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Estimating the natural rate of unemployment for Ukraine

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Abstract

In this study, we apply the Kalman filter to estimate the set of reduced-form Phillips curves for different types of inflation in Ukraine. Based on the estimated models, we derive a number of series of non-accelerating inflation rate of unemployment (NAIRU) that provide information about the general trajectory and last tendencies of trend unemployment. To better identify the unemployment trend, we include indicators of long-term unemployment and the Beveridge curve shifts as exogenous variables in the NAIRU equation. Both variables demonstrate a significant impact on NAIRU dynamics. Our estimates show that the Phillips curve slope in Ukraine lies in a standard range of -0.3 to -0.5, with high statistical significance. The median value of estimated NAIRUs was at its lowest at 7.2% at the end of 2008, after which it gradually increased to 9.4% by the end of 2021.

Keywords: *Phillips curve*, *unemployment*, *NAIRU*, *Kalman filter, Beveridge curve.*

JEL: *E24*, *E31*.

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1. Introduction

The supply side of an economy is usually presented as a Phillips curve (PC) equation in macroeconomic models. The Phillips curve assumes that inflation starts to accelerate after unemployment surpasses the equilibrium level (and vice versa). Therefore, equilibrium unemployment is usually called the non-accelerating inflation rate of unemployment (NAIRU).[1] Phillips's original paper argued that high unemployment, meaning the excess supply of the labour force, negatively affects money wage growth.[2] Low unemployment, characterised by excessive labour demand, accelerates the growth of monetary wages (Phillips, 1958). The subsequent development of the Phillips curve was induced by the divergence of its predictions and economic reality. The high unemployment and inflation in the US in the 1970s could not be explained by existing theories. Resolving this puzzle, Phelps (1967) and Friedman (1968) showed that a true negative relationship can be detected between *real* wage inflation and unemployment, which fluctuates around its natural rate. They explained the strong relationship between *nominal* wage growth and unemployment detected in Phillips (1958) by anchored inflation expectations in the gold standard period. Low volatility of inflation reduces the importance of inflation for labour supply and labour demand decisions. After the collapse of the gold standard system, inflation volatility increased dramatically, and this increase in inflation volatility made the role of inflation expectations important for decisions of workers and firms on employment. Under new conditions, firms and workers attempt to adjust nominal wage growth to inflation. If actual inflation is lower than inflation expectations (given the restricted bargaining power of workers, real wages are expected to decrease), unemployment temporarily exceeds its natural level and vice versa. With time, expectations adjust to lower inflation and unemployment decreases to the natural level. This logic led to the introduction of the *expectation-augmented Philips curve*.[3] According to this concept, unemployment deviates from its natural level when expected inflation deviates (accelerates) from the actual inflation. Therefore, the level of unemployment for which inflation expectations are equal to actual inflation is called the NAIRU.

Measuring the unemployment gap, the deviation of actual unemployment from NAIRU, is important for monetary policy as it explains current inflation dynamics and provides insights into future inflation developments. The NAIRU is also helpful in determining the cyclical position of an economy and understanding the structural transformations of the labour market.

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[1] "Natural rate of unemployment" is often used interchangeably with NAIRU.

[2] The evolution of the Phillips curve concept can be found in Farmer (2013).

[3] Expectations-augmented Philips curve underlines the importance of the monetary policy, which has to anchor inflation expectations. It facilitates the mechanism of reverting unemployment to its NAIRU level and reduces the amplitude of deviations from it.

The NAIRU is not constant over time, which makes its estimation more challenging. Time variation requires an economically sound model to explain movements in the NAIRU and their impact on the rest of the economy. According to Stiglitz (1997), the main forces shifting the NAIRU are changes in demography, asymmetry in workers' expectations of productivity growth and actual productivity, changes in the competitiveness of the labour and product markets, and hysteresis. Changes in the demographic structure of the population affect the NAIRU because each population group has its own natural rate of unemployment. Workers' expectations of productivity growth require real wages, which can be in disequilibrium with the actual perspectives of productivity. Equilibrium is achieved by shifts in the NAIRU. The rise in the competitiveness of the labour and product markets decreases the NAIRU while having an ambiguous effect on real wages. Hysteresis effects preserve the NAIRU at a high level because long periods of unemployment lead to a loss of skills and the desire of the unemployed to search for a new job.

There are four traditional approaches for estimating the NAIRU. The first two approaches involve structural and semi-structural models of aggregate wage and price-setting behaviour, which are then used to derive the NAIRU from the estimated system of equations under the assumption that markets are in equilibrium (Benes et al., 2010; Benes and N'Diaye, 2004; Alichi et al., 2018). The third approach is based on pure statistical methods, such as the Hodrick–Prescott filter. These atheoretical statistical filters split the actual unemployment rate into a cyclical component and a trend component, the NAIRU. The fourth approach uses reduced-form estimates of the NAIRU based on a behavioural equation, the expectation-augmented Phillips curve, which determines inflation. This approach applies statistical techniques to identify the constraints on the path of the estimated NAIRU (Laubach, 2001; Rusticelli, 2014; Turner et al., 2001).

In this study, we follow the latter approach and apply the Kalman filter technique to estimate the reduced-form Phillips curve for Ukraine. The main advantage of this approach over full-scale structural models is its relative simplicity. The Kalman filter technique also outperforms pure statistical filters in ability to highlight the economic variables that affect inflation and illustrate the links between inflation and unemployment in the Ukrainian economy.

The remainder of this paper is organised as follows. Section 2 presents a review of the literature on the NAIRU estimates. Section 3 presents the study's methodology and data characteristics. Section 4 reports the main results of our estimations for Ukraine. The conclusions are presented in Section 5.

2. Literature review

Laubach (2001) presented the pioneering study with reduced-form estimations of the NAIRU. The author tested several specifications of state-space models to obtain estimates of the NAIRU for Australia and G7 countries, except Japan, over a long period. Laubach shows that a Phillips curve-type regression can provide highly imprecise results in modelling the natural unemployment rate. However, using bivariate specification with information about the behaviour of unemployment, in addition to inflation, significantly improves the estimations.

There is a set of studies in which the standard OECD approach to NAIRU estimation was applied and expanded. For instance, Turner et al. (2001) applied this standard methodology to estimate Phillips curves for OECD countries. Using a similar methodology, Guichard and Rusticelli (2011) estimate how the Great Recession affected the NAIRU. They conclude that the increase in unemployment observed after the crisis reversed the reduction in structural unemployment, which has been estimated to have occurred in most OECD countries since the late 1990s. In this study, we describe the OECD approach and use it as a baseline Phillips curve modification.

Reduced-form estimations of the Phillips curve using the Kalman filter are popular in country-level studies. With small modifications in the specification of supply determinants of inflation, such models have been applied to data from the UK (Greenslade et al., 2003), Latvia (Meļihovs and Zasova, 2009), Poland (Kierzenkowski et al., 2008), and New Zealand (Jacob and Wong, 2018). Phillips curve specifications often differ in the formulation of the unemployment gap, which can be expressed in a nonlinear form. Nonlinearity is usually introduced by the ratio of the unemployment gap to the unemployment rate, instead of the simple difference between unemployment and its natural rate. In these studies, expectations of some form are also accounted for. The standard approach is to include adaptive expectations (lags of inflation), expected inflation from surveys, or expectations derived from the term structure of the bond rates in the model. Another popular way to improve estimates is to include exogenous observed variables[4] in the equation for the NAIRU. Sometimes, it helps improve the model's fit to the data and determine the unobserved components of the model.

Rusticelli et al. (2014) note that inflation has become less sensitive to movements in unemployment in recent decades, and a large share of the literature on the Phillips curve focuses on curve flattening. A flatter Phillips curve can be explained by the fact that inflation expectations are better anchored because of central banks' inflation targeting. Rusticelli et al. (2014) evaluate this hypothesis by comparing two competing empirical specifications across OECD economies. The first approach is based on a traditional backward-looking Phillips curve, where current inflation is partly explained by an autoregressive distributed lag process of past inflation, representing both inertia and inflation expectations formed on the basis of recent inflation outcomes. The second

[4] For instance, national statistics on long-term unemployment, data on migration, and so on.

approach adjusts the standard one and allows inflation expectations to be anchored around the central bank's inflation objectives. Rusticelli et al. (2014) showed that the latter approach tends to outperform the traditional backward-looking Phillips curve. Changes in productivity and globalization are among the potential factors for Phillips curve flattening. Higher trend productivity, caused by the technological revolution of the last few decades, could reduce the inflationary component of wage-setting decisions. Globalization gives workers more choice for employment abroad and reduces firms' bargaining power.

Rusticelli (2014) notes that estimating NAIRU is particularly difficult when changes in unemployment are both very large and rapid, as in the aftermath of the Great Recession. This study proposes a refinement that strengthens the relationship between inflation and labour market developments by considering the risk of hysteresis effects associated with changes in long-term unemployment. Formally, the author includes a long-term unemployment indicator in the equation for the NAIRU. The revised methodology improves the statistical properties of the reduced-form Phillips curve for a group of countries in the euro area (Greece, Ireland, Italy, Portugal, and Spain).

The reduced-form model provides estimates of time-varying NAIRU. However, it states nothing about the drivers of natural unemployment. To address this, Gianella et al. (2008) suggested a two-stage approach. First, they estimated time-varying NAIRUs for a panel of OECD economies using standard Kalman filter techniques. In the second stage, the estimated NAIRUs are regressed on the selected policy and institutional variables. According to the obtained results, the level of the tax wedge and user cost of capital are found to be important drivers of structural unemployment. The level of product market regulation, union density, and unemployment benefit replacement rate also play an important role in explaining changes in the NAIRU, although there is considerable variation in estimates across countries.

Crump et al. (2019) combined two popular approaches to estimate the natural rate of unemployment. In line with the first approach, they analysed detailed labour market indicators such as labour market flows, cross-sectional data on unemployment and vacancies, and various measures of demographic changes. In the framework of the second approach, they used the aggregate price and wage Phillips curve relationships. To estimate the natural rate of unemployment in the United States were used data on labour market flows and a forward-looking Phillips curve that links inflation to current and expected deviations of unemployment from its unobserved natural rate. The estimates identify a secular downward trend in the unemployment rate, driven solely by the inflow rate. Factors decreasing the inflow rate were identified as the increase in women's labour force attachment, decline in job destruction and reallocation intensity, and dual aging of workers and firms.

Brauer (2007) is another example of the application of alternative labour market indicators to better identify the natural rate of unemployment. In this study, cumulative shifts of the Beveridge curve were used to estimate the natural rate of unemployment in the US. It was shown that Beveridge curve movements are effective for the explanation of natural rate of unemployment dynamics.

In this study, we closely followed the OECD methodology, considering its relative simplicity, well-documented properties, and robustness checked in a series of papers.

