Price Authority under Competing Organizations[*]

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Abstract

This paper characterizes the degree of price discretion that competing organizations (principals) award to their sales managers (agents) and examines how such discretion is affected by principals' competitive conduct, market concentration, product substitutability, and demand volatility. We lay down a model in which firms sell differentiated products and sales managers own private information about demand and incur positive marketing costs to sell products to final consumers. Principals cannot internalize these costs through monetary incentives and need to design 'permission sets' from which their representatives choose prices. The objective is to understand the forces shaping this set and the constraints (if any) imposed on equilibrium prices. We find that when principals behave non-cooperatively, sales managers are biased towards excessively high prices owing to their will to pass on marketing costs to consumers. Hence, in equilibrium, the permission set requires a list price that caps agents' pricing choices. Such list price is more likely to bind in less concentrated industries, when products are closer substitutes, in industries where distribution requires sufficiently high costs and demand is not too volatile. Instead, when principals behave cooperatively and maximize industry profit, the optimal delegation scheme is more complex. Because principals want to raise prices to the monopoly level, in this regime, the optimal permission features a price floor rather than a list price when the marketing cost is sufficiently low, it features instead full discretion for moderate values of this cost, and only when it is sufficiently high, a list price is optimal. Interestingly, while competition hinders delegation in the noncooperative regime, the opposite happens when principals maximize industry profit.

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1 Introduction

In many prominent sectors, firms delegate prices to their sales managers (Zoltners et al., 2008). The reasons behind this phenomenon have been extensively debated in academic and policy circles on both sides of the Atlantic (Stephenson et al., 1979). Sales managers typically possess market knowledge (e.g., about consumers' willingness to pay) critical for pricing and marketing decisions (Lai, 1986 and Joseph, 2001). However, the preferences of these agents do not always align with profit maximization. For example, when contracts are incomplete, sales managers may exploit their informational advantage to abuse discretion and make suboptimal choices from the principals' standpoint. In some circumstances, monetary incentives may not even work (Holmstrom, 1997-1984). Principals, therefore, often face a 'delegation dilemma': giving up authority to gain flexibility, or retain price control and implement rigid rules unresponsive to the competitive environment?

Recent developments in the delegation literature have investigated this dilemma in-depth, characterizing the optimal mix of discretion and authority within vertical organizations (see, e.g., Bendor and Meirowitz, 2004, and Huber and Shipan, 2006, for surveys of these models). Stemming from the seminal work by Holmstrom (1977-1984), this literature has developed the idea of constrained (or interval) delegation (see, e.g., Aghion and Tirole, 1997, who developed the concepts of formal and real authority within vertical organizations). Essentially, when agents do not respond to monetary incentives — e.g., because contracts are incomplete — decision-makers (principals) can only influence their agents' behavior by constraining what they are entitled to do — i.e., the so-called 'permission set'. In these models, agents' discretion is determined by the trade-off between loss of information and loss of control. On the one hand, principals would like to tailor decisions to the agents' private information, thereby granting them more discretion. On the other hand, however, this loss of control is costly because agents' and principals' objectives typically do not coincide. The tension between these two forces leads to an interesting form of delegation: principals allow agents to choose their 'ideal' action in the states of nature where the conflict of interest is not too pronounced and impose a rigid rule, ceiling or flooring the agents' action depending on the direction of the conflict of interest, otherwise (see, e.g., Amador and Bagwell, 2013, Dessein, 2002, Martimort and Semenov, 2006, Melumad and Shibano, 1991).

In this paper, we characterize the degree of price discretion that, in equilibrium, N competing firms (principals) grant to their exclusive sales managers (agents). We consider a competitive environment in which principals compete by producing differentiated products (in the baseline model we consider a demand system à la Singh and Vives, 1984) and need to choose how much pricing discretion to grant their representatives that are privately informed about an aggregate demand shock — i.e., being closer to the final market, sales mangers are better informed than principals about consumer preferences. To introduce a wedge between upstream and downstream objectives, we assume that sales managers incur a positive marketing cost to finalize a sale — e.g., costs related to the effort they invest in persuading perspective customers into buying their products.[1] Following the literature, we posit that principals

[1]Sales managers can persuade perspective customers in many different ways — e.g., through informative advertising

cannot internalize this cost through monetary incentives. This misalignment of preferences creates a conflict of interest: agents have an incentive to pass on the marketing cost to consumers. Hence, they charge excessive prices compared to what upstream profit maximization would mandate: a friction that echoes the classical double marginalization phenomenon arising in vertical contracting where principals (e.g., manufacturers) are bound to offer linear contracts.

Existing models have neglected price delegation within such competing-organizations framework. The objective of the analysis, therefore, is twofold. First, we characterize the equilibrium permission set from which sales managers are entitled to choose prices, determine what type of constraints (if any) principals impose on their agents' pricing choices, and examine how these constraints respond to market concentration, product substitutability, and demand uncertainty. Second, we investigate whether principals award agents more or less discretion when they behave non-cooperatively than when they act cooperatively (i.e., they maximize aggregate profits). We, therefore, consider two alternative decision-making regimes. In the first regime, principals choose the permission set granted to their agents non-cooperatively — i.e., each principal maximizes its profit, taking as given the rivals' behavior. In the second regime, principals maximize industry profits, which reflects the case where they cooperate or merge into a single multiproduct monopolist delegating every product line to a single representative.

We find that when principals behave non-cooperatively, agents are biased towards excessively high prices compared to the price that principals would choose non-cooperatively if they were informed about demand. The reason is that agents' pricing choices reflect their marketing cost, thereby creating a sort of double marginalization problem that hurts principals (and, of course, consumers). Hence, in the non-cooperative regime, the equilibrium permission set only requires a price cap (i.e., a list price) that limits from above the set of prices that agents can charge to final consumers so to avoid that they appropriate excessive margins when demand conditions are favorable. Interestingly, this cap is more likely to bind in less concentrated industries, when products are relatively closer substitutes and in sectors where marketing costs are sufficiently high — e.g., when product quality is not or partly observable only and, therefore, sales managers' marketing activity can severely impact consumers' choices. Instead, list prices are less likely to bind in industries featuring higher demand volatility. Thus, the first important implication of our model is that when principals behave non-cooperatively, competition hinders price delegation while uncertainty favors it.

We then turn to the cooperative regime in which the coalition of principals behaves as a unique entity — i.e., it maximizes industry profit — and chooses the same permission set for all agents (which captures the idea of upstream coordination to exert downstream control). In this case, the optimal permission set becomes considerably more complex relative to the non-cooperative benchmark: a counterbalancing force shapes the conflict of interest between principals and agents. This is because industry profit maximization requires prices to be increased up to the monopoly level — i.e., other things being equal, the principals' ideal point rises compared to the non-cooperative regime. As a result, the presence of a moderate marketing cost may, ceteris paribus, align incentives within the competing organizations. The

campaigns, website maintenance, time spent on social media, etc.

optimal permission set features a price floor rather than a list price when the marketing cost is sufficiently low, it features complete discretion for moderate values of this cost, and only when it is sufficiently large it features a list price. Interestingly, the region of parameters in which agents are granted full pricing discretion expands when demand is more uncertain and when the market becomes more competitive, as implied by lower market concentration and products being relatively closer substitutes. Therefore, the second novel implication of our model is that, when principals behave cooperatively, competition favors price delegation.

Notably, these findings question the traditional view that upstream coordination is associated with tighter vertical price control, and suggest that observed patterns of price delegation may be partly explained by principals' cooperative conduct rather than an individual decision making behavior. Moreover, in addition to shedding new light on the rationale for list prices and their determinants, these results also provide important insights to better understand suppliers' cartels, their strategies and organization.

All our predictions are empirically testable and are robust to alternative specifications of the demand functions, to the introduction of intra-brand competition and alternative types of competition and industry structure. Specifically, the introduction of both intra- and inter-brand competition does not alter the conclusion that the non-cooperative game only features partial delegation implemented through list prices while full delegation may occur in the cooperative regime. However, while inter-brand competition tends to exacerbate the conflict of interest between principals and agents, intra-brand competition tends to align incentives. Moreover, results are robust to alternative linear and non-linear demand specifications — i.e., Shubick-Levitan and CES preferences. We also consider quantity competition. In this case, because of their marketing costs, sales managers tend to produce too little compared to what principals would like. Hence, the optimal permission set features minimal production requirements, which can be implemented through the imposition of list prices. Finally, when principals deal with a common representative rather than delegating price authority to exclusive sales managers, list prices are still optimal. However, in this scenario, list prices becomes relatively more binding as competition intensifies. This is because, when the number of products in the market grows or the existing products become closer substitutes, the common agent has an incentive to increase prices in order to soften competition between principals.

Literature review. Our analysis borrows from and contributes to several strands of the managerial economics, marketing, and IO literature dealing with price delegation, competition, and incentives within firms.

Stemming from Bonanno and Vickers (1988), Fershtman and Judd (1987), and Sklivas (1987), several scholars have investigated the strategic value of managerial delegation. In this literature, delegating pricing decisions to sale managers can be a credible mechanism to soften competition. The crucial feature is the ability of firms (shareholders) to disclose managerial contracts to rivals and therefore influence their conduct through the choice of these contracts. The model developed in our paper differs in two fundamental ways from this literature. First, it considers a framework in which agency conflicts cannot be solved by monetary incentives. Second, in our model, delegation is not explained by its strategic commitment value since, as in the literature on secret contracting, principals' permission sets are unobservable.

A few papers have extended the idea of strategic delegation to environments with secret contracts and different forms of information asymmetries. Pagnozzi and Piccolo (2011), for example, show that vertical delegation may occur at equilibrium even when contracts are secret provided that agents do not hold passive beliefs off-equilibrium. Bhardwaj (2001), instead, shows that with competition in prices and effort the strategic nature of delegation depends on the relative intensity of competition. In contrast to us, with unobservable contracts and risk-averse sales representatives, he finds that firms delegate the pricing decision when price competition is intense (see also Gal-Or, 1991, Blair and Lewis, 1994, Martimort and Piccolo, 2007-2010, for models with adverse selection, and Mishra and Prasad, 2005, for a moral hazard set-up).

In all these models, principals are able to align incentives (partially or in full) through monetary incentives, in contrast we focus on cases in which monetary incentives are not enforceable. In this sense, our model is more closely related and builds on the partial delegation literature initiated by Holmstrom (1977-1984). Following his seminal work, many scholars have investigated the determinants of delegation in the absence of monetary incentives and the conditions under which interval allocation is optimal (see, e.g., Amador and Bagwell, 2013, Alonso and Matouschek, 2008, Armstrong and Vickers, 2010, Dessein, 2002, Dessein and Santos, 2006, Frankel, 2014-2016, Martimort and Semenov, 2006, Melumad and Shibano, 1991, among many others). These models characterize the trade-off between loss of information and loss of control in several different environments. We contribute to this bulk of work by considering competing organizations and by characterizing the equilibrium interval delegation under the principals' cooperative and noncooperative behavior (Kräkel and Schöttner, 2019, also look at cooperative equilibria but focus on collusion between sale managers and consumers rather than collusion among principals). Our novel result is that upstream cooperation does necessarily imply tighter vertical control, especially in very competitive environments.

