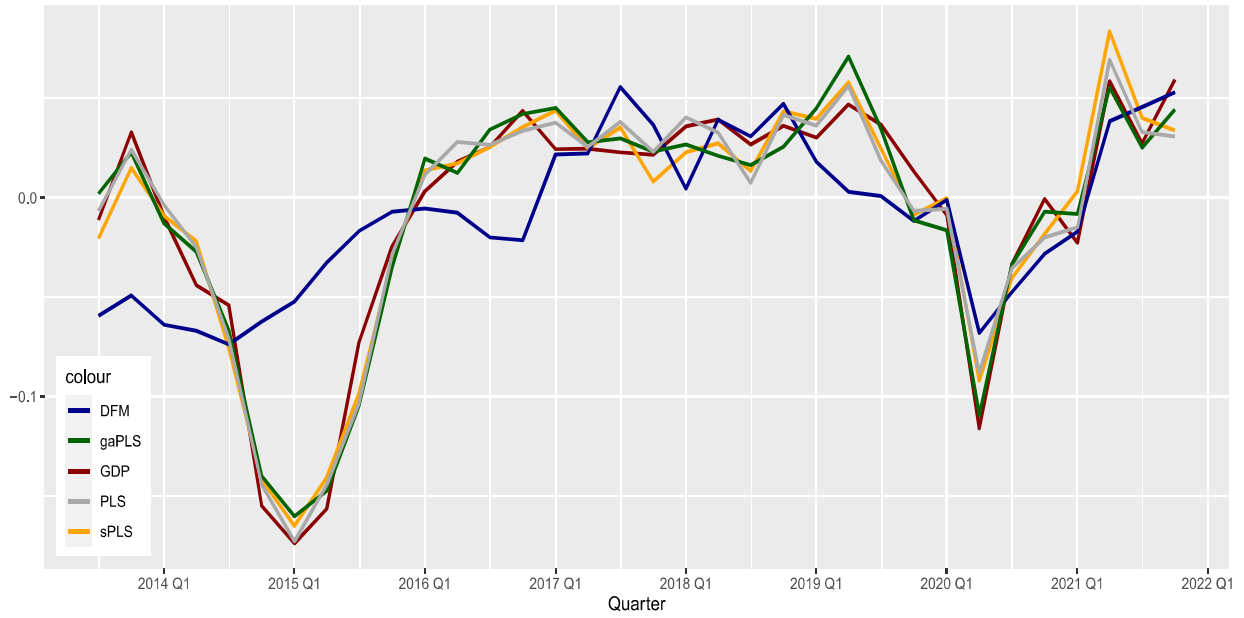
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A Appendix

Figure 8 – Insample fit 2013-2022



R code comparing performance of PLS, sPLS and PCR. Two latent factors F_1 and F_2 drive the first four variables, X_1 to X_4 and the target variable y . Six additional irrelevant variables X_5 to X_{10} , highly correlated, are included in the model. Subsequently, the PLS and PCR regression are estimated using the *spls* and [Mevik and Wehrens \(2007\)](#) *pls* R packages.

```
generate_data <- function(n, seed = NULL) {
  set.seed(seed)

  # True latent factors
  L1 <- rnorm(n)
  L2 <- rnorm(n)

  # Explanatory variables driven by the latent factors
  X1 <- 0.9*L1 + rnorm(n, sd = 0.5)
  X2 <- -0.3*L1 + rnorm(n, sd = 0.8)
  X3 <- 0.5*L2 + rnorm(n, sd = 0.7)
  X4 <- L2 + rnorm(n, sd = 0.7)
  X5 <- rnorm(n) # Independent noise variable
  X6 <- 0.7*X5 + rnorm(n, sd= 0.3)
  X7 <- -0.7*X5 + rnorm(n, sd= 0.3)
  X8 <- 0.5*X5 + rnorm(n, sd= 0.3)
  X9 <- 0.3*X5 + rnorm(n, sd= 0.3)
  X10 <- 0.1*X5 + rnorm(n, sd= 0.3)
  # Response variable
  Y <- 0.8 * L1 - 0.6 * L2 + rnorm(n, sd = 0.5)

  data.frame(L1, L2, X1, X2, X3, X4, X5, X6, X7, X8, X9, X10, Y)
}

SimData <- generate_data(100, 42)
cor(SimData[,c('L1', 'L2', 'X1', 'X2', 'X3', 'X4', 'X5', 'Y')])

plsr_model <- plsr(Y ~ X1 + X2 + X3 + X4 + X5 +
  X6 + X7 + X8 + X9 + X10, data = SimData,
  ncomp = 5, validation = "LOO")
pcr_model <- pcr(Y ~ X1 + X2 + X3 + X4 + X5 +
  X6 + X7 + X8 + X9 + X10, data = SimData, ncomp = 5,
  validation = "LOO")
spls_cv <- cv.spls(x = SimData[,3:(ncol(SimData)-1)], y = SimData[, "Y"],
  eta = seq(0.3, 0.9, 0.001), K = c(1:5))
spls_model <- spls(x = SimData[,3:(ncol(SimData)-1)], y = SimData[, "Y"],
  eta = spls_cv$eta.opt, K = spls_cv$K.opt)
summary(plsr_model)
plot(RMSEP(plsr_model)) # min RMSEP at 2 components
```

```

summary(pcr_model)
plot(RMSEP(pcr_model)) # min RMSEP at 3 components
spls_model

# Plotting True Latent vs. Estimated Latent
par(mfrow = c(2, 1))
plot(SimData[1:70, 'L1'], type = "l", col = "darkgrey")
#title("True Latent vs. Estimated")
lines(plsr_model$scores[1:70, 1], col = "blue")
#lines(pcr_model$scores[1:70, 1], col = "red")
plot(SimData[1:70, 'L2'], type = "l", col = "grey")
lines(plsr_model$scores[1:70, 2], col = "blue")

#
plot(SimData[1:70, 'Y'], type = "l", col = "darkgrey")
lines(plsr_model$fitted.values[1:70], col = "blue")

```

Figure 9 – sPLS Variable Selection