3. Methodology and Data

The general fit of the unobserved component model[5], presenting the reduced-form Phillips curve, can be found in Laubach (2001). Two basic specifications model the unobserved variable (the NAIRU) as a random walk or a random walk with a drift process:

$$\pi_t - \pi_t^e = \beta(L)(\pi_{t-1} - \pi_{t-1}^e) + \gamma(L)(u_{t-1} - u_{t-1}^*) + \delta(L)X_{t-1} + \epsilon_t, \qquad (1)$$

$$u_t^* = u_{t-1}^* + \varepsilon_t, \qquad (2, \text{RW})$$

$$u_t^* = u_{t-1}^* + \mu_{t-1} + \varepsilon_t, \qquad (2, \text{RW with drift})$$

$$\mu_t = \mu_{t-1} + \zeta_t, \qquad (3)$$

where π_t and π_t^e denote actual and expected inflation, u_t^* is the NAIRU at time *t*, X_t is a vector of supply side controls (changes in the nominal exchange rate and commodity prices). All disturbance terms $(\epsilon_t, \varepsilon_t, \zeta_t)$ are assumed to be i.i.d. normal with mean zero, respective variances $(\sigma_\epsilon^2, \sigma_\varepsilon^2, \sigma_\zeta^2)$, and uncorrelated with each other.

The OECD approach (Guichard and Rusticelli, 2011) assumes a more generous specification of the Phillips curve with autoregressive terms and more complex supply parts:

$$\Delta\pi_t = \beta(u_t - u_t^*) + \sum_{j=1}^{A} \alpha_j \Delta\pi_{t-j} + \sum_{j=1}^{L} \mu_j * MGS_{t-j}\left(\pi_{t-j}^{imp} - \pi_{t-j}\right) + \cdots$$

$$+ \textstyle\sum_{j=1}^{G} \gamma_j * OIL_{t-j}(\pi_{t-j}^{oil} - \pi_{t-j}) + \epsilon_t, \qquad (4)$$

$$u_t = u_t^* + u_t^{gap}, \qquad (5)$$

$$u_t^{gap} = \varphi_1 u_{t-1}^{gap} + \varphi_2 u_{t-2}^{gap} + \delta_t, \qquad (6)$$

$$u_t^* = u_{t-1}^* + \varepsilon_t. \qquad (7)$$

Supply side shocks driving inflation are identified by the changes in real import price inflation $(\pi_{t-j}^{imp} - \pi_{t-j})$ weighted by import penetration[6] and the changes in real oil

[5] The logic of the Kalman filter, which is applied to estimate the unobserved component model, is presented in Annex C.
[6] *MGS* – import content of consumption (Import/(GDP + Import - Export)), *OIL* – oil intensity of production (oil supply/domestic output).

price inflation $(\pi_{t-j}^{oil} - \pi_{t-j})$ weighted by the oil intensity of production. The unobserved components, NAIRU, and unemployment gap, are modelled as random walk and AR(2) processes, respectively.

Given the data availability, we start with a somewhat simplified version of the OECD specification:

$$\Delta\pi_t^c = \beta(u_t - u_t^*) + \sum_{j=1}^{A=4} \alpha_j \Delta\pi_{t-j}^c + \sum_{j=1}^{L=4} \mu_j \Delta\pi_{t-j}^{neer} + \sum_{j=1}^{G=4} \gamma_j \Delta\pi_{t-j}^{oil} + \cdots$$

$$+ \textstyle\sum_{w=1} \theta_w D_w + \epsilon_t, \tag{8}$$

$$u_t = u_t^* + u_t^{gap}, \tag{9}$$

$$u_t^{gap} = \varphi_1 u_{t-1}^{gap} + \delta_t, \tag{10}$$

$$u_t^* = u_{t-1}^* + \varepsilon_t, \tag{11}$$

where π_t^c is the core inflation, π_t^{neer} denotes the nominal effective exchange rate (NEER) growth[7], and π_t^{oil} is the oil price index growth[8], D_w is a dummy variable, taking the value of one in quarters with extremely large swings of inflation[9]. In the model, changes in NEER control for the effects of exchange rate movements, whereas the oil price index incorporates shocks to commodity markets. Considering that Ukraine is a small, open economy with a large share of energy commodities in its imports, both of these factors are important for price development. The inclusion of dummies is a standard way to control for outliers in inflation related to abrupt changes in taxation, price controls, or exogenous shocks (for instance Gianella et al. (2008)). All variables in equation (8), except unemployment and dummies, are included in the model with up to four lags. To achieve parsimony of the model, we apply a backward elimination procedure to leave the lags that are significant at the 10% level or close to it.

Equations (8)–(11) present a simple starting specification, which we extend in multiple ways. Considering the variety of specifications for the reduced-form Phillips curve in the literature, we estimate the number of alternatives.

Choice of inflation measure

According to the Phillips curve concept, the dependent variable could be either headline inflation or wage inflation, adjusted for productivity growth. Headline inflation is a natural choice, as agents usually form their expectations considering the changes in consumer prices. Central banks usually target headline inflation, which is

[7] In all cases we use y-o-y growth.
[8] Data from the Primary Commodity Price System of IMF. Crude Oil (petroleum), Price index, 2016 = 100, simple average of three spot prices: Dated Brent, West Texas Intermediate, and the Dubai Fateh.
[9] Inflation "jumps" lead to the skewness of error term distribution. We add a minimal number of dummies just to achieve the normality of errors in the signal equation. In some specifications, dummies are not added at all.

also an argument for its use. However, the CPI data in Ukraine are very noisy. Estimates based on this approach are not robust and theoretically consistent. Therefore, we used core inflation for the estimates.

We also closely follow Ruberl et al. (2021) in estimating several specifications of the Phillips curve for wage inflation in Ukraine:

$$(\Delta w_t - \Delta z_t - \pi_{t-1}) = \beta_1 \left(\frac{u_t - u_t^*}{u_t}\right) + \beta_2 \left(\frac{\Delta u_{t-1}}{u_t}\right) + \sum_{j=1}^{A=4} \alpha_j (\pi_t - \pi_{t-1})_{t-j} + \cdots$$

$$+ \sum_{j=1}^{L=4} \mu_j \Delta w_{t-j} + \sum_{j=1}^{G=4} \gamma_j (\Delta z_{t-j}^* - \Delta z_{t-j}) + \sum_{w=1} \theta_w D_w + \epsilon_t, \tag{12}$$

$$u_t^* = u_{t-1}^* + \varepsilon_t, \tag{13}$$

where Δw_t is nominal wage growth, Δz_t is productivity growth, π_{t-1} is headline inflation, and Δz_t^* is growth in trend productivity. Nominal wage growth was estimated based on the statistics of average nominal wages of employed persons provided by the State Statistical Service of Ukraine (SSSU). Productivity is the ratio of seasonally adjusted real GDP to the seasonally adjusted number of employed persons[10]. Trend productivity was derived using the Hodrick–Prescott filter[11]. The inclusion of lags of productivity trend growth and growth of productivity itself reflects the assumption that productivity changes are gradually incorporated in wages over time.

Supply-side controls

For supply side identification in signal equation (8), we try to add changes in real import price inflation (import deflator minus CPI) weighted by import penetration and changes in real oil price inflation (oil price index changes minus CPI) weighted by oil intensity of production[12]. This specification mimics an OECD-type model. As other proxies of imported inflation, we also use an import deflator (weighted/unweighted by import penetration) and the weighted average of the main trading partners' CPIs (weighted/unweighted by import penetration).

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[10] Time series for hours worked, which are usually used for productivity estimates, are too short for Ukraine. This is why we apply a simplified approach to the measurement of productivity.

[11] In line with Ruberl et al. (2021), we constructed a state-space model to extract the trend from the time series of productivity. In the model, cyclical productivity is specified as an AR(2) process, and trend productivity is modeled as a random walk with time-varying drift. The resulting estimates of the Phillips curve for wage inflation are not markedly different from what we got using the Hodrick–Prescott filtering. However, there can be an issue with calibrating the state-space model for Ukraine. Ruberl et al. (2021) have a benchmark for the estimates of trend productivity taken from the Australian Treasury's macro-econometric model of the Australian economy. With respect to this benchmark, they calibrate the state-space model, which provides them with longer time series of productivity trend than the Treasury's model does. For Ukraine, such a convenient benchmark is absent. That is the reason why we stopped on the Hodrick–Prescott filter.

[12] Oil intensity of production was estimated as a ratio to GDP of "overall supply of primary energy" for oil, oil products and natural gas from the Energy Balance of Ukraine (State Statistical Service of Ukraine, SSSU). The data were interpolated to quarterly frequency afterwards.

Nonlinearities

We tested two types of specifications to control for possible nonlinearities. The first specification includes a proxy for speed-limit effects ($\frac{\Delta u_{t-1}}{u_t}$), which allows the rate of change in unemployment to have different impacts on inflationary pressure. The second type of nonlinearity is in the unemployment gap equation (9), which is redefined as ($\frac{(u_t - u_t^*)}{u_t}$). This means that a 1% gap when the NAIRU equals 2% has a different impact on inflation than a 1% gap when the NAIRU equals 10%. Such specification of the NAIRU is a reasonable choice for economies with high volatility of unemployment and for those in which unemployment had risen considerably over the past periods.

Lag structure of the state variables

As an alternative, we modify the lag structure of signal equation (8) by substituting the first and second lags of the unemployment gap into it ($(u_{t-1} - u_{t-1}^*), (u_{t-2} - u_{t-2}^*)$). The lag structure of state equation (10) was also verified to determine the most appropriate fit for the model. Specifically, we examine the AR(2) structure of the unemployment gap against AR(1) in the baseline variant. Finally, we test several alternative NAIRU specifications in the following form:

$$u_t^* = \alpha * u_{t-1}^* + \varepsilon_t, \tag{11.1}$$

$$u_t^* = \theta + u_{t-1}^* + \varepsilon_t, \tag{11.2}$$

$$u_t^* = \theta + \alpha * u_{t-1}^* + \varepsilon_t. \tag{11.3}$$

Exogenous variables in the NAIRU equation

Following Rusticelli (2014), we conduct experiments including exogenous variables affecting unemployment in the long run in the NAIRU equation. Specification (11.4) includes a proxy variable for long-term unemployment. We assume that the inclusion of this information can help determine the hysteresis effects and improve the fit of the model.

$$u_t^* = u_{t-1}^* + \omega * \Delta ltu^{13}{}_{t-(0,1,2)} + \varepsilon_t. \tag{11.4}$$

As an exogenous determinant of the NAIRU, we also try the cumulative shift in the Beveridge curve of the Ukrainian labour market (11.5). Brauer (2007) shows that this indicator could have a statistically significant effect on the estimates of natural unemployment. The non-cyclical shifts of the Beveridge curve indicate the average job-matching efficiency of the labour market. Outwards movements of the curve

[13] To get ltu_t we used annual data of SSSU on the percentage of the unemployed population aged 15-70 years searching for work for more than 12 months. The data were interpolated to the quarterly frequency. After that, we extracted the trend by the Hodrick–Prescott filter and used its first difference in (11.4). Smoothing by Hodrick–Prescott filter was made to provide close frequency domains of NAIRU and ltu_t.

indicate a reduction in labour market efficiency and the accumulation of disproportions. More information on this variable is given in Annex B.

$$u_t^* = u_{t-1}^* + \omega * \Delta bev^{14}{}_{t-(0,1,2)} + \varepsilon_t. \quad (11.5)$$

Calibrating signal-to-noise ratio and initial values

The application of the Kalman filter provides an opportunity to estimate all the parameters of the model. However, the strategy of the free estimation of parameters usually yields counterintuitive results. The common practice is to impose restrictions on the signal-to-noise ratio, which determines the relative volatility of the trend (NAIRU) to the gap. There is no strict guideline on how the signal-to-noise parameter should be calibrated; nevertheless, general recommendations suggest a gradual selection of this parameter to obtain smooth NAIRU dynamics without jumps or drops. We follow this logic and calibrate the signal-to-noise parameter to achieve both smoothness of the NAIRU and a statistically significant Phillips curve slope with the correct sign.