The competitive aspects of our analysis also share common features with the marketing and managerial literature investigating the link between sales managers incentives and competition (see Ruzzier, 2009, for a survey of the literature across management and economics). In line with Mishra and Prasad (2005), we also find that competitive product markets may favor centralized pricing, but only when principals behave non-cooperatively. An opposite relationship is found in the cooperative case.

Finally, our model is related to the burgeoning literature on list prices, their determinants, and competitive effects. Harrington and Ye (2017) develop a theory to explain how coordination on list prices can raise transaction prices. The model developed in our paper differs in two fundamental ways from Harrington and Ye (2017). First, they assume a deterministic link between list and transaction prices, while in our model, there is a stochastic relationship between them driven by demand uncertainty. Second, their model assumes that principals are privately informed about a common product technology; we consider the equally plausible, polar case in which, being closer to customers, sales managers own private information on demand. In this context, we find that industry profit maximization may well require principals to grant agents full pricing discretion, even though this discretion would be constrained by the presence of list prices in a non-cooperative equilibrium. Gill and Thanassoulis (2016) also consider upstream cooperation but, in contrast to Harrington and Ye (2017), assume that firms can coordinate on both list and transaction prices because both are verifiable (see also Raskovich, 2007, Lester et al., 2015, and Mallucci et al., 2019). In these models, coordination on transaction prices is viable because they do not consider privately informed salespeople; in our model, instead, coordination on transaction prices is feasible only when principals give up the benefits of flexibility. Finally, while all these papers emphasize the adverse effects of list prices on consumer surplus, our model shows that list prices limit agents' incentive to pass on to consumers their marketing costs, thereby avoiding the standard double marginalization loss. A result in this spirit is also offered by Myatt and Ronayne (2019) who show that list prices tend to intensify competition and reduce search costs, thereby benefitting consumers. Our model abstracts from search costs and focuses more on the internal agency conflict between firms and sales managers.

The rest of the paper is organized as follows. Section 3 lays down the model. In Section 3.1, we characterize the benchmark with informed principals. Section 3.2, instead, characterizes, the equilibrium permission under each decision-making regime. In Section 4 we perform some robustness checks of the baseline model. Section 5 summarizes the managerial implications of our analysis and concludes. Proofs are in the Appendix.

2 The baseline model

Environment. Consider N firms (principals), each denoted by P_i, with $i = 1, .., N$, selling to a single (representative) consumer whose preferences are described by a quadratic utility function à la Singh and Vives (1984)

$$U(\cdot) \triangleq (1+\theta)\sum_{i=1}^{N} q_i - \frac{1}{2}\sum_{i=1}^{N} q_i^2 - \frac{\gamma}{2}\sum_{i=1}^{N}\sum_{j=1, j\neq i}^{N} q_i q_j - \sum_{i=1}^{N} p_i q_i + I, \tag{1}$$

where, as standard, $I \geq 0$ is the representative consumer's income. The parameter θ is uniformly distributed over the support $\Theta \triangleq [-\sigma, \sigma]$ and captures the consumer's stochastic willingness to pay, with σ being its volatility. The parameter $\gamma \in [0, 1)$ is an inverse measure of the degree of differentiation between products: the larger γ, the more homogenous (less differentiated) products are.

Differentiating (1) with respect to q_i $(i = 1, ..., N)$ and inverting the implied system of first-order conditions, we obtain the following demand functions

$$D_i(\theta, p_i, p_{-i}) \triangleq \frac{(1-\gamma)(1+\theta) - (1+(N-2)\gamma) p_i + \gamma \sum_{j=1, j\neq i}^{N} p_j}{(1-\gamma)(1+(N-1)\gamma)} \qquad \forall i = 1, .., N. \tag{2}$$

Principals sell through exclusive agents (dealers, sales managers, retailers, etc.) to whom they can grant pricing authority. Agents, each denoted by A_i $(i = 1, .., N)$, are privately informed about the representative consumer's willingness to pay θ and condition their pricing decision (if they are entitled to do so) on its realization (Lai, 1986; Joseph, 2001; Mishra and Prasad, 2004). Principals are uninformed about θ.

Payoffs and conflict of interest. Principals and agents have misaligned preferences. P_i maximizes profit, that is

$$\pi_i(\cdot) \triangleq D_i(\theta, p_i, p_{-i}) p_i \qquad \forall i = 1, .., N,$$

with technologies being linear and marginal costs normalized to zero. A_i's objective function is, instead,

$$u_i(\cdot) \triangleq \pi_i(\cdot) - cD_i(\theta, p_i, p_{-i}) = D_i(\theta, p_i, p_{-i})(p_i - c) \qquad \forall i = 1, ..N,$$

where $c \geq 0$ can be interpreted as the marketing cost that an agent incurs to finalize a sale — e.g., the opportunity cost of the time required to convince a buyer to purchase the product, the cost of effort that an agent must invest in this negotiation process, the cost of engaging in informative and/or promotional activities, etc. Hence, de facto, c represents A_i's bias vis-à-vis P_i. If $c = 0$ their preferences are fully aligned; otherwise, A_i sets a price higher than P_i's ideal price because it has an incentive to pass on the marketing cost to consumers (i.e., due to a sort of double marginalization phenomenon). Although we shall stick with the marketing cost interpretation, the parameter c can represent any other friction capturing retailers' incentive to appropriate excessive margins compared to what is optimal from the principals' standpoint.

We assume that whenever $p_i < c$ agent A_i $(i = 1, .., N)$ prefers not sell. The implicit hypothesis is that whenever downstream margins are negative, agents do not exert the necessary effort to finalize a sale.

Decision-making. We consider and compare two alternative decision-making regimes:

a A *non-cooperative* regime in which each principal maximizes its own profit;

b A *cooperative* regime in which principals maximize industry profits.

The non-cooperative regime reflects the standard case in which competing principals, each dealing with an exclusive agent, behave non-cooperatively. On the contrary, the cooperative regime can either be interpreted as upstream coordination with myopic agents that do not internalize their principals' objectives, or the case of a multiproduct monopolist delegating the marketing of each product line to a self-interested representative.

Interval delegation. Following Holmstrom (1977-1984), we rule out monetary incentives — i.e., there are contractual frictions (e.g., moral hazard) that prevent principals to fully extract the downstream profits through monetary incentives.[2] Hence, P_i can only limit A_i's discretion by determining the permission set $\mathcal{P}_i$ within which the price p_i must be chosen. More (less) discretion, as reflected by a larger (smaller) $\mathcal{P}_i$, means more (less) authority delegated to A_i. To simplify exposition, we focus on interval delegation by assuming that permission sets are connected — i.e., $\mathcal{P}_i \triangleq [\underline{p}_i, \overline{p}_i]$, with $\overline{p}_i \geq \underline{p}_i$ for every $i = 1, .., N$. This

[2] For example, this is the case when c is a random variable not verifiable in Court, so that monetary incentives contingent on c cannot be enforced.

restriction hinges on Martimort and Semenov (2006) who show that, in a linear quadratic framework like ours, assuming a connected permission set is equivalent to allow principals to use continuous, truthful revelation mechanisms requiring each agent to report the state of demand. Specifically, each principal P_i chooses a direct mechanism $\mathcal{M}_i \triangleq \{p_i(m_i)\}_{m_i \in \Theta}$, with the mapping $p_i(\cdot) : \Theta \to \Re$ specifying a pricing rule $p_i(m_i)$ for any report $m_i \in \Theta$ made by A_i to P_i about the state of the world θ. Under the hypothesis of passive beliefs off the equilibrium path (which we shall discuss below), each principal takes as given the rivals' equilibrium behavior; therefore, the results of Martimort and Semenev (2006) directly apply to our competing-organizations framework, implying that the restriction to connected permission sets is without loss of generality.

In the cooperative regime, we assume that principals are able to coordinate on a single permission set $\mathcal{P} \triangleq [\underline{p}, \overline{p}]$, so that all agents face the same pricing constraints. This assumption if made for expositional purposes only and is meant to rule out any other friction due to principals' lack of coordination — i.e., if principals could, they would certainly agree to set a common permission set which grants the upstream coalition commitment power.

The upper bound of the permission set can be interpreted as a list price, while the lower bound is a price floor that determines the maximal rebate that agents can offer to consumers.

Timing. Within each decision-making regime, the timing of the game is as follows:

1. Principals choose permission sets;
2. The demand shock θ realizes, and agents choose prices simultaneously given their permission sets;
3. Demand is allocated between the N rivals, and profits are made.

Equilibrium concept. The equilibrium concept is Perfect Bayesian Nash Equilibrium. In the non-cooperative regime we impose the refinement of 'passive beliefs', which is the one most widely used in the vertical contracting literature (Hart and Tirole, 1990; McAfee and Schwartz, 1994; Rey and Tirole, 2007). With passive beliefs, an agent's conjecture about the permission sets offered to its rivals is not influenced by an out-of-equilibrium offer he receives. In the cooperative regime, all agents receive the same permission set. Hence, out of equilibrium beliefs require that agents best reply to the permission set chosen by the principals' coalition and to their expectations about the rivals' pricing behavior.

Assumptions. Throughout we make the following assumptions.

A1 Demand is not too dispersed — i.e., $\sigma \leq \frac{1}{2}$ — and the marketing cost is not too large — i.e.,

$$c \leq \min\{1-\gamma, 1-\sigma\}.$$

This assumption guarantees that agents' profits are always positive.

2.1 Informed principals

To gain insights, before characterizing the equilibrium of the game with uninformed principals, it is useful to briefly describe the case of informed principals, who can force a price contingent on θ.

Non-cooperative behavior. In the non-cooperative regime, P_i solves

$$\max_{p_i} D_i\left(\theta, p_i, p_{-i}\right) p_i,$$

whose first-order condition requires

$$\underbrace{D_i\left(\theta, p_i, p_{-i}\right)}_{\text{Profit-margin effect}} + \underbrace{\frac{\partial D_i\left(\theta, p_i, p_{-i}\right)}{\partial p_i} p_i}_{\text{Volume effect}} = 0 \qquad \forall i = 1, .., N. \tag{3}$$

This condition reflects the standard trade-off between the positive profit-margin effect and the negative volume effect — i.e., a higher price benefits P_i because it increases its profit on the 'infra-marginal' consumers, but it also reduces the 'marginal' consumer thereby reducing P_i's sale volume. It is immediate to show that the game has a unique equilibrium in which all firms (principals) charge

$$p^N\left(\theta\right) \triangleq \frac{\left(1-\gamma\right)\left(1+\theta\right)}{2+\left(N-3\right)\gamma} \qquad \forall \theta \in \Theta, \tag{4}$$

which, as expected, increases in the consumers' willingness to pay θ and decreases in the degree of product market competition, as reflected by higher product substitutability (γ) and/or a higher number of competitors (N).

Cooperative behavior. Suppose now that principals maximize industry profit — i.e.,

$$\max_{(p_1,\ldots,p_N)} \sum_{i=1}^{N} D_i\left(\theta, p_i, p_{-i}\right) p_i.$$

The first-order condition is

$$\underbrace{D_i\left(\theta, p_i, p_{-i}\right)}_{\text{Margin effect } (+)} + \underbrace{\frac{\partial D_i\left(\theta, p_i, p_{-i}\right)}{\partial p_i} p_i}_{\text{Volume effect } (-)} + \underbrace{\sum_{j=1, j\neq i}^{N} \frac{\partial D_j\left(\theta, p_j, p_{-j}\right)}{\partial p_i} p_j}_{\text{Competition softening } (+)} = 0 \qquad \forall i = 1, .., N. \tag{5}$$

This condition reflects an additional effect compared to (3). When prices are chosen cooperatively, principals internalize the effect of changing a price on the rivals' profits: a competition softening effect. Solving equation (5), we have the monopoly price

$$p^M\left(\theta\right) \triangleq \frac{1+\theta}{2} \qquad \forall \theta \in \Theta,$$

with

$$\Delta p(\theta) \triangleq p^M(\theta) - p^N(\theta) = \frac{(N-1)\gamma}{2+(N-3)\gamma} p^M(\theta) \geq 0 \qquad \forall \theta \in \Theta.$$

As intuition suggests, with informed principals, prices are higher in the cooperative regime than in the noncooperative regime. The difference $\Delta p(\theta)$ between the monopoly and the noncooperative price is increasing in θ: the monopoly price is relatively more responsive to consumers' willingness to pay than the noncooperative price.

2.2 Uninformed principals

We can now consider the case of uninformed principals. Throughout, we assume (and verify ex-post) that it is never optimal to set a floor below c (recall that A_i will prefer not to sell at any price $p_i < c$).

Noncooperative behavior. Suppose that principals behave non-cooperatively. We will look for a symmetric equilibrium in which all principals choose a permission set $\mathcal{P}^\star \triangleq [\underline{p}^\star, \overline{p}^\star]$. Let

$$D_i(\theta, p_i, p^\star(\theta)) \triangleq \frac{(1-\gamma)(1+\theta) - (1+(N-2)\gamma) p_i + \gamma (N-1) p^\star(\theta)}{(1-\gamma)(1+(N-1)\gamma)},$$

be A_i's demand when his $N-1$ rivals face $\mathcal{P}^\star$ and charge $p^\star(\theta) \in \mathcal{P}^\star$. A_i's unconstrained maximization problem is

$$\max_{p_i} D_i(\theta, p_i, p^\star(\theta))(p_i - c),$$

whose first-order condition yields a best reply

$$p_i(\theta, p^\star(\theta)) \triangleq \frac{c}{2} + \frac{(1-\gamma)(1+\theta) + \gamma (N-1) p^\star(\theta)}{2(1+(N-2)\gamma)} \qquad \forall \theta \in \Theta, \tag{6}$$

which, as expected, is increasing in the price $p^\star(\theta) \in \mathcal{P}^\star$ that A_i expects rivals to charge, and increasing in c since A_i will (partly) pass on to consumers the marketing cost c.

Lemma 1 *Consider a demand state (if it exists) in which agents' pricing decisions are not constrained by the permission sets chosen by their principals. In this state, the equilibrium price is*

$$\hat{p}(\theta) = \underbrace{p^N(\theta)}_{P_i\text{'s ideal point}} + \underbrace{\frac{1+(N-2)\gamma}{2+(N-3)\gamma} c}_{A_i\text{'s bias}} \qquad \forall \theta \in \Theta.$$

This proposition shows that when agents' pricing decisions are unconstrained, the prevailing price exceeds the principals' ideal point — i.e., there is a misalignment of preferences between principals and agents measured by the bias

$$b^\star = \frac{1+(N-2)\gamma}{2+(N-3)\gamma} c.$$

As intuition suggests, this bias is increasing in c: as marketing becomes more costly, agents have an incentive to pass on this cost to a greater extent to consumers, thereby exacerbating double marginalization at the principals' expense. More interestingly, since $b^\star$ is increasing in N and γ, the agency conflict is also exacerbated by intensified competition. The intuition is as follows. When the number of firms increase and/or products become closer substitutes, each principal would like to reduce its price not to loose business to its rivals, but agents are more reluctant to do so because they need to pass on their marketing cost to consumers. This suggests that, in more competitive environments, delegation becomes harder to sustain, as we will argue below.

We can therefore turn to characterize some important properties of the equilibrium of the game in the cooperative regime. Given the candidate equilibrium $\mathcal{P}^\star \subseteq [\hat{p}(-\sigma), \hat{p}(\sigma)]$, there exist two thresholds $\overline{\theta}^\star$ and $\underline{\theta}^\star$ such that $\overline{\theta}^\star \triangleq \hat{p}^{-1}(\overline{p}^\star) > \underline{\theta}^\star \triangleq \hat{p}^{-1}(\underline{p}^\star)$. Hence, the equilibrium price has the following step-wise shape

$$p^\star(\theta) \triangleq \begin{cases} \overline{p}^\star & \text{if } \theta \geq \overline{\theta}^\star \\ \hat{p}(\theta) & \text{if } \theta \in [\underline{\theta}^\star, \overline{\theta}^\star] \\ \underline{p}^\star & \text{if } \theta \leq \underline{\theta}^\star \end{cases},$$

with $p^\star(\theta)$ being (weakly) increasing in θ.

Hence, we can show the first important preliminary result.

Lemma 2 *A_i's best reply $p_i(\theta, p^\star(\theta))$ is increasing in θ. As a result, for any permission set $\mathcal{P}_i$ chosen by P_i, the solution of A_i's constrained maximization problem is*

$$\tilde{p}_i(\theta, p^\star(\theta)|\mathcal{P}_i) \triangleq \begin{cases} \overline{p}_i & \text{if } \theta \geq \overline{\theta}_i, \\ p_i(\theta, p^\star(\theta)) & \text{if } \theta \in (\underline{\theta}_i, \overline{\theta}_i), \\ \underline{p}_i & \text{if } \theta \leq \underline{\theta}_i. \end{cases}$$

The threshold $\overline{\theta}_i$ (resp. $\underline{\theta}_i$) is the unique solution of $\overline{p}_i = p_i(\theta, p^\star(\theta))$ (resp. $\underline{p}_i = p_i(\theta, p^\star(\theta))$), with $\overline{\theta}_i \geq \underline{\theta}_i$.

This results shows that the price chosen by A_i is increasing in θ. It then follows immediately that the best reply $p_i(\theta, p^\star(\theta))$ hits the constraints imposed by the permission set when demand is sufficiently low (the floor $\underline{p}_i$ binds) and when it is too high (the list price $\overline{p}_i$ binds). Clearly, $\overline{\theta}_i = \overline{\theta}^\star$ and $\underline{\theta}_i = \underline{\theta}^\star$ if $\mathcal{P}_i = \mathcal{P}^\star$.

Hence, assuming interior solutions and operating a simple change of variables to optimize with respect to $\underline{\theta}_i$ and $\overline{\theta}_i$ rather than over the boundaries of the permission set, it is immediate to show that P_i's maximization problem is

$$\max_{(\underline{\theta}_i, \overline{\theta}_i) \in \Theta^2} \int_{-\sigma}^{\sigma} D_i(\theta, \tilde{p}_i(\theta, p^\star(\theta)|\mathcal{P}_i), p^\star(\theta))\, \tilde{p}_i(\theta, p^\star(\theta)|\mathcal{P}_i) \frac{d\theta}{2\sigma},$$

subject to $\overline{\theta}_i \geq \underline{\theta}_i$, with

$$\int_{-\sigma}^{\sigma} D_i\left(\theta, \tilde{p}_i\left(\theta, p^{\star}(\theta)|\mathcal{P}_i\right), p^{\star}(\theta)\right) \tilde{p}_i\left(\theta, p^{\star}(\theta)|\mathcal{P}_i\right) \frac{d\theta}{2\sigma} \triangleq \underbrace{\int_{-\sigma}^{\underline{\theta}_i} D_i\left(\theta, \hat{p}(\underline{\theta}_i), p^{\star}(\theta)\right) \hat{p}(\underline{\theta}_i) \frac{d\theta}{2\sigma}}_{\text{Binding floor}} +$$

$$\underbrace{\int_{\underline{\theta}_i}^{\overline{\theta}_i} D_i\left(\theta, p_i\left(\theta, p^{\star}(\theta)\right), p^{\star}(\theta)\right) p_i\left(\theta, p^{\star}(\theta)\right) \frac{d\theta}{2\sigma}}_{A_i \text{ is delegated pricing authority}} + \underbrace{\int_{\overline{\theta}_i}^{\sigma} D_i\left(\theta, \hat{p}(\overline{\theta}_i), p^{\star}(\theta)\right) \hat{p}(\overline{\theta}_i) \frac{d\theta}{2\sigma}}_{\text{Binding list price}}.$$

We shall neglect and verify ex post the constraint $\overline{\theta}_i \geq \underline{\theta}_i$. Let $\overline{\lambda}_i$ (resp. $\underline{\lambda}_i$) be the Lagrangian multiplier associated to the constraint $\overline{\theta}_i \leq \sigma$ (resp. $\underline{\theta}_i \geq -\sigma$). Differentiating with respect to $\underline{\theta}_i$ and $\overline{\theta}_i$, respectively, and imposing symmetry — i.e., $\underline{\theta}_i = \underline{\theta}^{\star} < \overline{\theta}_i = \overline{\theta}^{\star}$ — we have the following first-order conditions

$$\int_{\theta \leq \underline{\theta}^{\star}} \left[D_i(\theta, \underline{p}^{\star}, \underline{p}^{\star}) + \frac{\partial D_i(\theta, \underline{p}^{\star}, \underline{p}^{\star})}{\partial p_i} \underline{p}^{\star}\right] \frac{d\theta}{2\sigma} = \underline{\lambda}^{\star}, \tag{7}$$

$$\int_{\theta \geq \overline{\theta}^{\star}} \left[D_i(\theta, \overline{p}^{\star}, \overline{p}^{\star}) + \frac{\partial D_i(\theta, \overline{p}^{\star}, \overline{p}^{\star})}{\partial p_i} \overline{p}^{\star}\right] \frac{d\theta}{2\sigma} = -\overline{\lambda}^{\star}. \tag{8}$$

with

$$\overline{p}^{\star} \triangleq \hat{p}(\overline{\theta}^{\star}) \geq \underline{p}^{\star} \triangleq \hat{p}(\underline{\theta}^{\star}),$$

and associated complementary slackness conditions

$$\underline{\lambda}^{\star} \geq 0, \quad \underline{\lambda}^{\star}\left[\underline{\theta}^{\star} + \sigma\right] = 0,$$

$$\overline{\lambda}^{\star} \geq 0, \quad \overline{\lambda}^{\star}[\overline{\theta}^{\star} - \sigma] = 0.$$

The firs-order condition conditions in (7) and (8) reflect (in expected terms) the standard trade-off between the positive profit margin effect and the negative volume effect that each principal P_i faces when increasing the price floor and/or the list price. Specifically, by rising $\underline{\theta}_i$ (resp. $\overline{\theta}_i$) principal P_i increases the price floor (resp. ceiling) imposed on A_i's choice in all demand states $\theta \in [-\sigma, \underline{\theta}_i]$ (resp. $\theta \in \left[\overline{\theta}_i, \sigma\right]$), thereby increasing the profit margin on the infra-marginal consumers but reducing the volume of sales. Let

$$c^{\star} \triangleq \frac{\sigma\left(1-\gamma\right)}{1+\gamma\left(N-2\right)},$$

solving the above first-order conditions we can characterize the symmetric equilibrium of the game with non-cooperative behavior.