The initial values of the unobserved components (NAIRU and unemployment gap) are usually also calibrated to provide more information to the Kalman filter algorithm and help it converge. We set the average unemployment for the first three years of a sample as the starting value for the NAIRU (and the respective gap). The variance-covariance matrix is calibrated to provide an opportunity for NAIRU to deviate within a reasonably wide range from its starting values (+/-2%).

Inflation expectations

By using the first difference in inflation as a depending signal variable in the unobserved component model, we implicitly assume adaptive (backward-looking) inflation expectations[15]. However, the reliability of this assumption is questionable. Coibion et al. (2019) argue that the backward-looking Phillips curve is often unsuccessful in linking inflation and economic tightness. The logic of the New Keynesian Phillips curve requires the inclusion of forward-looking expectations, which are usually taken directly from surveys or econometrically estimated from a set of inflation surveys and forecasts (Ruberl et al., 2021). Estimates with the inclusion of survey data are problematic, as such time series are not very long, and their results can be biased because of poor representativeness. Another problem is that surveys are usually conducted for households and professional forecasters, while for modelling goals, firms' inflation expectations are more desirable as they are price-setters. The horizon of expectations is questionable. In general, surveys contain information on

[14] To estimate the cumulative Beveridge curve shifts we followed Valletta (2005) with some simplifications, described in Annex B. As in the case of ltu_t, the estimated time series was interpolated to quarterly frequency and smoothed by the Hodrick–Prescott filter.

[15] $\Delta\pi_t^c = \pi_t^c - \pi_{t-1}^c$ in (8) instead of $\pi_t - \pi_t^e$ in (1).

short-run expectations with horizons of up to 1 year. However, the horizon for expected inflation used in the decision-making process may be much longer.

For Ukraine, data from the surveys are limited. The longest time series is a 1-year-ahead inflation expectations of businesses from surveys conducted by the National Bank of Ukraine (NBU). These surveys have been conducted by the NBU on a quarterly basis since 2006. Participants were non-financial sector enterprises representing the economy in terms of main economic activities, enterprise size, and number of employees[16]. We used these surveys to construct the expectation-augmented Phillips curve in the manner suggested by Laubach (2001) and Ruberl et al. (2021) [17].

4. Results

Core inflation

Considering a large number of tested specifications, we report the variants of Phillips curve estimates that gave a statistically significant coefficient for the unemployment gap and smooth dynamics of NAIRU. In Annex A, we group the reported models based on signal variables.

The coefficients of the unemployment gap in the core inflation model (Table A. 1) have the correct signs and are of reasonable magnitude for the linear specifications of NAIRU. The nonlinear variants of the Phillips curve indicate extremely low slope magnitudes. In the estimations, we multiply the nonlinear unemployment gap ($\frac{(u_t - u_t^*)}{u_t}$) by 100 to be in the percentage point dimension. This transformation increases the volatility of the unemployment gap in the model relative to inflation variations, thus reducing the coefficient.

Plots of the NAIRUs specified in Table A.1 (Figure A.1) demonstrate that for the Ukrainian economy with high statistical significance, we can identify episodes of overheating in the period 2005–2008 and at the end of 2013. The periods when the unemployment rate was significantly higher than its natural level were 2001, 2014, and 2020–2021. A common feature of the derived NAIRUs is a gradual decrease in the 2000s and an increase after 2014.

[16] Amount of enterprises: near 700 (excluding temporarily occupied Crimea, Donetsk, and Luhansk regions). In 2014 sample size was shortened due to the occupation (from 1300 firms in 2006).
[17] General fit of the signal equations is presented in (1) and (12). We use up to four lags of $(\pi_t - \pi_t^e)$ and a standard set of supply-side variables described earlier.

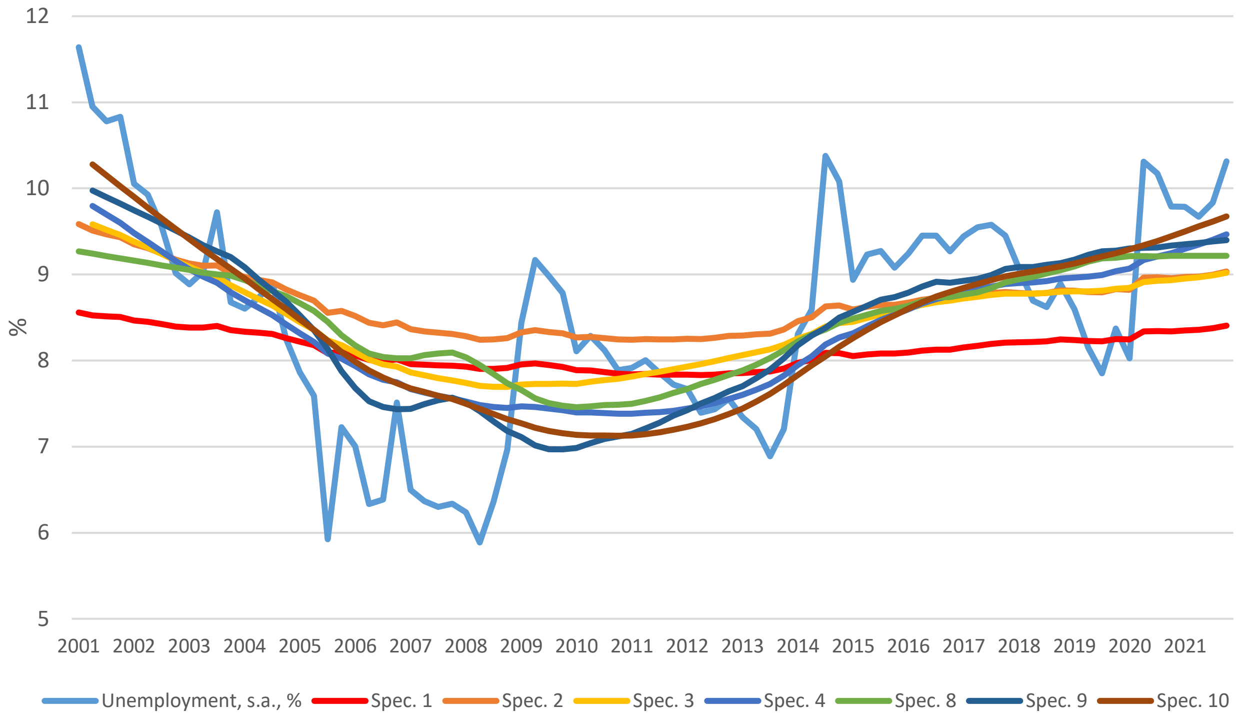
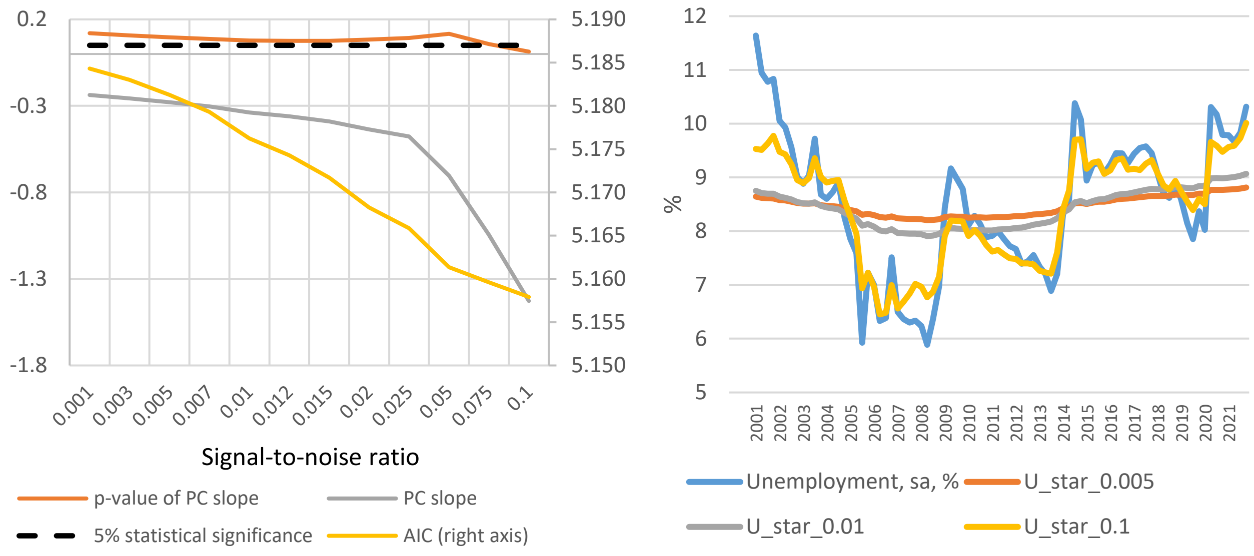


Figure 1. NAIRU estimates from the core inflation Phillips curve

A visual comparison (Figure 1) indicates a marked difference in volatility between the NAIRUs. This is partially explained by the calibration of the signal-to-noise ratio for each specific model. This parameter determines the volatility of unobserved components relative to observed components. Our estimates are low compared to those of analogous studies. We calibrate this ratio to obtain a statistically significant Phillips curve slope with the correct sign, a NAIRU estimate with appropriate smoothness, and the model fitting the data well. Figure 2(a) shows the evolution of the PC slope, its statistical significance, and the Akaike information criterion (AIC) of the model when we change the signal-to-noise ratio from 0.001 to 0.1.[18] An increase in the signal-to-noise ratio, meaning higher volatility of the NAIRU, makes the Philips curve slope steeper, while the statistical significance gradually moves from 12% to 1.3%. AIC improved only marginally. Together with improvements in the statistical significance of the PC slope and its abnormal level, the volatility of the NAIRU also increases, making its dynamics much less reasonable (see Figure 2(b)). Considering this, we chose a compromise calibration of the signal-to-noise ratio, in the range of 0.005–0.01.

[18] This exercise was conducted for Specification 2, which can be considered as a baseline.



(a) **(b)**

Figure 2. The evolution of PC parameters with the signal-to-noise changes*

* PC slope denotes the coefficient of the unemployment gap in the Phillips curve equation, while its statistical significance is presented by p-value. U_star_0.005 denotes the NAIRU derived from the model were signal-to-noise ratio was set at the level of 0.005.