Proposition 3 *When principals behave non-cooperatively, the game has a unique symmetric equilibrium in which all principals choose the same permission set $\mathcal{P}^{\star}$ with the following features:*

- *If $c < c^\star$, then $\mathcal{P}^\star$ features a list price only — i.e.,*

$$\hat{p}(\sigma) > \overline{p}^\star \triangleq \hat{p}(\overline{\theta}^\star) > \underline{p}^\star \triangleq \hat{p}(-\sigma),$$

  with

$$\overline{\theta}^\star \triangleq \hat{p}^{-1}\left(\overline{p}^\star\right) = \sigma - 2c\frac{1+(N-2)\gamma}{1-\gamma}.$$

  $\overline{p}^\star$ and $\overline{\theta}^\star$ are decreasing in c, N and γ, and increasing in σ.

- *If $c \geq c^\star$, then $\mathcal{P}^\star$ is a singleton — i.e., $\overline{p}^\star = \underline{p}^\star = \hat{p}(-\sigma)$.*

This proposition shows that the equilibrium of the noncooperative game features partial delegation implemented through a list price. The reason is that, with full delegation, agents would always choose an excessive price compared to the price that informed principals would charge. Two contrasting forces shape the equilibrium list price. On the one hand, reducing agents' discretion is costly to principals because they would like the price to reflect demand conditions. On the other hand, granting discretion to the agents is equivalent to induce excessive prices and thus double marginalization, which harms principals. Figure 1 illustrates the equilibrium permission set: the horizontal blue line corresponds to the list price.

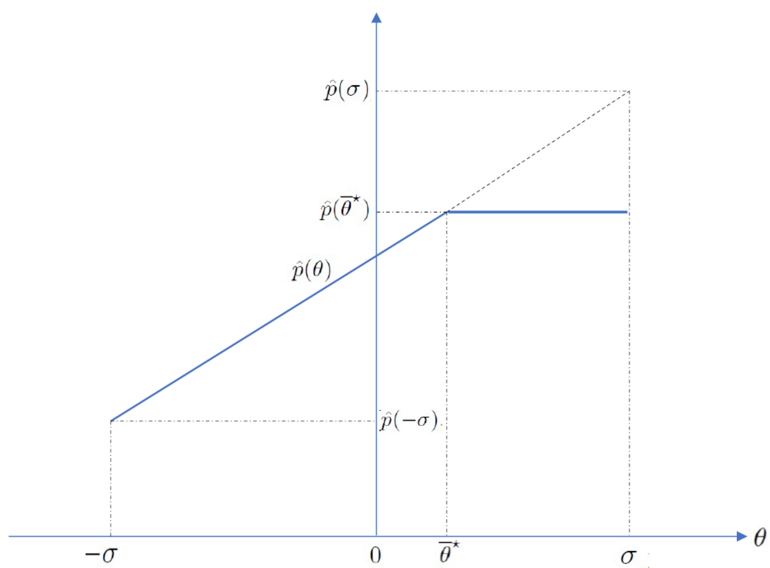

Figure 1: Equilibrium permission set in the non-cooperative regime

Notice that for every $\theta \leq \overline{\theta}^\star$ the difference $d(\theta) \triangleq \overline{p}^\star - \hat{p}(\theta)$ can be interpreted as the discounts off list prices. A more binding (lower) list price, can therefore be interpreted as a lower maximal discount that agents can offer consumers.

In order to understand how the model's parameters affect the amount of discretion granted to the agents in equilibrium, it is useful to compute the probability with which the (equilibrium) list price does not bind — i.e.,

$$\Pr[\theta \leq \overline{\theta}^{\star}] = \frac{\overline{\theta}^{\star} + \sigma}{2\sigma} = 1 - \frac{c}{c^{\star}}.$$

We can thus state the following:

Corollary 4 *In the noncooperative regime, the probability of price delegation increases with σ and decreases with N, γ and c.*

Hence, list prices are relatively less likely to impact final prices in less concentrated industries (higher N), when products are closer substitutes (higher γ) and if marketing costs are sufficiently high (higher c). By contrast, list prices are more likely to be binding in industries with low demand volatility. Fiercer competition, as reflected by an increase in the number of firms or by increased product substitutability, reduces the agents' ideal point, thereby exacerbating the conflict of interest within each firm. Instead, low demand volatility hinders price delegation: the lower uncertainty, the easier it is for principals to control agents' behavior with rigid price control. Of course, the higher the marketing cost, the more severe is the conflict of interest between principals and agents because this exacerbates the double marginalization effect.

Cooperative behavior. Consider now the case in which principals behave cooperatively. Hence, while agents still play non-cooperatively (they choose prices to maximize individual profits net of the marketing costs) the permission set that each of them faces maximizes expected industry profits — i.e.,

$$\int_{-\sigma}^{\sigma} \sum_{i=1}^{N} D_i (\cdot) \, p_i \frac{d\theta}{2\sigma}.$$

Accordingly, A_i's unrestricted problem is

$$\max_{p_i} D_i \left(\theta, p_i, p(\theta|\mathcal{P})\right) (p_i - c) \, ,$$

where $p(\theta|\mathcal{P})$ is the price that each agent expects its rivals to charge in state θ given the permission set $\mathcal{P}$. The solution of this problem yields the same best-reply function as before — i.e.,

$$p_i \left(\theta, p(\theta|\mathcal{P})\right) \triangleq \frac{c}{2} + \frac{(1-\gamma)(1+\theta) + \gamma (N-1) \, p(\theta|\mathcal{P})}{2 \left(1 + (N-2) \gamma\right)} \qquad \forall \theta \in \Theta.$$

The prevailing price in every state θ where agents' choices are not restricted by $\mathcal{P}$ is again $\hat{p}(\theta)$.

To gain insights on the structure of the optimal permission set in the cooperative regime, it is useful to construct a measure of the agents' bias

$$b^{\star\star}(\theta) \triangleq \hat{p}(\theta) - p^{M}(\theta) = b^{\star} - \Delta p(\theta) \, .$$

This bias reflects two opposite forces. On the one hand, since agents have an incentive to mark up their cost, there is an incentive to price above the monopoly price, which calls for the introduction of an endogenous list price as seen in the non-cooperative regime. On the other hand, however, when principals cooperate they would like to increase prices above the non-cooperative level to soften competition. Hence, other things being equal, agents set too low price compared to what principals would prefer, thereby creating scope for the introduction of a price floor. This second effect is zero when products are sufficiently independent — i.e., γ is close to zero — and becomes stronger as products become closer substitutes. Moreover, this effect depends on the state of nature θ: the higher consumers' willingness to pay, the higher the wedge between the monopoly and the noncooperative price. Hence, for higher values of θ agents are biased towards excessively low prices in the cooperative regime.

Lemma 5 *The bias $b^{\star\star}(\theta)$ is decreasing in θ. Moreover, there exist two thresholds $\underline{c}$ and $\overline{c}$, with*

$$\underline{c} \triangleq \frac{\gamma(N-1)(1-\sigma)}{2(1+(N-2)\gamma)} < \overline{c} \triangleq \frac{\gamma(N-1)(1+\sigma)}{2(1+(N-2)\gamma)},$$

such that

- *If $c \leq \underline{c}$, then $\hat{p}(\theta) \leq p^M(\theta)$ for every $\theta \in \Theta$;*
- *If $c \in (\underline{c}, \overline{c})$, there exits a unique demand state*

$$\hat{\theta} \triangleq 2c\frac{1+(N-2)\gamma}{(N-1)\gamma} - 1 \in (-\sigma, \sigma),$$

  such that $\hat{p}(\theta) \geq p^M(\theta)$ if and only if $\theta \leq \hat{\theta}$;
- *If $c \geq \overline{c}$, then $\hat{p}(\theta) \geq p^M(\theta)$ for every $\theta \in \Theta$.*

This result shows that the structure of the optimal permission set in the cooperative regime will be different than in the non-cooperative regime. The reason is as follows. When c is sufficiently low, if agents can choose prices without restrictions, they would prefer a too low price compared to what principals would like. Hence, the optimal permission set features a price floor and potentially converges to a singleton with a fixed price (as we will argue below). By contrast, when c is sufficiently large, the agents' ideal point exceeds the principals' one, so that the optimal permission set may still feature a list price. Clearly, when c takes intermediate values, since $b^{\star\star}(\theta)$ is decreasing in θ, agents would like to price below the principals' ideal point when θ is sufficiently large, and above otherwise.

Since $\hat{p}(\theta)$ is increasing in θ and agents face the same permission set, for every non-empty $\mathcal{P}$, we can define $\overline{\theta} \triangleq \hat{p}^{-1}(\overline{p}) > \underline{\theta} \triangleq \hat{p}^{-1}(\underline{p})$. Hence, in the cooperative regime, the equilibrium price schedule induced by $\mathcal{P}$ must be such that

$$p(\theta|\mathcal{P}) \triangleq \begin{cases} \overline{p} & \text{if } \theta \geq \overline{\theta} \\ \hat{p}(\theta) & \text{if } \theta \in [\underline{\theta}, \overline{\theta}] \\ \underline{p} & \text{if } \theta \leq \underline{\theta} \end{cases}.$$

Assuming interior solutions — i.e., $\mathcal{P} \subset [\hat{p}(-\sigma), \hat{p}(\sigma)]$ — which we will check ex post, and operating a simple change of variables to optimize over $\underline{\theta}$ and $\overline{\theta}$ rather than $\underline{p}$ and $\overline{p}$, the maximization problem solved by the principals' coalition is

$$\max_{(\underline{\theta},\overline{\theta})\in\Theta^2} \sum_{i=1}^{N} \int_{-\sigma}^{\sigma} D_i\left(\theta, p\left(\theta|\mathcal{P}\right), p\left(\theta|\mathcal{P}\right)\right) p\left(\theta|\mathcal{P}\right) \frac{d\theta}{2\sigma}, \tag{9}$$

subject to $\overline{\theta} \geq \underline{\theta}$, where

$$\int_{-\sigma}^{\sigma} D_i\left(\theta, p\left(\theta|\mathcal{P}\right), p\left(\theta|\mathcal{P}\right)\right) p\left(\theta|\mathcal{P}\right) \frac{d\theta}{2\sigma} \triangleq \underbrace{\int_{-\sigma}^{\underline{\theta}} D_i\left(\theta, \hat{p}(\underline{\theta}), \hat{p}(\underline{\theta})\right) \hat{p}(\underline{\theta}) \frac{d\theta}{2\sigma}}_{\text{Binding floor}} +$$
$$\underbrace{\int_{\underline{\theta}}^{\overline{\theta}} D_i\left(\theta, \hat{p}(\theta), \hat{p}(\theta)\right) \hat{p}(\theta) \frac{d\theta}{2\sigma}}_{\text{Agents are delegated pricing authority}} + \underbrace{\int_{\overline{\theta}}^{\sigma} D_i\left(\theta, \hat{p}(\overline{\theta}), \hat{p}(\overline{\theta})\right) \hat{p}(\overline{\theta}) \frac{d\theta}{2\sigma}}_{\text{Binding list price}}.$$

We shall neglect and verify ex post the constraint $\overline{\theta} \geq \underline{\theta}$. Let $\overline{\lambda}$ (resp. $\underline{\lambda}$) the multiplier associated to the constraint $\overline{\theta} \leq \sigma$ (resp. $\underline{\theta} \geq -\sigma$). Differentiating with respect to $\underline{\theta}$ and $\overline{\theta}$, respectively, the optimal permission set in the cooperative regime, hereafter $\mathcal{P}^{\star\star} \triangleq [\underline{p}^{\star\star}, \overline{p}^{\star\star}]$, solves the following first-order conditions

$$\int_{\theta \leq \underline{\theta}^{\star\star}} \left[\left(\frac{\partial D_i(\theta, \underline{p}^{\star\star}, \underline{p}^{\star\star})}{\partial p_i} + \sum_{j \neq i} \frac{\partial D_i(\theta, \underline{p}^{\star\star}, \underline{p}^{\star\star})}{\partial p_j} \right) \underline{p}^{\star\star} + D_i(\theta, \underline{p}^{\star\star}, \underline{p}^{\star\star}) \right] \frac{d\theta}{2\sigma} = \overline{\lambda}^{\star\star}, \tag{10}$$

$$\int_{\theta \geq \overline{\theta}^{\star\star}} \left[\left(\frac{\partial D_i(\theta, \overline{p}^{\star\star}, \overline{p}^{\star\star})}{\partial p_i} + \sum_{j \neq i} \frac{\partial D_i(\theta, \overline{p}^{\star\star}, \overline{p}^{\star\star})}{\partial p_j} \right) \overline{p}^{\star\star} + D_i(\theta, \overline{p}^{\star\star}, \overline{p}^{\star\star}) \right] \frac{d\theta}{2\sigma} = -\overline{\lambda}^{\star\star}, \tag{11}$$

with

$$\overline{p}^{\star\star} \triangleq \hat{p}(\overline{\theta}^{\star\star}) \geq \underline{p}^{\star\star} \triangleq \hat{p}(\underline{\theta}^{\star\star}),$$

and associated complementary slackness conditions

$$\underline{\lambda}^{\star\star} \geq 0, \quad \underline{\lambda}^{\star\star}\left[\underline{\theta}^{\star\star} + \sigma\right] = 0,$$

$$\overline{\lambda}^{\star\star} \geq 0, \quad \overline{\lambda}^{\star\star}[\overline{\theta}^{\star\star} - \sigma] = 0.$$

The intuition for these first-order conditions is similar to what we have seen in the non-cooperative regime, the only difference being that here the coalition of principals internalizes the impact of each product's price on the demand of the competing products. Hence, there is (in expected terms) the same competition softening effect discussed in equation (5).

Solving the first-order conditions above, we can finally state the main result of the section.

Proposition 6 *The permission set $\mathcal{P}^{\star\star}$ that maximizes industry profit has the following features:*

- *Suppose that $N \leq \overline{N} \triangleq \frac{2-\gamma}{\gamma}$ and $\gamma \leq \frac{2}{3}$ (so that $\overline{N} > 2$). Then, there exist two thresholds $\underline{c}^-$ and $\overline{c}^+$, with*

$$\underline{c}^- \triangleq \underline{c} - \frac{\gamma\sigma\left(\overline{N}-N\right)}{2\left(1+\gamma\left(N-2\right)\right)} < \overline{c}^+ \triangleq \overline{c} + \frac{\gamma\sigma\left(\overline{N}-N\right)}{2\left(1+\gamma\left(N-2\right)\right)},$$

such that:

  - *If $c \leq \underline{c}^-$, $\mathcal{P}^{\star\star}$ is a singleton — i.e., $\overline{p}^{\star\star} = \underline{p}^{\star\star} = \hat{p}\left(\sigma\right)$.*
  - *If $c \in \left(\underline{c}^-, \underline{c}\right)$, $\mathcal{P}^{\star\star}$ features only a binding price floor — i.e.,*

$$\overline{p}^{\star\star} \triangleq \hat{p}\left(\sigma\right) > \underline{p}^{\star\star} \triangleq \hat{p}\left(\theta_F^{\star\star}\right) > \hat{p}\left(\sigma\right),$$

with

$$\theta_F^{\star\star} \triangleq -\sigma + \frac{4\left(1+\gamma\left(N-2\right)\right)}{\gamma\left(\overline{N}-N\right)}\left(\underline{c}-c\right) \in \left(-\sigma,\sigma\right).$$

$\underline{p}^{\star\star}$ and $\theta_F^{\star\star}$ are decreasing in σ and c and increasing in γ and N.

  - *If $c \in \left[\underline{c}, \overline{c}\right]$, $\mathcal{P}^{\star\star}$ features full delegation — i.e., $\mathcal{P}^{\star\star} = \left[\hat{p}\left(-\sigma\right), \hat{p}\left(\sigma\right)\right]$.*
  - *If $c \in \left(\overline{c}, \overline{c}^+\right)$, $\mathcal{P}^{\star\star}$ features only a binding list price — i.e.,*

$$\underline{p}^{\star\star} \triangleq \hat{p}\left(-\sigma\right) < \overline{p}^{\star\star} \triangleq \hat{p}(\theta_L^{\star\star}) < \hat{p}\left(\sigma\right),$$

with

$$\theta_L^{\star\star} \triangleq \sigma - \frac{4\left(1+\gamma\left(N-2\right)\right)}{\gamma\left(\overline{N}-N\right)}\left(c-\overline{c}\right) \in \left(-\sigma,\sigma\right).$$

$\overline{p}^{\star\star}$ and $\theta_L^{\star\star}$ are decreasing in N, γ and c and increasing in σ.

  - *If $c \geq \overline{c}^+$, $\mathcal{P}^{\star\star}$ is a singleton — i.e., $\overline{p}^{\star\star} = \underline{p}^{\star\star} = \hat{p}\left(-\sigma\right)$.*

- *Suppose that $N > \overline{N}$:*

  - *If $c < \underline{c}$, $\mathcal{P}^{\star\star}$ is a singleton — i.e., $\overline{p}^{\star\star} = \underline{p}^{\star\star} = \hat{p}\left(\sigma\right)$.*
  - *If $c \in \left[\underline{c}, \overline{c}\right]$, $\mathcal{P}^{\star\star}$ features full delegation — i.e., $\mathcal{P}^{\star\star} = \left[\hat{p}\left(-\sigma\right), \hat{p}\left(\sigma\right)\right]$.*
  - *If $c > \overline{c}$, $\mathcal{P}^{\star\star}$ is a singleton — i.e., $\overline{p}^{\star\star} = \underline{p}^{\star\star} = \hat{p}\left(-\sigma\right)$*

The optimal permission set in the cooperative regime changes considerably compared to the non-cooperative outcome. Interestingly, while a non-cooperative behavior by the principals always requires partial delegation or no delegation at all, full price delegation is an equilibrium outcome of the cooperative version of the game when marketing costs are neither too low nor too high, so that industry profit maximization does not require the imposition of binding list prices nor a binding price floor.

When the number of firms is not too high ($N \leq \overline{N}$) and products are not too close substitutes ($\gamma \leq \frac{2}{3}$), the equilibrium may still features forms of partial delegation. A price floor is imposed for medium-low values of c, a list price only occurs when this cost takes medium-high values. The intuition for these properties follows directly from the discussion of Lemma 5. When c is sufficiently low, agents would like to price competitively — i.e., close to $p^N(\theta)$, while while joint-profit maximization mandates the monopoly price $p^M(\theta)$. Hence, in contrast to the non-cooperative benchmark, agents are biased towards excessively low prices. This explains why in this region of parameters the optimal permission set features either a price floor or no delegation at all. By contrast, for large values of c, agents are upward biased as in the non-cooperative benchmark. Therefore, a list price is still optimal in this region of parameters. Of course, the conflict of interest tends to weaken for intermediate values of the marketing cost since, in this case, the two forces illustrated above tend to balance out.

Figure 2 illustrates the case in which the optimal permission set features a floor (the case with a list price is analogous to Figure 1).

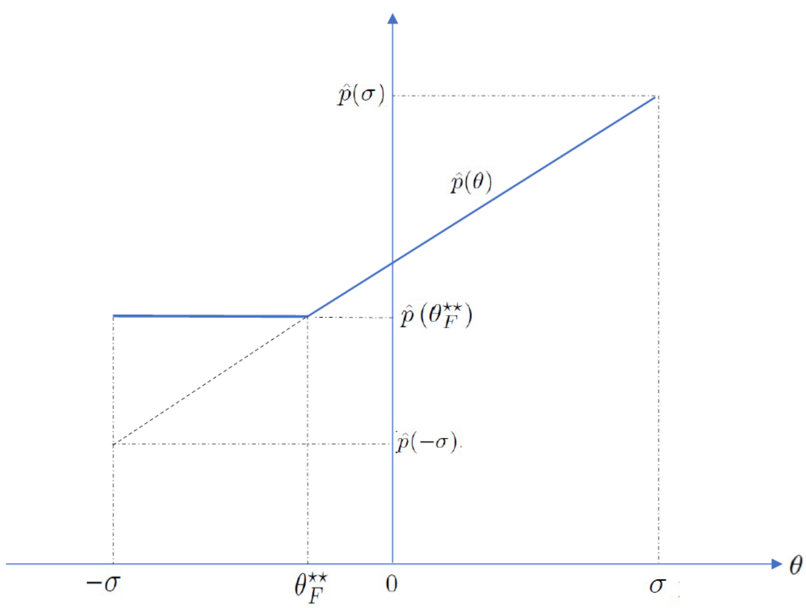

Figure 2: Price-floor in the cooperative regime

By contrast, when the number of rivals in the market is sufficiently high — i.e., $N > \overline{N}$ — the cooperative equilibrium either always features full delegation or not delegation at all. This is because competition exacerbates the conflict of interest between principals and agents when agents would like to price below the monopoly price (as reflected by the fact that $\underline{c}^- > \underline{c}$) and softens the conflict of interest when they would like to price above the monopoly price (as reflected by the fact that $\underline{c}^{+} < \overline{c}$). Formally, this is implied by

$$\frac{\partial b^{\star\star}(\theta)}{\partial N} = -\frac{\gamma(1-\gamma)(1+\theta-c)}{(2+\gamma(N-3))^2} < 0 \quad \forall \theta \in \Theta,$$

so that when $b^{\star\star}(\theta)$ is positive it falls with N, but when it is negative it increases in absolute value with N. Therefore, in this region of parameters, principals are more willing to delegate for higher values of c and less willing to delegate for lower values of c.

We can now turn to examine how the likelihood of delegation responds to changes in the model's parameters. As before, under the assumption that θ is uniformly distributed, the probability that an agent is allowed to pick its ideal point and the width of the range of θ in which this occurs are two equivalent metrics of price delegation. Let us focus on the most interesting region of parameters where the number of firms is not too high ($N \leq \overline{N}$) and products are not too close substitutes ($\gamma \leq \frac{2}{3}$), so that partial delegation is a possible equilibrium outcome of the game. It is immediate to verify that

$$\Pr\left[\theta \geq \theta_F^{\star\star}\right] = 1 - \frac{2(1+\gamma(N-2))}{\sigma\gamma\left(\overline{N}-N\right)}(\underline{c}-c),$$

and

$$\Pr\left[\theta \leq \theta_L^{\star\star}\right] = 1 - \frac{2(1+\gamma(N-2))}{\sigma\gamma\left(\overline{N}-N\right)}(c-\overline{c}).$$

The following holds:

Corollary 7 *Suppose that $N \leq \overline{N}$ and $\gamma \leq \frac{2}{3}$. In the cooperative regime, the likelihood of (partial) delegation is:*

- *Increasing in N, γ and σ, and decreasing in c when a list price is imposed — i.e., for $c \in (\overline{c}, \overline{c}^{+})$;*
- *Increasing in σ and c, and decreasing in N and γ when a price floor is imposed — i.e., for $c \in (\underline{c}^{-}, \underline{c})$.*

The intuition for the above comparative statics is the same as in the noncooperative case when a list price is imposed at equilibrium. By contrast, when it is optimal to impose a floor, the likelihood of delegation decreases if competition intensifies (as reflected by a higher number of firms in the market and/or greater product substitutability) and increases when demand is more uncertain, and the marketing cost grows large. The reason is as follows. First, fiercer competition reduces the agents' ideal point, thereby exacerbating the conflict of interest with principals when the monopoly price lies above the agents' ideal point, which drops as N and/or γ rise. Second, an increase in the marketing cost tends to align incentives because agents' bias towards higher prices increases, moving their ideal point towards the (higher) monopoly price. Finally, as in the noncooperative regime, high demand uncertainty tends to align incentives even with upstream coordination: when uncertainty diminishes, principals can control more easily agents' behavior via rigid price control.