The estimates presented in Table A.1 show that among the different alternatives, imported inflation is well approximated by the product of the import deflator and import penetration ($\Delta\pi_t^{imp}$). Specifications with inclusion of the import deflator times import penetration have a high statistical significance. The nonlinear specification of the NAIRU demonstrates statistical significance as well as its linear presentation. However, the confidence intervals for the nonlinear NAIRU (specifications 8–10) are much wider than those for linear variants (specifications 1–4). Exogenous variables introduced in the NAIRU equations have the correct signs[19] and are statistically significant in both linear (specifications 3–4) and nonlinear specifications (9–10). The inclusion of these variables reduces the confidence intervals, which are particularly visible for nonlinear specifications. The trend growth of the Beveridge curve shifts (Δbev_t) affects the NAIRU more than long-term unemployment (Δltu_t).

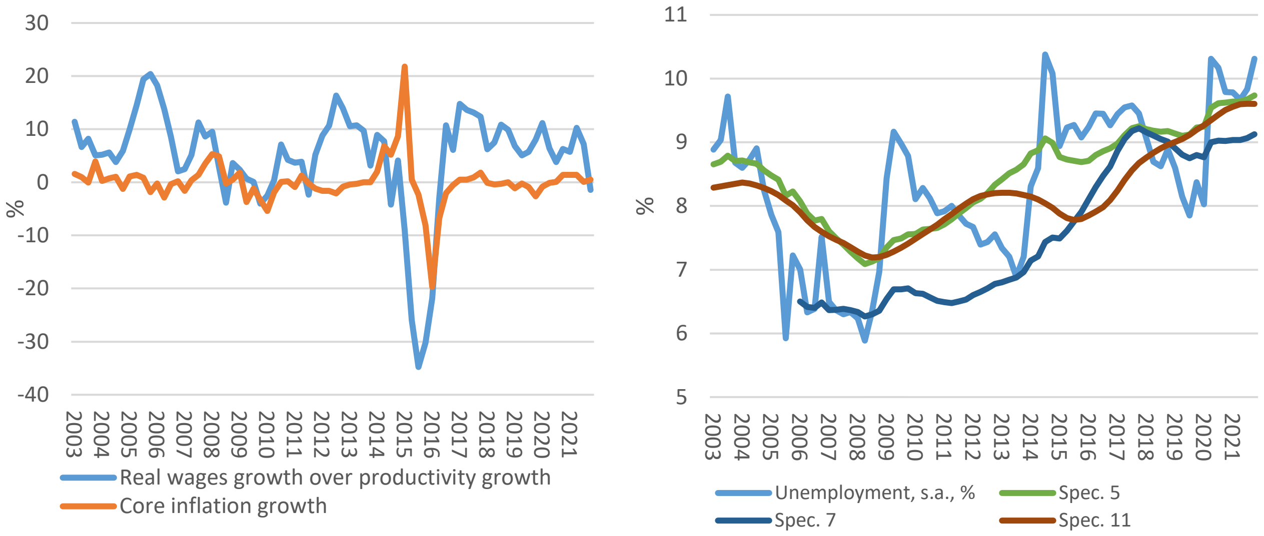
Wage growth

The estimates of the Phillips curve for wage growth are presented in Table A.2. The coefficients of the unemployment gap are significant and with the correct sign, but their magnitudes for linear specifications (5,7) are large compared to the reference values for the Phillips curve slope. This can be explained by the high volatility of real wage growth over productivity growth ($\Delta w_t - \Delta z_t - \pi_{t-1}$) relative to the volatility of core inflation growth ($\Delta\pi_t^c$)[20] (Figure 3). In Ruberl et al. (2021), where similar models were

[19] A positive coefficient of Δltu_t means that when long-term unemployment increases, the NAIRU also goes up. This effect catches the impact of hysteresis. A positive coefficient of Δbev_t means that when the Beveridge curve moves inward (indicating improvement of the job-matching efficiency of the market), the NAIRU goes down.

[20] Standard deviations of ($\Delta w_t - \Delta z_t - \pi_{t-1}$) and ($\Delta\pi_t^c$) for 2003–2021 are 9.7 and 4.2 respectively.

applied to Australian data, the resulting coefficient fluctuated from (-1) to (-5) for different specifications.



(a) **(b)**

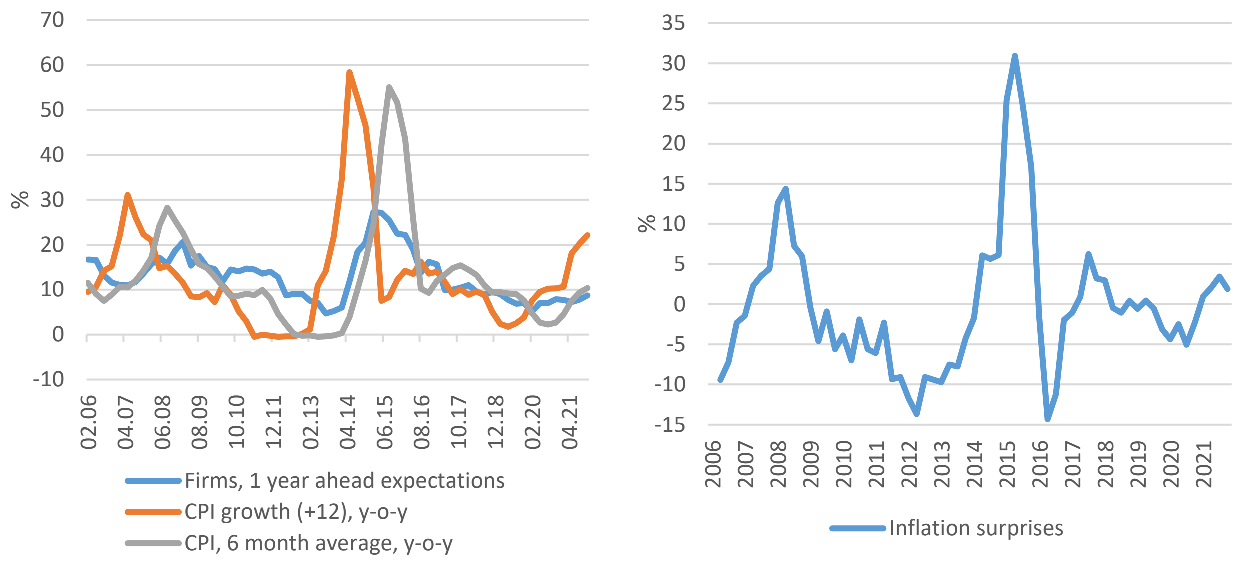
Figure 3. Phillips curve for wage growth: dependent variable and extracted NAIRUs

The NAIRUs derived were volatile. However, they indicate a negative correlation between unemployment gap and real wage growth (see Figure A.3 as an example). In periods of high growth in wages (2005–2007, 2012–2013, 2018–2019), there was a large negative unemployment gap in Ukraine, provided by the respective movements of the NAIRU. Drops in wage growth have led to the opening of positive unemployment gaps. The best example is the crisis of 2014–2015, when the NAIRU temporarily went down to increase the gap. In the model with nonlinearity (specification 11), the NAIRU reacts much more to the swings of wage growth (Figure 3(b)), indicating that such a specification can be inappropriate in the case of high volatility of the dependent variable. As for 2020–2021 wages growth moderately declined while unemployment markedly increased, causing a slight upward shift of NAIRUs while preserving a positive unemployment gap.

The NAIRU from specification 7, containing the 1-year-ahead inflation expectations of firms, behaves like an outlier. This can be explained by the shorter sample for this model, which means that it uses other initial values for unobserved components. Another possible reason is the large divergence between inflation and expectations that occurs during periods of economic turbulence.

Expectations-augmented Phillips curve

We use firm surveys of inflation expectations from a 1-year perspective. The time series of expectations from these surveys covers the longest period (2006–2021) available for Ukraine. Figure 4. demonstrates some characteristics of expected inflation and plots “inflation surprises” that happened in the Ukrainian economy. Firms are poor forecasters of inflation, as actual inflation after 12 months of the survey is weakly correlated with expectations (see Figure 4 (a)). The adaptive (backward-looking) component of expectations was rather large. The correlation between the average inflation for six previous months (including the month of conducting the survey) and expectations was 0.8. There were extremely large differences between actual inflation and expectations, which makes the dependent variable of the expectation-augmented Phillips curve extremely volatile (Figure 4 (b)).



(a) **(b)**

Figure 4. Inflation expectations of firms and actual inflation

In the surveys, firms were asked about headline inflation. This is the reason why we use CPI inflation for the estimates of Table A.3. The NAIRUs derived from the expectation-augmented Phillips curves were close to each other (Figure 5). Somewhat more volatile is natural unemployment from specification 14, which is explained by fluctuations in the Beveridge curve shifts included in the state equation. Both long-term unemployment and the Beveridge curve shifts have significant coefficients with correct signs.

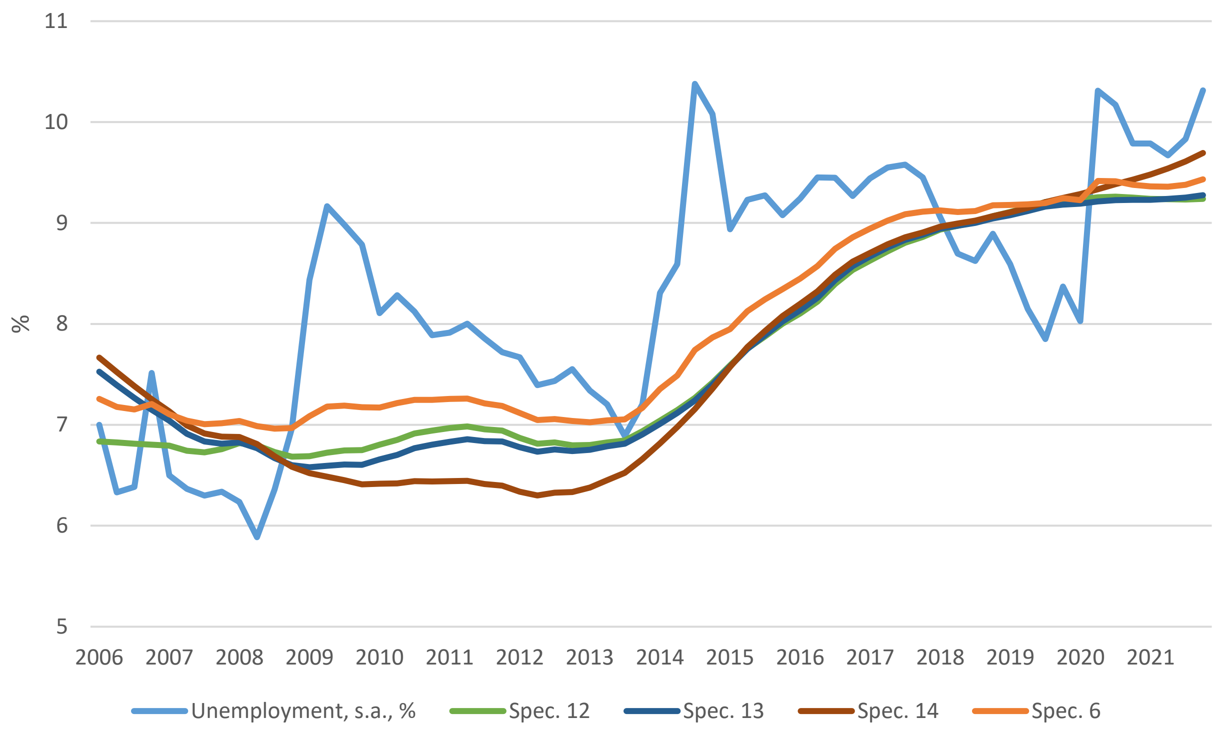


Figure 5. NAIRU estimates from the expectations-augmented Phillips curve

In general, the derived NAIRUs show what is predicted by the theory: higher than expected inflation goes together with a negative unemployment gap (unemployment is too low), and vice versa. An important exclusion from this narrative is the crisis period of 2014–2015 when both inflation and unemployment were high. These years were characterised by numerous large supply side shocks related to political crisis and war in Ukraine.

5. Conclusions

The estimates of reduced-form models of the Phillips curve for different types of inflation gave us a set of time series for the NAIRU and an understanding of the relationship between the unemployment gap and inflation in Ukraine. The meaning of the Phillips curve slope depends on the specifications of the model, particularly in terms of its linearity. In general, with an appropriate calibration of the signal-to-noise ratio, the slope is statistically significant. For core inflation models with the linear specification of the unemployment gap, the values are in the range of (-0.3)–(-0.5), which is the standard result for similar estimates.

The indicators of long-term unemployment and the Beveridge curve shifts are statistically significant in explaining NAIRU changes. The proxy for long-term unemployment captures the hysteresis effects that drive the unemployment trend. Beveridge curve shifts reflect changes in the labour market's ability to match job seekers and job suppliers. Long-term moves in job-matching efficiency also affect NAIRU in our models.

On average, NAIRUs derived from estimated models were at the lowest level in 2008 Q4 (7.2%), after which the median level gradually increased to 9.4% as of 2021 Q4 (Figure A.5). A number of factors can explain NAIRU growth: the growth of the minimal-to-average wage ratio; hysteresis after the crises in 2008–2009, 2014–2015, and the COVID-19 pandemic; widening of opportunities for labour migration; and aging. The positive unemployment gap (0.3% in 2020 and 0.6% in 2021 as median estimates) opened in 2020 and created disinflation pressure on Ukraine's economy.

References

Benes, J., N'Diaye P. (2004). A Multivariate Filter for Measuring Potential Output and the NAIRU: Application to The Czech Republic IMF Working Papers, 04/45.

Benes, J., Clinton, K., Garcia-Saltos, R., Johnson, M., Laxton, D., Manchev, P., Matheson, T., (2010). Estimating Potential Output with a Multivariate Filter. IMF Working Papers, 10/285.

Blake, A., Mumtaz, H. (2012) Applied Bayesian Econometrics for Central Bankers. CCBS Technical Handbook No. 4. Bank of England.

Brauer, D. (2007). The Natural Rate of Unemployment. Congressional Budget Office Working Papers, 18566.

Coibion, O., Gorodnichenko, Y., Ulate, M. (2019). Is Inflation Just Around the Corner? The Phillips Curve and Global Inflationary Pressures. AEA Papers and Proceedings, vol. 109, 465–469.

Crump, R., Eusepi, S., Giannoni, M., Şahin, A. (2019). A Unified Approach to Measuring u*. Brookings Papers on Economic Activity, BPEA Conference Drafts, March 7–8.

Farmer, R. (2013). The Natural Rate Hypothesis: An idea Past Its Sell By Date. Bank of England Quarterly Bulletin, 53(3), 244–256.

Friedman, M. (1968). The Role of Monetary Policy. The American Economic Review, 58(1), 1–17.

Gianella, C., Koske, I., Rusticelli, E., Chatal O. (2008). What Drives the NAIRU? Evidence from a Panel of OECD Countries. OECD Economics Department Working Papers, 649.

Greenslade, J., Pierse, R., Saleheen, J. (2003). A Kalman Filter Approach to Estimating the UK NAIRU. Bank of England Working Paper, 179.

Guichard, S., Rusticelli, E. (2011). Reassessing the NAIRUs after the Crisis. OECD Economics Department Working Papers, 918.

Jacob, P., Wong, M. (2018). Estimating the NAIRU and the Natural Rate of Unemployment for New Zealand. Reserve Bank of New Zealand Analytical Note Series, AN2018/04.

Kalman, R. (1960). A New Approach to Linear Filtering and Prediction Problems. Journal of Basic Engineering, Series D(82), 35–45.

Kalman, R. (1963). New Methods in Wiener Filtering Theory, in Bogdanoff, J. L. and Kozin, F. (eds). Proceedings of the first symposium on engineering applications of random function theory and probability. New York, Wiley, 270–388.

Kalman, R. and Bucy, R. (1961). New Results in Linear Filtering and Prediction Theory. Journal of Basic Engineering, Series D(83), 95–108.

Kierzenkowski, R., Ollivaudi, P., Sédilloti, F., Briardi P. (2008). Estimating a Supply Block for Poland. OECD Economics Department Working Papers, 601.

Laubach, T. (2001). Measuring the NAIRU: Evidence From Seven Economies. The Review of Economics and Statistics, 83(2), 218–231.

Meļihovs, A., Zasova A. (2009). Assessment of the Natural Rate of Unemployment and Capacity Utilization in Latvia. Baltic Journal of Economics, 9(2), 25–46.

Phelps, E. (1967). Phillips Curves, Expectations of Inflation and Optimal Employment over Time. Economica, 34(3), 254–281.

Phillips, A. (1958). The Relation Between Unemployment and the Rate of Change of Money Wage Rates in the United Kingdom, 1861–1957. Economica, 25(100), 283–299.

Ruberl, H., Ball, M., Lucas, L., Williamson, T. (2021). Estimating the NAIRU in Australia. Treasury Working Paper, 2021-01.

Rusticelli, E. (2014). Rescuing the Phillips Curve: Making Use of Long-term Unemployment in the Measurement of the NAIRU. OECD Journal: Economics Studies, 2014(1), 109–127.

Rusticelli, E., Turner, D., Cavalleri, M. (2014). Incorporating Anchored Inflation Expectations in the Phillips Curve and in the Derivation of Measures of OECD Equilibrium Unemployment. OECD Economics Department Working Papers, 1231.

Stiglitz, J. (1997). Reflections on the Natural Rate Hypothesis. Journal of Economic Perspectives, 11 (1), 3–10.

Turner, D., Boone, L., Giorno, C., Meacci, M., Rae, D., Richardson, P. (2001). Estimating the Structural Rate of Unemployment for the OECD Countries. OECD Economic Studies, 33, 171–216.

Valletta, R. (2005). Why Has the U.S. Beveridge Curve Shifted Back? New Evidence Using Regional Data. Federal Reserve Bank of San Francisco, Working Paper Series, 2005-25.

Annex A

Table A.1 Parameters of the Phillips curve for core inflation (Sample: 2001Q1–2021Q4)

	Spec. 1	Spec. 2	Spec. 3	Spec. 4	Spec. 8	Spec. 9	Spec. 10
Dependent signal variable	$\boldsymbol{\Delta\pi_t^c}$	$\boldsymbol{\Delta\pi_t^c}$	$\boldsymbol{\Delta\pi_t^c}$	$\boldsymbol{\Delta\pi_t^c}$	$\boldsymbol{\Delta\pi_t^c}$	$\boldsymbol{\Delta\pi_t^c}$	$\boldsymbol{\Delta\pi_t^c}$
u_{t-1}^{gap}	-0.49 0.02*	-0.34 0.09	-0.49 0.03	-0.48 0.05			
$(u_t - u_t^*)/u_t$					-0.02 0.03	-0.04 0.02	-0.04 0.01
$\Delta u_{t-1}/u_t$	5.21 0.11	4.01 0.13	4.43 0.14	4.66 0.11	3.86 0.10	4.49 0.06	4.50 0.06
$\Delta\pi_{t-1}^c$	0.34 0.00	0.20 0.04		0.19 0.04			
$\Delta\pi_{t-2}^c$			0.13 0.04		0.13 0.02	0.12 0.04	0.12 0.03
$\Delta\pi_{t-3}^c$		0.18 0.00		0.18 0.00			
$\Delta\pi_{t-4}^c$	-0.51 0.00	-0.55 0.00	-0.46 0.00	-0.54 0.00	-0.47 0.00	-0.46 0.00	-0.46 0.00
$\Delta\pi_t^{neer}$	-0.06 0.08						
$\Delta\pi_{t-1}^{neer}$	-0.09 0.00	-0.05 0.15		-0.05 0.14			
$\Delta\pi_{t-2}^{neer}$	-0.05 0.13						
$\Delta\pi_{t-3}^{neer}$			0.12 0.00		0.12 0.00	0.12 0.00	0.12 0.00
$\Delta\pi_t^{oil}$			0.01 0.14		0.01 0.16	0.01 0.18	0.01 0.16
$\Delta\pi_{t-2}^{oil}$			0.01 0.14		0.01 0.16	0.01 0.17	0.01 0.15
$\Delta\pi_{t-3}^{oil}$	0.01 0.22	0.01 0.06	0.02 0.04	0.01 0.09	0.02 0.03	0.02 0.03	0.02 0.03
$\Delta\pi_{t-4}^{oil}$	0.01 0.26						
$\Delta\pi_t^{imp}$		0.22 0.00	0.29 0.00	0.22 0.00	0.29 0.00	0.29 0.00	0.29 0.00
$\Delta\pi_{t-1}^{imp}$		0.14 0.02	0.30 0.00	0.15 0.02	0.30 0.00	0.30 0.00	0.30 0.00
$\Delta\pi_{t-2}^{imp}$		0.13 0.00	0.17 0.01	0.14 0.00	0.16 0.00	0.17 0.00	0.17 0.00
$\Delta\pi_{t-3}^{imp}$			0.22 0.00		0.22 0.00	0.23 0.00	0.22 0.00
dum_15_1	19.70 0.90	13.71 0.41	11.98 0.0724	13.92 0.58	11.94 0.11	11.90 11.90	12.07 0.27
dum_17_1	-8.28 0.68	-8.63 0.73	-8.29 0.4109	-8.34 0.79	-8.63 0.33	-8.46 -8.46	-8.44 0.58
dum_14_4	5.84 0.97						
$\sigma_\in$	2.70 0.00	2.07 0.00	1.70 0.00	1.94 0.00	1.63 0.00	1.52 0.00	1.56 0.00
Unobserved states	$u_t^{gap} = \varphi_1 u_{t-1}^{gap} + \delta_t$, $u_t^* = u_{t-1}^* + \varepsilon_t$		$u_t^{gap} = \varphi_1 u_{t-1}^{gap} + \delta_t$, $u_t^* = u_{t-1}^* + \tau_1 \Delta ltu_t \vert \Delta bev_t + \varepsilon_t$		$u_t^* = u_{t-1}^* + \varepsilon_t$	$u_t^* = u_{t-1}^* + \tau_1 \Delta ltu_t \vert \Delta bev_t + \varepsilon_t$	
u_{t-1}^{gap}	0.89 0.00	0.84 0.00	0.83 0.00	0.81 0.00			
Δltu_t			0.05 0.07			0.08 0.08	
Δbev_t				0.83 0.05			1.02 0.01
σ_δ	0.31 0.00	0.30 0.00	0.30 0.00	0.29 0.00			
σ_ε	0.01 0.00	0.02 0.00	0.01 0.00	0.01 0.00	0.08 0.00	0.08 0.00	0.02 0.00
u_t^* final state	8.40 0.00	9.07 0.00	9.03 0.00	9.51 0.00	9.21 0.01	9.42 0.00	9.73 0.00
u_t^{gap} final state	1.67 0.02	1.05 0.14	1.07 0.09	0.69 0.29			
Log-likelihood	-217.62	-202.40	-192.49	-198.49	-122.66	-121.48	-120.26
AIC	5.54	5.18	5.07	5.17	3.82	3.82	3.78
Signal-to-noise ratio	0.01	0.01	0.01	0.01	0.05	0.05	0.01