Another interesting comparative statics exercise is to study how the model's parameters affect the region of parameters in which full delegation occurs — i.e., the difference

$$\Delta c \triangleq \overline{c} - \underline{c} = \frac{\sigma (N-1) \gamma}{1 + (N-2) \gamma}.$$

Differentiating Δc with respect to σ, γ and N, we have:

Corollary 8 *The range of parameters in which full delegation occurs expands as σ, γ and N increase.*

Hence, in sharp contrast with results obtained in the noncooperative regime, not only the cooperative regime features an equilibrium with full delegation, but it also turns out that the region of parameters in which this equilibrium exists expands with the intensity of competition.

3 Extensions

Building on the insights gained above, in this section, we extend the baseline model to account for some potentially important aspects that have been neglected so far. The objective is to examine circumstances in which our results are robust and when they are, instead, reverted.

3.1 Intra-brand competition

Until now, we focused on the traditional competing-organizations framework (see. e.g., Martimort, 1996), where every principal deal with one exclusive sales manager. We have, therefore, neglected intra-brand competition. How would our results change when introducing both intra- and inter-brand competition?

Suppose (for simplicity) that there are only two principals each dealing with a pair of sales mangers, so that there are two competing distribution networks in the market. Let the two principals be denoted by $k = 1, 2$ and denote by $i = 1, 2$ the agents within each distribution network. The pair (i, k) then indicates agent i dealing with principal k. Consider a representative consumer with the following modified version of the Singh-Vives (1984) utility function

$$U(\cdot) \triangleq (1+\theta) \sum_{k=1,2} \sum_{i=1,2} q_{i,k} - \frac{1}{2} \sum_{k=1,2} \sum_{i=1,2} q_{i,k}^2 - \mu \sum_{k=1,2} q_{1,k} q_{2,k} + \\ - \gamma \sum_{i=1,2} q_{i,1} \sum_{i=1,2} q_{i,2} - \sum_{k=1,2} \sum_{i=1,2} q_{i,k} p_{i,k} + I,$$

where, as before, $I > 0$ is the representative consumer's income. The parameter $\mu \in [0, 1)$ is a measure of *intra-brand competition*: the larger μ, the more homogeneous (less differentiated) the products distributed by the agents within each distribution network. The parameter $\gamma \in [0, \mu]$ is a measure of the degree of *inter-brand competition*: the larger γ, the more homogeneous the products distributed by the two principals. Differentiating with respect to quantities and inverting the resulting first-order conditions, we obtain the following system of demand functions

$$D_{i,k}(\cdot) = \frac{(1+\theta)(1-\mu)(1+\mu-2\gamma) - \left(1+\mu-2\gamma^2\right)p_{1,1} + \left(\mu(1+\mu)-2\gamma^2\right)p_{2,1} + (1-\mu)\gamma\sum_{i=1,2} p_{i,-k}}{(1-\mu)\left((1+\mu)^2-4\gamma^2\right)},$$

for every $i, k = 1, 2$.

Focusing on a symmetric equilibrium (which is also unique under the above demand function), in the non-cooperative regime where θ is common knowledge, each principal solves the following maximization problem

$$\max_{p_{1,k}, p_{2,k}} \sum_{i=1,2} D_{i,k}(\cdot)\, p_{i,k},$$

so that the equilibrium price is

$$p^N(\theta) \triangleq \frac{(1+\theta)(1+\mu-2\gamma)}{2(1+\mu-\gamma)} \qquad \forall\theta.$$

By contrast, for given principal $k = 1, 2$, each sales manager $i = 1, 2$ solves

$$\max_{p_{i,k}} D_{i,k}(\cdot)\, p_{i,k},$$

so that, the agents' equilibrium ideal price is

$$\hat{p}(\theta) \triangleq p^N(\theta) + \frac{1+\mu}{2(1+\mu-\gamma)}c \qquad \forall\theta.$$

Hence, as intuition suggests, sales managers still have an incentive to price above the principals' non-cooperative ideal price, so that the non-cooperative regime will feature again partial delegation implemented by list prices. The agents' bias, however, now depends on both the degree of intra- and inter-brand competition — i.e.,

$$b^\star \triangleq \frac{1+\mu}{2(1+\mu-\gamma)}c,$$

which is increasing in γ and decreasing in μ. Hence, while inter-brand competition exacerbates the conflict of interest between principals and agents — i.e., $b^\star$ is increasing in γ — intra-brand competition tends to align downstream and upstream incentives — i.e., $b^\star$ is decreasing in μ. As a result, the equilibrium list price will be less binding in industries featuring stronger intra-brand competition.

Finally, it can be shown that the cooperative equilibrium still features the monopoly price $p^M(\theta) = \frac{1+\theta}{2}$. Hence, the bias in this regime is measured by the following difference

$$\hat{p}(\theta) - p^M(\theta) = \frac{1}{2(1+\mu-\gamma)} \times [1+\mu-\gamma(1+\theta)c] \geq 0 \quad \Leftrightarrow \quad \theta \leq \frac{1+\mu}{c\gamma} - 1.$$

This condition implies that the structure of the optimal permission set in the cooperative regime has the same qualitative properties than in the baseline. Notice, however, that the above condition becomes easier to satisfy for higher values of μ and harder to satisfy for higher values of γ. Hence, while stronger inter-brand competition (higher γ) makes it more likely that sales managers price above the monopoly price, thereby calling for a price cap, more intense intra-brand competition (higher μ) tends to induce sales managers to price below the monopoly price, which instead calls for a price floor. The intuition is as follows. As seen in the baseline model, when inter-brand competition becomes more intense, sales managers tend to pass on to a greater extent their marketing cost to consumers, hence there they tend to charge excessively high prices to consumers. On the contrary, when intra-brand competition becomes more intense, sales managers' bias drops because as μ rises the demand function of each agent becomes more responsive to the own price, thereby mitigating the incentive to pass on costs to consumers.

3.2 Alternative demand specification

To check robustness, we now consider an alternative specifications for the demand function.

Shubick-Levitan preferences. Assume for simplicity that each principal deals with an exclusive agent as in the baseline model. Consider a demand system determined by the standard Shubik-Levitan quadratic utility function (see, e.g., Motta, 2004, Ch. 8.4.2.)

$$U(\cdot) \triangleq (1+\theta)\sum_{i=1}^{N} q_i - \frac{1}{2}\left(\sum_{i=1}^{N} q_i\right)^2 - \frac{N}{2(1+\gamma)}\left[\sum_{i=1}^{N} q_i^2 - \frac{1}{N}\left(\sum_{i=1}^{N} q_i\right)^2\right] - \sum_{i=1}^{N} q_i p_i.$$

The parameter $\gamma \geq 0$ captures the degree of differentiation between products: the larger γ, the less differentiated products are. Differentiating with respect to quantities and inverting the system of first-order conditions we have

$$D_i(\cdot) = \frac{1}{N}\left[1+\theta-p_i-\gamma\left(p_i - \frac{\sum_{j=1}^{N} p_j}{N}\right)\right] \qquad \forall i = 1,..,N.$$

Under this demand specification, principals' ideal point in the non-cooperative regime is

$$p^N(\theta) \triangleq \frac{N(1+\theta)}{2N+(N-1)\gamma} \qquad \forall\theta,$$

while in an unrestricted equilibrium agents would set

$$\hat{p}(\theta) \triangleq p^N(\theta) + \underbrace{\frac{N+(N-1)\gamma}{2N+(N-1)\gamma}c}_{\text{Agents' bias}} \qquad \forall\theta.$$

As intuition suggests, even under this alternative demand specification agents feature a positive bias toward prices higher than the principals' ideal point — i.e.,

$$b^{\star} = \frac{N + (N-1)\gamma}{2N + (N-1)\gamma} c.$$

This bias is increasing in N and γ as in the baseline model. Hence, the results of the baseline model are robust to alternative specifications for the demand function. The same holds true in the cooperative regime since the monopoly price is still $p^M(\theta) = \frac{1+\theta}{2}$.

CES preferences. Above we have considered only linear demand functions and assumed that sales managers were privately informed about consumers' willingness to pay. We now study show that our results are robust to the case of non-linear demand functions where sales managers are privately informed about the degree of substitution between products. To this purpose, consider a system of demand functions obtained by CES preferences (Dixit and Stiglitz, 1977)

$$D_i(\cdot) \triangleq \frac{p_i^{-\theta-1}}{\sum_{j=1}^{N} p_j^{-\theta}} \qquad \forall i = 1, ..., N. \tag{12}$$

The parameter $\theta \in [0, 1]$ represents a measure of product substitutability: the larger θ the closer substitutes products are. Suppose that sales managers are informed about θ and assume that c_0 is the principals' production cost, while c is the agents marketing cost.[3]

It is easy to show that when principals are informed about θ, their ideal non-cooperative price is

$$p^N(\theta) \triangleq c_0 \frac{N + (N-1)\theta}{\theta(N-1)} \qquad \forall\theta$$

whereas sales managers would rather choose

$$\hat{p}(\theta) \triangleq p^N(\theta) + \underbrace{c\frac{N + (N-1)\theta}{\theta(N-1)}}_{\text{Agents' bias}} \qquad \forall\theta.$$

In line with the baseline model, agents are biased towards excessively high prices in the non-cooperative equilibrium. Hence, principals will still find it optimal to discipline their behavior through the imposition of a price cap. Notice that the agents' bias decreases in θ, meaning that, in the spirit of the baseline model, imposing a price cap is more valuable when profit margins are high — i.e., when products are relatively more differentiated (low values of θ).