* - p-value of the parameter

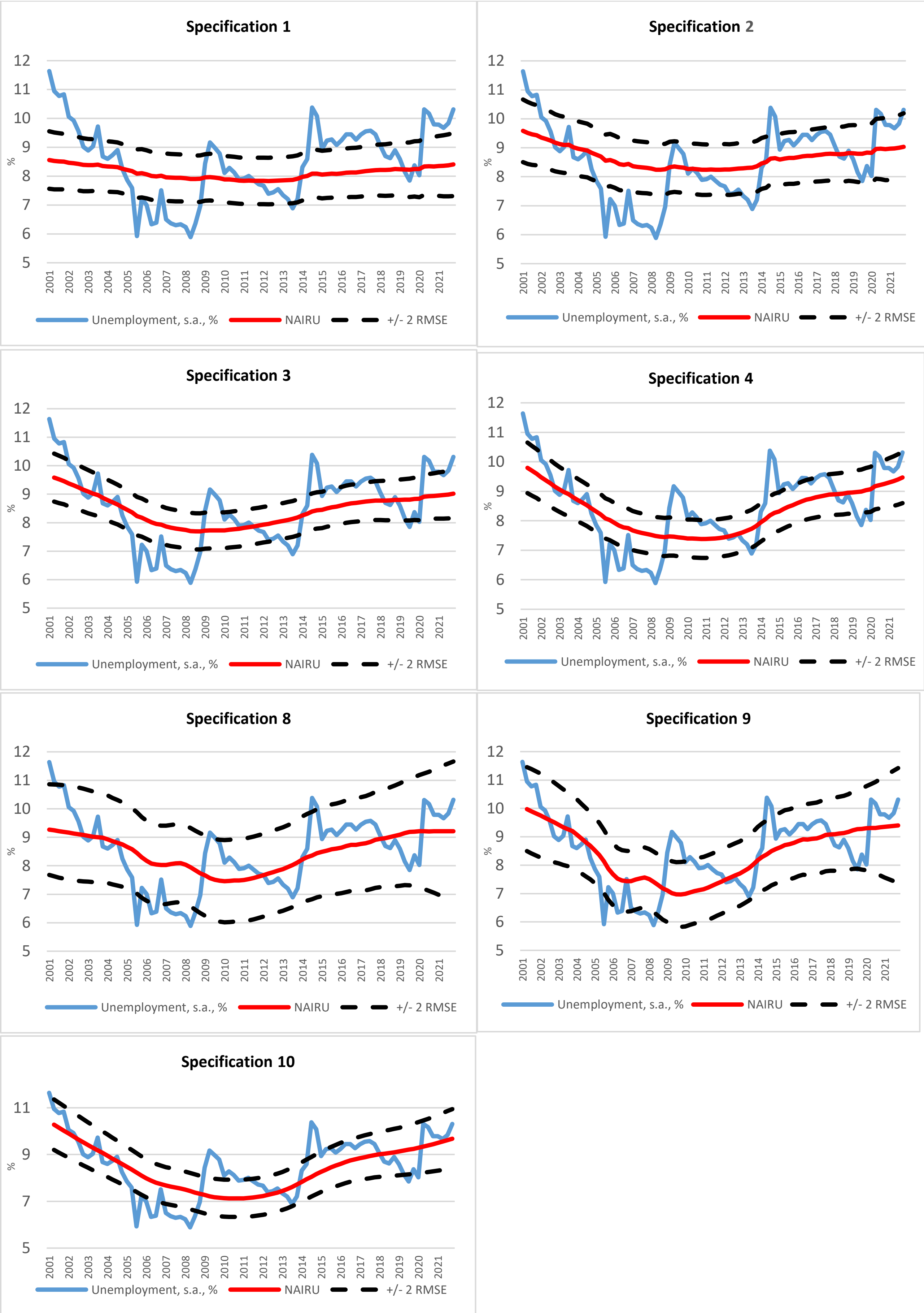


Figure A.1. Estimates of NAIRU from the Phillips curve for core inflation

Table A.2 Parameters of the Phillips curve for wage growth

	Spec. 5 (Sample:2003–2021)	Spec. 7 (Sample:2006–2021)	Spec. 11 (Sample:2003–2021)
Dependent signal variable	$\boldsymbol{\Delta w_t - \Delta z_t - \pi_{t-1}}$	$\boldsymbol{\Delta w_t - \Delta z_t - \pi_{t-1}}$	$\boldsymbol{\Delta w_t - \Delta z_t - \pi_{t-1}}$
u_{t-1}^{gap}	-3.16 0.00*	-4.09 0.00	
$(u_t - u_t^*)/u_t$			-0.32 0.00
$\Delta u_{t-1}/u_t$	3.85 0.67	12.91 0.45	-1.28 0.93
$\Delta\pi_t$	0.51 0.00		0.39 0.02
$\Delta\pi_{t-1}$	-0.70 0.00		-0.41 0.04
$\Delta\pi_{t-2}$	-0.81 0.00		-0.38 0.05
$\Delta\pi_{t-3}$	-0.82 0.00		-0.32 0.08
$\Delta\pi_{t-4}$	-0.55 0.00		-0.84 0.00
$(\pi_t - \pi_{t-1}^e)_t$		1.10 0.00	
$(\pi_t - \pi_{t-1}^e)_{t-3}$		0.29 0.00	
$(\pi_t - \pi_{t-1}^e)_{t-4}$		-0.22 0.04	
$\Delta z_t^* - \Delta z_t$	0.54 0.00	0.56 0.00	0.66 0.00
$\Delta z_{t-4}^* - \Delta z_{t-4}$	-0.46 0.00		
Δw_{t-1}	0.63 0.00	0.54 0.00	
Δw_{t-2}			0.60 0.00
Δw_{t-4}	-0.36 0.00	-0.13 0.29	-0.36 0.04
dum_16_4	-32.35 0.03		
dum_16_3	-29.65 0.06		
dum_16_2	-20.60 0.22		
dum_15_1		-11.89 0.17	
σ_ϵ	13.42 0.00	10.52 0.00	31.52 0.00
Unobserved states	$u_t^{gap} = \varphi_1 u_{t-1}^{gap} + \delta_t$, $u_t^* = u_{t-1}^* + \varepsilon_t$		$u_t^* = u_{t-1}^* + \varepsilon_t$
u_{t-1}^{gap}	0.81 0.00	0.90 0.00	
σ_δ	0.29 0.00	0.27 0.00	
σ_ε	0.04 0.00	0.03 0.00	0.03 0.00
u_t^* final state	9.74 0.00	9.12 0.00	9.60 0.00
u_t^{gap} final state	0.47 0.49	1.07 0.11	
Log-likelihood	-256.38	-204.90	-230.69
AIC	7.19	6.78	6.71
Sig-to-noise	0.005	0.005	0.005

* - p-value of the parameter

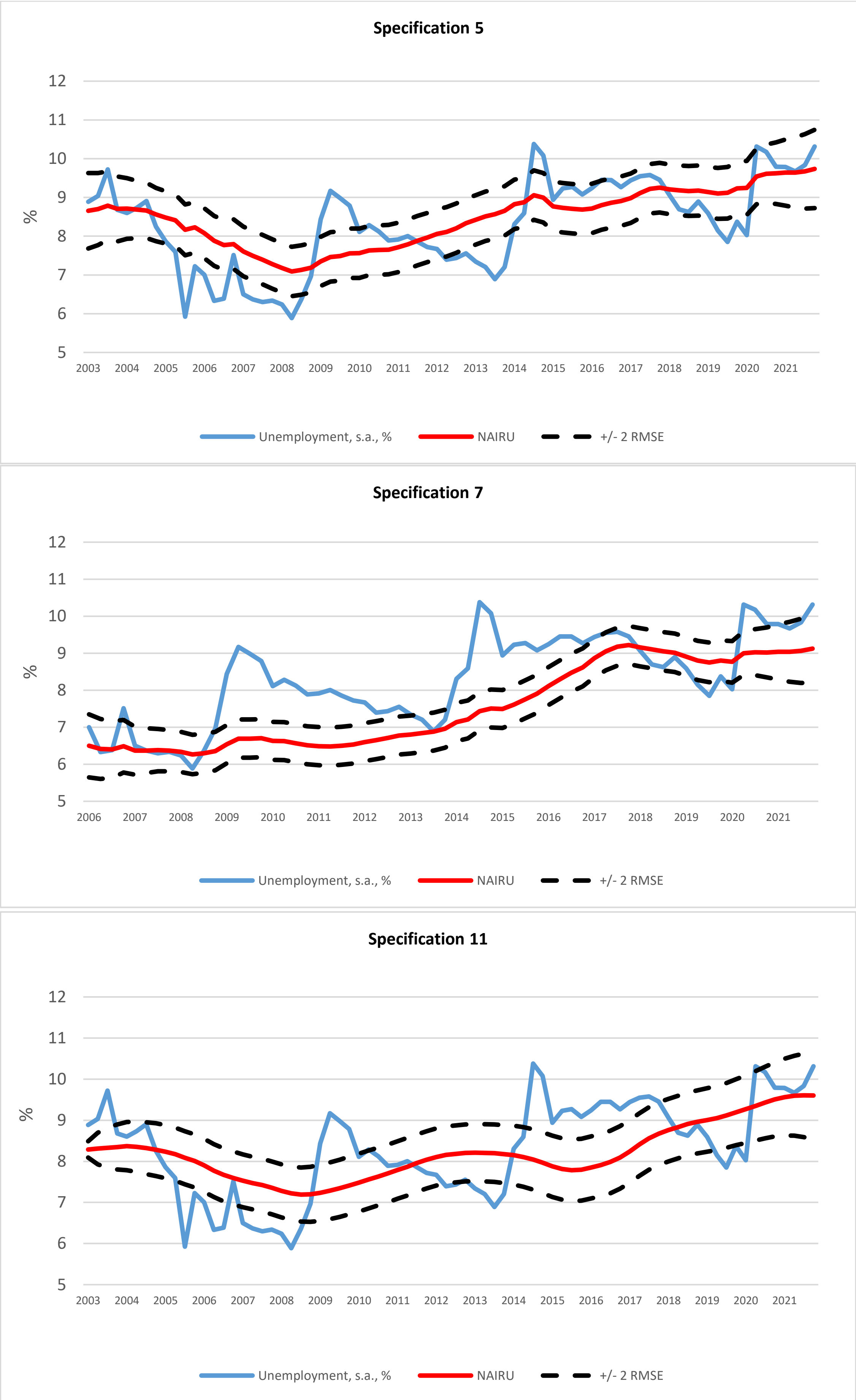


Figure A.2. Estimates of NAIRU from the Phillips curve for wage growth

Figure A.3. Estimates of NAIRU from the Phillips curve for wage growth

Table A.3 Parameters of expectations-augmented Phillips curve

	Spec. 6 (Sample: 2006–2021)	**Spec. 12 (Sample: 2006–2021)**	**Spec. 13 (Sample: 2006–2021)**	**Spec. 14 (Sample: 2006–2021)**
Dependent signal variable	$\boldsymbol{\pi_t - \pi_{t-1}^e}$	$\boldsymbol{\pi_t - \pi_{t-1}^e}$	$\boldsymbol{\pi_t - \pi_{t-1}^e}$	$\boldsymbol{\pi_t - \pi_{t-1}^e}$
u_{t-1}^{gap}	-3.06 0.01*			
$(u_t - u_t^*)/u_t$		-0.25 0.00	-0.23 0.02	-0.20 0.01
$\Delta u_{t-1}/u_t$	-14.12 0.16	-14.29 0.04	-15.00 0.07	-16.46 0.02
$(\pi_t - \pi_{t-1}^e)_{t-1}$	0.70 0.00	0.67 0.00	0.67 0.00	0.66 0.00
$(\pi_t - \pi_{t-1}^e)_{t-3}$	-0.26 0.00	-0.28 0.00	-0.28 0.00	-0.29 0.00
$\Delta\pi_t^{neer}$	-0.18 0.01	-0.20 0.01	-0.19 0.01	-0.19 0.01
$\Delta\pi_{t-2}^{neer}$	0.18 0.01	0.17 0.01	0.17 0.01	0.17 0.01
$\Delta\pi_{t-4}^{neer}$	-0.30 0.00	-0.32 0.00	-0.32 0.00	-0.32 0.00
$\Delta\pi_{t-1}^{oil}$	-0.04 0.03	-0.04 0.02	-0.04 0.03	-0.04 0.02
$\Delta\pi_{t-4}^{oil}$	-0.04 0.03	-0.04 0.05	-0.04 0.05	-0.03 0.11
$\Delta\pi_t^{imp}$	0.40 0.00	0.37 0.00	0.38 0.00	0.36 0.00
$\Delta\pi_{t-1}^{imp}$	0.29 0.00	0.26 0.00	0.26 0.00	0.24 0.00
$\Delta\pi_{t-2}^{imp}$	0.43 0.00	0.40 0.00	0.40 0.00	0.38 0.00
$\Delta\pi_{t-3}^{imp}$	0.29 0.00	0.28 0.00	0.29 0.00	0.29 0.00
$\sigma_\in$	5.00 0.00	4.93 0.00	4.91 0.00	4.38 0.00
Unobserved states	$u_t^{gap} = \varphi_1 u_{t-1}^{gap} + \delta_t$, $u_t^* = u_{t-1}^* + \varepsilon_t$	$u_t^* = u_{t-1}^* + \varepsilon_t$	, $u_t^* = u_{t-1}^* + \tau_1 \Delta ltu_t \vert \Delta bev_t + \varepsilon_t$	
u_{t-1}^{gap}	0.84 0.00			
Δltu_t			0.10 0.33	
Δbev_t				1.61 0.03
σ_δ	0.27 0.00			
σ_ε	0.02 0.00	0.02 0.00	0.02 0.00	0.02 0.00
u_t^* final state	9.43 0.00	9.24 0.00	9.30 0.00	9.79 0.00
u_t^{gap} final state	0.74 0.20			
Log-likelihood	-189.35	-139.90	-139.22	-135.14
AIC	6.42	5.13	5.14	5.00
Sig-to-noise	0.005	0.005	0.005	0.005

* - p-value of the parameter

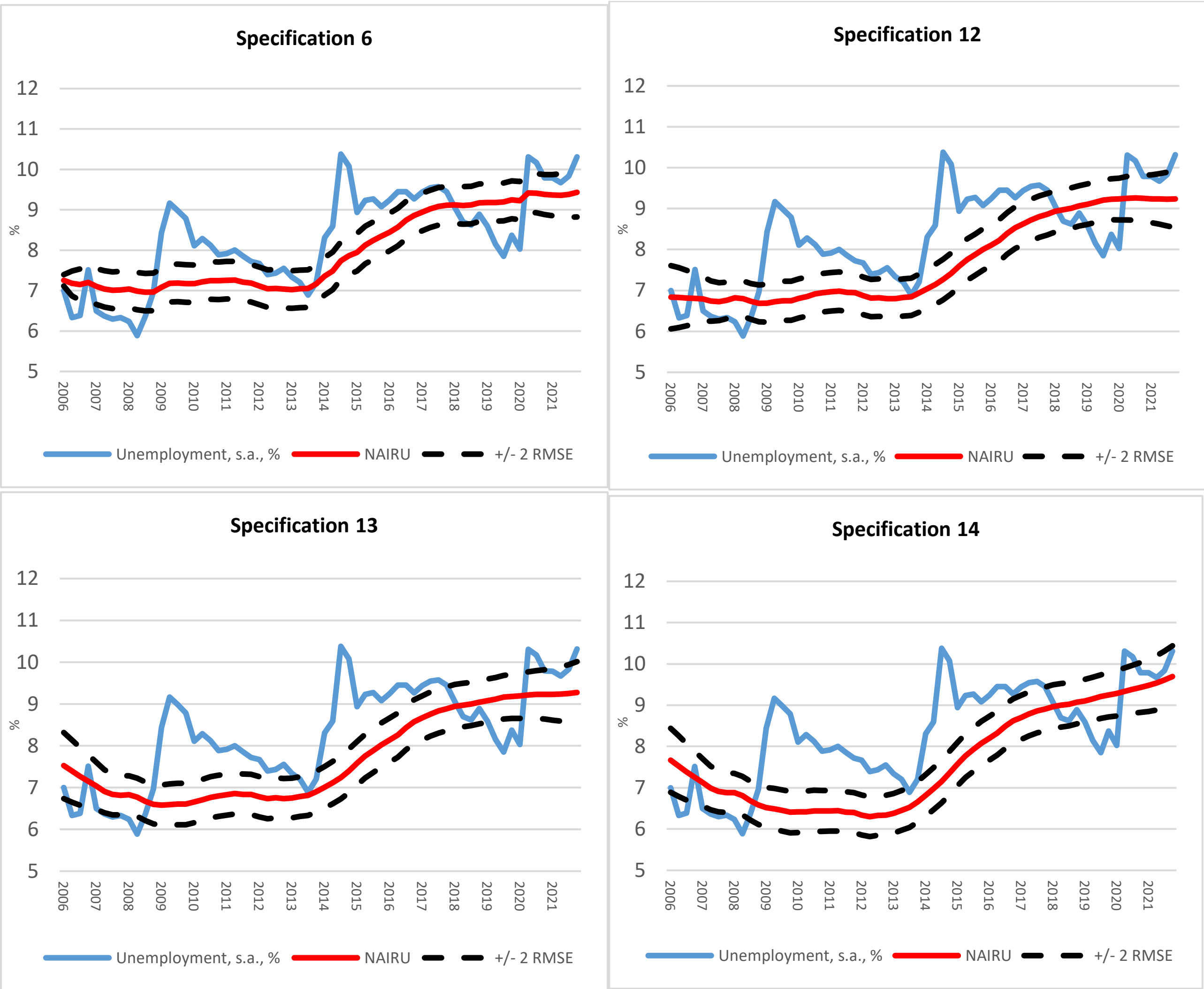


Figure A.4. Estimates of NAIRU from the expectations-augmented Phillips curve

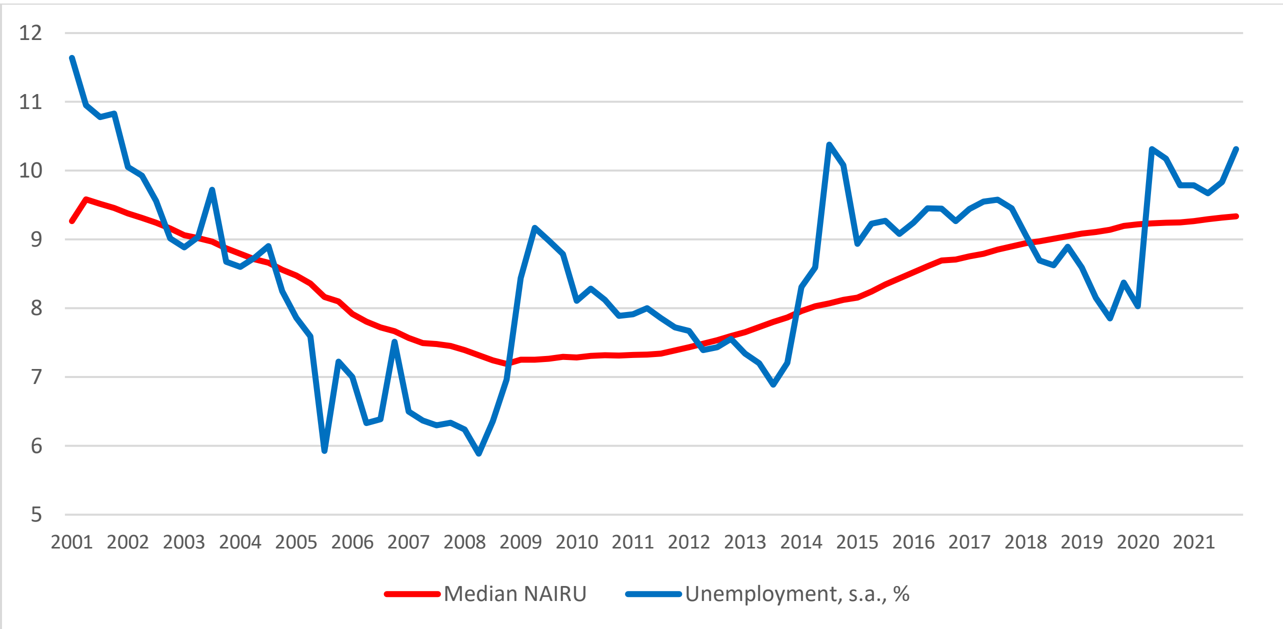


Figure A.5. Median of estimated NAIRUs

Annex B

The cumulative shifts of the Beveridge curve

To better identify NAIRU in the unobserved component model, we add the trend growth of the cumulative shift in the Beveridge curve (Δbev_t) as an exogenous variable to the u* equation. To estimate the Beveridge curve shifts, we follow Valletta (2005).

The Beveridge curve contains information on cyclical fluctuations in the labour market. The periods of economic slack are characterised by a low level of vacancies and high unemployment. Abnormally high economic growth leads to a significant reduction in unemployment and increase in the vacancy rate. Figure B.1 demonstrates the Beveridge curve for Ukraine, where two points are selected: 2006Q2, from the period when the Ukrainian economy was overheated, and 2014Q4, from the period of economic crisis. However, the combination of vacancies and unemployment can result in both "outward" and "inward" deviations. "Outward" deviations mean that job seekers can not meet with job suppliers. This is why the efficiency of the job matching process in an economy is low. The same logic is applicable to "inward" movements of the Beveridge curve.