3.3 Quantity competition

Would the results change if competition is in quantity rather than prices? Suppose that sales managers decide how much to distribute instead of setting prices. Consider, accordingly, the following linear (inverse)

[3] We introduce c_0 in order to avoid that in the non cooperative equilibrium principals choose $p^N(\theta) = 0$.

demand function

$$P(Q) \triangleq \max\{0, A + \theta - \gamma Q\},$$

where $Q \triangleq \sum_{i=1}^{N} q_i$ denotes aggregate output, and γ is a measure of the responsiveness of the inverse demand to aggregate output. A is the fixed demand intercept while θ is a random shifter as before. With Cournot competition, a direct way of approaching the problem is to consider a permission set that specifies how much each sales manager is entitled to distribute — i.e., every principal i chooses a permission set $\mathcal{Q}_i \triangleq [\underline{q}_i, \overline{q}_i]$, with $\overline{q}_i \geq \underline{q}_i$. Following the logic developed above, it is easy to show that, in the non-cooperative equilibrium, each principal would like to sell

$$q^N(\theta) \triangleq \frac{A + \theta}{\gamma(N+1)} \qquad \forall\theta,$$

while agents' would rather distribute

$$\hat{q}(\theta) \triangleq q^N(\theta) - \frac{c}{\gamma(N+1)} \qquad \forall\theta.$$

Hence, with Cournot competition, agents have an incentive to distribute less than what principals would like to sell. As a result, the optimal permission set will feature a minimal output requirement (i.e., a floor on the amount of output that agents must distribute), which will be less binding as N and γ rise — i.e., in more competitive environments. Interestingly, principals can implement this type of output restrictions through vertical price control. They can indeed impose a price cap above which agents cannot sell, which de facto implements a minimal output requirement. This cap will have the same features of the baseline model since upstream and downstream preferences align when N and γ rise and diverge as c rises.

As for the cooperative regime, notice that the monopoly solution implies $Q^M(\theta) \triangleq \frac{A+\theta}{2\gamma}$ with each agent distributing $q^M(\theta) \triangleq \frac{1}{N} Q^M(\theta)$. The bias is then measured by the following difference

$$\hat{q}(\theta) - q^M(\theta) = \frac{1}{\gamma(N+1)} \times \left[\frac{(N-1)(A+\theta)}{2N} - c\right] \geq 0 \quad \Leftrightarrow \quad c \leq \frac{(N-1)(A+\theta)}{2N}.$$

In line with the case of price competition, whether agents over- or under-distribute compared to the monopoly solution depends on the model's parameters. Specifically they tend to under-produce when the marketing cost is sufficiently small, while they tend to over-produce when this cost is large enough. This implies that, when c is relatively large, the cooperative solution entails a cap on the maximal quantity that sales mangers can distribute (or, alternatively, a price floor) and and minimal output requirement (or, alternatively, a list price) when c is relatively small.

3.4 A remark on common agency

Finally, following the vertical contracting literature studying the costs and benefits associated with centralization of the distribution function (see, e.g., Bernheim and Whinston, 1986-1990), it is interesting to provide a preliminary exploration of how upstream and downstream incentives align in a common agency setting where principals delegate distribution and pricing authority to a common agent. To this purpose, consider a framework in which the N principals introduced in the baseline model rely on a common sales manager to distribute their products. Consider the demand system (2) and, for simplicity, assume that the common agent either contracts with all principals or with none — i.e., an 'intrinsic common agency' game (see, e.g., Martimort, 1996, Martimort and Stole, 2009, Piccolo et al., 2016, among others). In this setting, principals' non-cooperative ideal point remains $p^N(\theta)$ in equation (4). The common agent, instead, solves the following maximization problem

$$\max_{p_1,p_2} \sum\nolimits_{i=1}^{N} D_i(\cdot)(p_i - c),$$

whose first-order conditions are

$$D_i(\cdot) + \frac{\partial D_i(\cdot)}{\partial p_i}(p_i - c) + \sum_{j \neq i} \frac{D_j(\cdot)}{\partial p_i}(p_j - c) = 0 \qquad \forall i = 1,..,N.$$

In a symmetric (unrestricted) equilibrium we have

$$\hat{p}^{CA}(\theta) = p^M(\theta) + \frac{c}{2} \qquad \forall \theta \in \Theta.$$

The conflict of interest between principals and agents is, therefore, measured by the following difference

$$\hat{p}^{CA}(\theta) - p^N(\theta) = \frac{\gamma(N-1)(1+\theta)}{2((N-3)\gamma + 2)} + \frac{c}{2} > 0 \qquad \forall \theta \in \Theta.$$

This expression is always positive. As a result, in the non-cooperative scenario, the common agent has an incentive to price above what principals would like. This is for two reasons. First, the common agent is biased towards excessive prices because it will pass on the marketing costs to the final consumers. Second, since the common agent distributes all products, it internalizes the positive price-externality created by each product on the others, and therefore tends to increase all prices relative to what principals would like if they were informed about θ and would act non-cooperatively. This means, once again, that even under common agency the equilibrium permission set will feature a price cap. Since the above difference is increasing in N and γ, in contrast with the exclusivity case, the list price however becomes relatively more binding in more competitive environments. This is because, when the number of products in the market grows or the existing products become closer substitutes, the common agent as an incentive to increase prices in order to soften competition between its principals.

Finally, it can also be easily shown that in the cooperative regime it is still the case that the common agent wants to price above the monopoly price. In this case, however, the bias towards excessive prices is fully driven by the presence of its marketing costs. Hence, in contrast to the case of competing hierarchies, the optimal permission set either features partial delegation with a price cap or no delegation at all.

The analysis becomes relatively more complex in a 'delegated common agency' game — i.e., the scenario in which the common sales manager can decide not to distribute some of the principals' products. As intuition suggests, this feature creates an additional dimension of competition between principals who must also ensure their products are not dropped. The agent will, indeed, tend to favor principals that offer delegation schemes that are relatively more aligned with its preferences — i.e., schemes that award more price discretion. As a result, the price cap will be less binding, and the equilibrium will feature more price discretion than in an intrinsic common agency game. We plan to explore this issue in future research.

4 Take-away and conclusions

Our findings offer new insights regarding how pricing authority should be delegated within competing organizations. We have shown that the instruments through which pricing decisions are delegated to sales managers depend on the industry competitive conduct, the severity of the conflict of interest within firms, the strength of competition, and the volatility of demand conditions. The paper takeaways can be summarized as follows.

First, the analysis suggests that the emergence of list prices implement the equilibrium delegation form in a noncooperative environment, reflecting principals' genuine need to disciplining the sales managers' incentive to pass on their marketing costs to consumers. Hence, the presence of list prices is, in principle, not a symptom of consumer harm compared to instances where list prices are not imposed at all.

Second, higher price dispersion should be associated with higher demand uncertainty, higher market concentration, and more significant product differentiation.

Third, our results also suggest that delegation becomes a considerably more complex phenomenon with upstream coordination relative to a non-cooperative setting. In particular, when marketing costs are sufficiently low industry profit maximization requires a price floor rather than a list price. In addition, and perhaps more surprisingly, for an intermediate value of these costs, upstream coordination may well require principals to grant their sales managers full pricing discretion, while the opposite never occurs in a non-cooperative equilibrium.

Fourth, contrary to what intuition would suggest, while increased competition hinders delegation when principals behave non-cooperatively, it typically facilitates managerial discretion in an environment where principals cooperate even though managers do not internalize this objective.

All these predictions are empirically testable and are robust to alternative specifications of the demand functions, to the introduction of intra-brand competition and alternative types of competition and industry structure.

Appendix

Proof of Lemma 1. Solving (6) for a symmetric equilibrium we immediately have $\hat{p}(\theta)$. ■

Proof of Lemma 2. Showing that $p_i(\theta, p^\star(\theta))$ is increasing in θ is immediate since $\hat{p}(\theta)$ is increasing in θ and

$$\frac{\partial p_i(\theta, p^\star(\theta))}{\partial\theta} = \frac{1-\gamma}{2(1+\gamma(N-2))} > 0.$$

Hence, for any given $\mathcal{P}_i$ the constrained pricing rule for A_i has a binding cap for $\theta \geq \overline{\theta}_i$ and a binding floor for $\theta \leq \underline{\theta}_i$. Finally, $\overline{\theta}_i \geq \underline{\theta}_i$ follows immediately from the hypothesis $\overline{p}_i \geq \underline{p}_i$. ■

Proof of Proposition 3. We first show that if a symmetric equilibrium exists, then it either features a list price or no delegation at all — i.e., there is no symmetric equilibrium with full delegation occurs or in which a price floor is imposed. Suppose that a symmetric equilibrium exists and that it features interior solutions — i.e., $\underline{\lambda}^\star = \overline{\lambda}^\star = 0$. Solving the first-order conditions (7) and (8) we have

$$\overline{\theta}^\star = \sigma - 2c\frac{1+(N-2)\gamma}{1-\gamma},$$

and

$$\underline{\theta}^\star = -\sigma - 2c\frac{1+(N-2)\gamma}{1-\gamma}.$$

It is immediate to show that $\underline{\theta}^\star < -\sigma$ and $\overline{\theta}^\star > -\sigma$ if and only if $c < c^\star$. Hence, if a symmetric equilibrium exists, it either features a list price — i.e., for $c < c^\star$ — with $\overline{p}^\star = \hat{p}(\overline{\theta}^\star)$ and $\underline{p}^\star = \hat{p}(-\sigma)$, or a fixed price — i.e., for $c \geq c^\star$ — with $\overline{p}^\star = \underline{p}^\star = \hat{p}(-\sigma)$.

We now check deviations — i.e., we show that given that its $N-1$ rivals set $\mathcal{P}^\star$, principal P_i cannot profit by choosing a permission set $\mathcal{P}_i \neq \mathcal{P}^\star$. First, suppose that $c \geq c^\star$ so that $\overline{p}^\star = \underline{p}^\star = \hat{p}(-\sigma)$. Then, A_i's best reply is

$$p_i(\theta, \hat{p}(-\sigma)) \triangleq \frac{c}{2} + \frac{(1-\gamma)(1+\theta) + \gamma(N-1)\hat{p}(-\sigma)}{2(1+\gamma(N-2))}.$$

P_i then solves

$$\max_{(\underline{\theta}_i, \overline{\theta}_i)\in\Theta} \int_{-\sigma}^{\underline{\theta}_i} D_i(\theta, p_i(\underline{\theta}_i, \hat{p}(-\sigma)), \hat{p}(-\sigma))\, p_i(\underline{\theta}_i, \hat{p}(-\sigma))\frac{d\theta}{2\sigma}$$
$$+ \int_{\underline{\theta}_i}^{\overline{\theta}_i} D_i(\theta, p_i(\theta, \hat{p}(-\sigma)), \hat{p}(-\sigma))\, \hat{p}(\theta)\frac{d\theta}{2\sigma}$$
$$+ \int_{\overline{\theta}_i}^{\sigma} D_i\left(\theta, p_i\left(\overline{\theta}_i, \hat{p}(-\sigma)\right), \hat{p}(-\sigma)\right) p_i\left(\overline{\theta}_i, \hat{p}(-\sigma)\right)\frac{d\theta}{2\sigma}.$$

Differentiating with respect to $\underline{\theta}_i$ and $\overline{\theta}_i$ respectively, in an interior solution we have:

$$\int_{-\sigma}^{\underline{\theta}_i}\left[\frac{\partial D_i\left(\theta, p_i\left(\underline{\theta}_i, \hat{p}(-\sigma)\right), \hat{p}(-\sigma)\right)}{\partial p_i} p_i\left(\underline{\theta}_i, \hat{p}(-\sigma)\right)+D_i\left(\theta, p_i\left(\underline{\theta}_i, \hat{p}(-\sigma)\right), \hat{p}(-\sigma)\right)\right] \frac{d\theta}{2\sigma}=0,$$

$$\int_{\overline{\theta}_i}^{\sigma}\left[\frac{\partial D_i\left(\theta, p_i\left(\overline{\theta}_i, \hat{p}(-\sigma)\right), \hat{p}(-\sigma)\right)}{\partial p_i} p_i\left(\overline{\theta}_i, \hat{p}(-\sigma)\right)+D_i\left(\theta, p_i\left(\overline{\theta}_i, \hat{p}(-\sigma)\right), \hat{p}(-\sigma)\right)\right] \frac{d\theta}{2\sigma}=0.$$

The solutions of these two equations yield $\overline{\theta}_i=\overline{\theta}^{\star} \leq -\sigma$ for $c \geq c^{\star}$ and $\underline{\theta}_i=\underline{\theta}^{\star}<-\sigma$. Hence, for $c \geq c^{\star}$ there exists the symmetric equilibrium in which all principals set $\overline{p}^{\star}=\underline{p}^{\star}=\hat{p}(-\sigma)$.