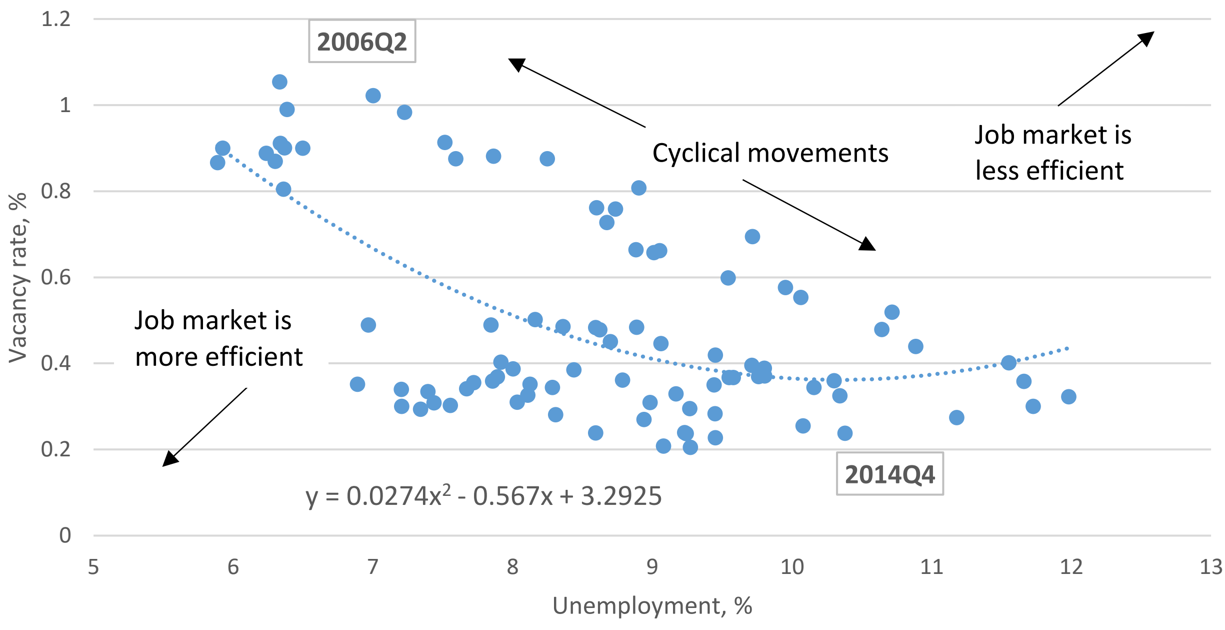


Figure B.1 Beveridge curve for Ukraine, 2001Q1–2021Q4

Note: Vacancies rate = (vacancies reported by the state service of employment of Ukraine/labour force) × 100.

The idea of Valletta (2005) is to quantify the "outward"/"inward" movements of the Beveridge curve. To do this, he suggested running a regression as follows:

$$u_t = \alpha + \beta_1 v_t + \beta_1 v_t^2 + \tau Y + \varepsilon_t, \qquad \text{(B.1)}$$

where u_t is the unemployment rate, v_t is the vacancy rate, and *Y is* the time effect captured by the year dummies. The series of estimated τ reflects the shifts in job matching efficiency relative to the base year, which is omitted from the dummies.

We performed this exercise on Ukrainian data for the sample 2000Q1–2021Q4 (Table B.1). The year dummy for 2000 was skipped, so cumulative shifts are presented relative to the beginning of the sample[21].

Table B.1. Beveridge curve regression, 88 obs.

Variable	Coefficient	P-value
v_t	-1.02	0.80
v_t^2	-1.48	0.62
Y01	-0.37	0.38
Y02	-1.32	0.02
Y03	-1.62	0.01
Y04	-1.71	0.01
Y05	-2.80	0.00
Y06	-2.81	0.00
Y07	-3.63	0.00
Y08	-4.07	0.00
Y09	-2.73	0.00
Y10	-3.47	0.00
Y11	-3.64	0.00
Y12	-4.11	0.00
Y13	-4.46	0.00
Y14	-2.41	0.00
Y15	-2.66	0.00
Y16	-2.38	0.00
Y17	-2.00	0.00
Y18	-2.49	0.00
Y19	-3.01	0.00
Y20	-2.02	0.00
Y21	-1.60	0.00
α	12.10	0.00
Adjusted R^2	0.90	

To obtain quarterly data, we converted the estimated annual shifts (Table B.1) from low to high frequencies using a quadratic method with average matching. The result is reflected in Figure B.2, where values that are more negative indicate "inward" shifts. It can be noted that the loss of job-matching efficiency occurred in crisis periods: 2008–2009, 2014–2015, and 2020–2021. In the last stage of data preparation, we smooth

[21] Valletta (2005) makes a series of adjustments to make data on vacancies and unemployment less biased. Unfortunately, we cannot do the same for Ukraine because of data limitations.

quarterly shifts using the Hodrick–Prescott filter to obtain the volatility of the potential explanatory variable comparable to the assumed volatility of NAIRU.

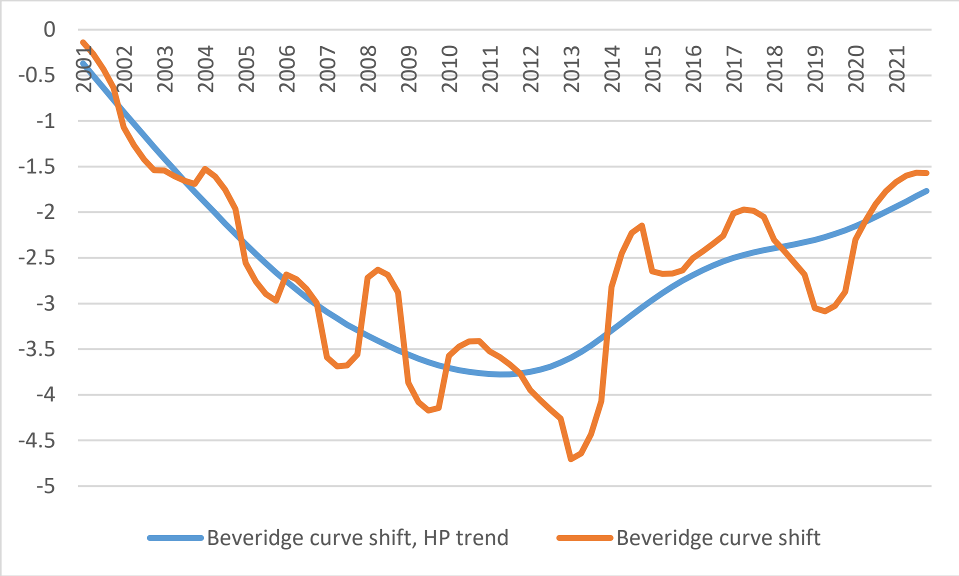


Figure B.2 Cumulative Beveridge curve shifts

Annex C

Kalman filter technique[22]

The Kalman filter, developed by Kalman (1960, 1963) and Kalman and Bucy (1961), is an algorithm for estimating parameters and unobserved variables in a state-space model. The general form of the state–space model is as follows:

$$y_t = H\beta_t + AZ_t + e_t\ , \tag{C.1}$$

$$\beta_t = \mu + F\beta_{t-1} + v_t\ , \tag{C.2}$$

where y_t is a vector of the observed data, H is a matrix of variables or coefficients depending on specification, β_t is a vector of unobserved components or state variables, A and F are matrices of coefficients, Z_t is a matrix of deterministic, lagged endogenous, or exogenous variables, μ is the vector of parameters, e_t and v_t is the error vector. The assumptions regarding the errors are as follows:

$$e_t \sim iid\ N(0, R),$$

$$v_t \sim iid\ N(0, Q),$$

$$E[e_t v_t] = 0.$$

The so-called "signal equation" (C.1.) connects the observed data to the unobserved state variable, which is modelled in Equation C.2 (state equation). In our study, the signal equation models inflation, which is observed and depends on lagged inflation, exogenous variables, and the unobserved unemployment gap. Equation C.2 specifies the dynamics of the NAIRU.

The Kalman filter is a recursive algorithm that estimates the parameters of the state-space model (H, A, μ, F, R, Q) and recovers the unobserved state variable (β_t) given the observed data in y_t and Z_t. The algorithm consists of the following equations evaluated recursively over time, starting from the initial values for $\beta_{0|0}$ and $P_{0|0}$:

$$\beta_{t|t-1} = \mu + F\beta_{t-1|t-1}, \tag{C.3}$$

$$P_{t|t-1} = FP_{t-1|t-1}F' + Q, \tag{C.4}$$

$$\eta_{t|t-1} = Y_t - H\beta_{t|t-1} - AZ_t, \tag{C.5}$$

$$f_{t|t-1} = HP_{t|t-1}H' + R, \tag{C.6}$$

$$\beta_{t|t} = \beta_{t|t-1} + K\eta_{t|t-1}, \tag{C.7}$$

$$P_{t|t} = P_{t|t-1} + KHP_{t|t-1}, \tag{C.8}$$

[22] In more details explanation of the Kalman filter technique can be found in Blake and Mumtaz (2012).

where $K = P_{t|t-1}H'f_{t|t-1}^{-1}$ denotes the Kalman gain. Equations C.3 and C.4 are referred to as prediction equations. Assuming that the parameters of the model (H, A, μ, F, R, Q) are known, C.3 predicts the value of the state variable one period ahead using the transition equation of the model. Equation (4) is the estimated variance of the state variable β, given information at time *t-1*. The prediction equations of the Kalman filter produce an estimate of the state variable based on the parameters of transition equation (C.2). Equation C. 5 of the Kalman filter calculates the prediction error. Equation (6) calculates the variance of the prediction error. The final two equations are referred to as updating equations. These equations update the initial estimates $\beta_{t|t-1}$ and $P_{t|t-1}$ by using the information contained in the prediction error $\eta_{t|t-1}$. The Kalman gain (K) can be considered the weight attached to the prediction error. Running these equations from $t = 1,2 \ldots T$ delivers $\beta_{t|t}$ and $P_{t|t}$ at the end of recursion.

The algorithm in C.3–C.8 assumes that the parameters of the state-space are known. In the general case, this is not true, and these parameters must also be estimated. For this, at the end of each iteration, the Kalman filter provides us with an input for a likelihood function that can be maximised with respect to the unknown parameters. This means that we also need initial values for parameters $H_0, A_0, \mu_0, F_0, R_0, Q_0$.

The Kalman filter predictors $\beta_{t|t-1}$ and $P_{t|t-1}$ provide the optimal predictors of the state vector β_t and its covariance matrix P_t based on information available at time *t-1*. This procedure for obtaining *filtered* estimates of the unobserved state does not use all the available information. The Kalman filter allows for a *smoothing* procedure, which is a backward recursion. The smoothed estimators $\beta_{t|T}$ and $P_{t|T}$ provide the optimal predictors of β_t and P_t based on all the information in the sample.