Second, suppose that $c<c^{\star}$. In this region of parameters the candidate equilibrium is such that $N-1$ principals set a list price $\overline{p}^{\star}=\hat{p}(\overline{\theta}^{\star})$. Hence, to define P_i's maximization problem we need to consider various types of deviations.

a. Suppose that P_i sets $-\sigma<\underline{\theta}_i \leq \overline{\theta}_i \leq \overline{\theta}^{\star} \leq \sigma$, its maximization problem is

$$\begin{aligned}\max_{(\underline{\theta}_i,\overline{\theta}_i)\in\Theta} &\int_{-\sigma}^{\underline{\theta}_i} D_i\left(\theta, p_i(\underline{\theta}_i, \hat{p}(\theta)), \hat{p}(\theta)\right) p_i(\underline{\theta}_i, \hat{p}(\theta)) \frac{d\theta}{2\sigma}+\int_{\underline{\theta}_i}^{\overline{\theta}_i} D_i\left(\theta, \hat{p}(\theta), \hat{p}(\theta)\right) \hat{p}(\theta) \frac{d\theta}{2\sigma}\\ &+\int_{\overline{\theta}_i}^{\overline{\theta}^{\star}} D_i(\theta, p_i(\overline{\theta}_i, \hat{p}(\theta)), \hat{p}(\theta)) p_i(\overline{\theta}_i, \hat{p}(\theta)) \frac{d\theta}{2\sigma}+\int_{\overline{\theta}^{\star}}^{\sigma} D_i(\theta, p_i(\overline{\theta}_i, \hat{p}(\overline{\theta}^{\star})), \hat{p}(\overline{\theta}^{\star})) p_i(\overline{\theta}_i, \hat{p}(\overline{\theta}^{\star})) \frac{d\theta}{2\sigma}.\end{aligned}$$

The first order condition with respect to $\underline{\theta}_i$ is

$$\int_{-\sigma}^{\underline{\theta}_i}\left[\frac{\partial D_i\left(\theta, p_i(\underline{\theta}_i, \hat{p}(\theta)), \hat{p}(\theta)\right)}{\partial p_i} p_i(\underline{\theta}_i, \hat{p}(\theta))+D_i\left(\theta, p_i(\underline{\theta}_i, \hat{p}(\theta)), \hat{p}(\theta)\right)\right] \frac{d\theta}{2\sigma}=0,$$

whose solution yields $\underline{\theta}_i=\underline{\theta}^{\star}<-\sigma$: a contradiction. It can be immediately shown that the same conclusion applies when P_i sets $\underline{\theta}_i \leq \overline{\theta}^{\star}$.

b. Suppose that P_i sets $-\sigma<\overline{\theta}^{\star} \leq \underline{\theta}_i \leq \overline{\theta}_i \leq \sigma$, its maximization problem is

$$\begin{aligned}\max_{(\underline{\theta}_i,\overline{\theta}_i)\in\Theta} &\int_{-\sigma}^{\overline{\theta}^{\star}} D_i(\theta, p_i(\underline{\theta}_i, \hat{p}(\theta)), \hat{p}(\theta)) p_i(\underline{\theta}_i, \hat{p}(\theta)) \frac{d\theta}{2\sigma}+\int_{\overline{\theta}^{\star}}^{\underline{\theta}_i} D_i(\theta, p_i(\underline{\theta}_i, \hat{p}(\overline{\theta}^{\star})), \hat{p}(\overline{\theta}^{\star})) p_i(\underline{\theta}_i, \hat{p}(\overline{\theta}^{\star})) \frac{d\theta}{2\sigma}\\ &+\int_{\underline{\theta}_i}^{\overline{\theta}_i} D_i\left(\theta, \hat{p}(\theta), \hat{p}(\theta)\right) \hat{p}(\theta) \frac{d\theta}{2\sigma}+\int_{\overline{\theta}_i}^{\sigma} D_i(\theta, p_i(\overline{\theta}_i, \hat{p}(\overline{\theta}^{\star})), \hat{p}(\overline{\theta}^{\star})) p_i(\overline{\theta}_i, \hat{p}(\overline{\theta}^{\star})) \frac{d\theta}{2\sigma}\end{aligned}$$

The derivative with respect to $\underline{\theta}_i$ is

$$\frac{\partial p_i(\underline{\theta}_i,\hat{p}(\theta))}{\partial \underline{\theta}_i}\int_{-\sigma}^{\overline{\theta}^\star}\left[\frac{\partial D_i(\theta,p_i(\underline{\theta}_i,\hat{p}(\theta)),\hat{p}(\theta))}{\partial p_i}p_i(\underline{\theta}_i,\hat{p}(\theta))+D_i(\theta,p_i(\underline{\theta}_i,\hat{p}(\theta)),\hat{p}(\theta))\right]\frac{d\theta}{2\sigma}$$
$$+\frac{\partial p_i(\underline{\theta}_i,\hat{p}(\overline{\theta}^\star))}{\partial \underline{\theta}_i}\int_{\overline{\theta}^\star}^{\underline{\theta}_i}\left[\frac{\partial D_i(\theta,p_i(\underline{\theta}_i,\hat{p}(\overline{\theta}^\star)),\hat{p}(\overline{\theta}^\star))}{\partial p_i}p_i(\underline{\theta}_i,\hat{p}(\overline{\theta}^\star))+D_i(\theta,p_i(\underline{\theta}_i,\hat{p}(\overline{\theta}^\star)),\hat{p}(\overline{\theta}^\star))\right]\frac{d\theta}{2\sigma}$$
$$+\frac{D_i(\underline{\theta}_i,p_i(\underline{\theta}_i,\hat{p}(\overline{\theta}^\star)),\hat{p}(\overline{\theta}^\star))p_i(\underline{\theta}_i,\hat{p}(\overline{\theta}^\star))}{2\sigma}-\frac{D_i\left(\underline{\theta}_i,\hat{p}(\underline{\theta}_i),\hat{p}(\underline{\theta}_i)\right)\hat{p}(\underline{\theta}_i)}{2\sigma},$$

Evaluating this derivative at $\underline{\theta}_i=\overline{\theta}^\star$, we have

$$\frac{\partial p_i(\overline{\theta}^\star,\hat{p}(\theta))}{\partial \underline{\theta}_i}\int_{-\sigma}^{\overline{\theta}^\star}\left[\frac{\partial D_i(\theta,p_i(\overline{\theta}^\star,\hat{p}(\theta)),\hat{p}(\theta))}{\partial p_i}p_i(\overline{\theta}^\star,\hat{p}(\theta))+D_i(\theta,p_i(\overline{\theta}^\star,\hat{p}(\theta)),\hat{p}(\theta))\right]\frac{d\theta}{2\sigma}$$
$$=\frac{1}{2}\frac{\left(1+(N-2)\gamma\right)c-\sigma\left(1-\gamma\right)}{\left(1+(N-2)\gamma\right)\left(1+(N-1)\gamma\right)},$$

which is (strictly) negative for $c<c^\star$: again a contradiction.

Therefore, $\mathcal{P}^\star$ is the unique symmetric equilibrium of the game. Finally, differentiating $\overline{\theta}^\star$ with respect to σ, c, N and γ, and exploiting the fact that $\hat{p}(\cdot)$ is increasing in θ, it is immediate to obtain that $\overline{p}^\star$ and $\overline{\theta}^\star$ are decreasing in c, N and γ, and increasing in σ. ■

Proof of Corollary 4. The proof follows immediately by definition of $c^\star$. ■

Proof of Lemma 5. The proof follows immediately from the fact that $b^{\star\star}(\theta)$ is decreasing in θ and that $b^{\star\star}(\theta)<0$ for $c=0$. The threshold $\hat{\theta}$ solves $b^{\star\star}(\theta)=0$, while $\underline{c}$ and $\overline{c}$ are the solutions with respect to c of $\hat{\theta}=-\sigma$ and $\hat{\theta}=\sigma$, respectively. ■

Proof of Proposition 6. Solving the first-order conditions (10) and (11), in an interior solution we have

$$\overline{\theta}^{\star\star}=\sigma-2\frac{2c\left(1+(N-2)\gamma\right)-\gamma\left(1+\sigma\right)\left(N-1\right)}{2-\gamma\left(N+1\right)},$$

and

$$\underline{\theta}^{\star\star}=-\sigma+2\frac{\gamma\left(1-\sigma\right)\left(N-1\right)-2c\left(1+(N-2)\gamma\right)}{2-\gamma\left(N+1\right)}.$$

To begin with, it can be immediately shown that this cannot be a solution for $N>\overline{N}$ since

$$\overline{\theta}^{\star\star}-\underline{\theta}^{\star\star}=\frac{2\sigma\left(2+(N-3)\gamma\right)}{\gamma\left(\overline{N}-N\right)},$$

which is negative in the region of parameters under consideration .

Next, consider $N \leq \overline{N}$ and $\gamma \leq \frac{2}{3}$ so that $\overline{N} > 2$. It can be shown that such a solution does not satisfy the second-order conditions for a maximum — i.e., the Hessian matrix is not negative semi-definite at $(\overline{\theta}^{\star\star}, \underline{\theta}^{\star\star}) \in (-\sigma, \sigma)$. Specifically, letting

$$\pi(\overline{\theta}, \underline{\theta}) \triangleq \int_{-\sigma}^{\underline{\theta}} \frac{1+\theta-\hat{p}(\underline{\theta})}{1+(N-1)\gamma}\hat{p}(\underline{\theta})\frac{d\theta}{2\sigma} + \int_{\underline{\theta}i}^{\overline{\theta}} \frac{1+\theta-\hat{p}(\theta)}{1+(N-1)\gamma}\hat{p}(\theta)\frac{d\theta}{2\sigma} + \int_{\overline{\theta}}^{\sigma} \frac{1+\theta-\hat{p}(\overline{\theta})}{1+(N-1)\gamma}\hat{p}(\overline{\theta})\frac{d\theta}{2\sigma}.$$

we have

$$\frac{\partial^2 \pi(\overline{\theta}^{\star\star}, \underline{\theta}^{\star\star})}{\partial \overline{\theta}^2} = -\frac{(2c(1+(N-2)\gamma) - \gamma(1+\sigma)(N-1))(1-\gamma)}{2\sigma(2+(N-3)\gamma)^2} \quad \Leftrightarrow \quad c \geq \overline{c}, \tag{13}$$

$$\frac{\partial^2 \pi(\overline{\theta}^{\star\star}, \underline{\theta}^{\star\star})}{\partial \underline{\theta}^2} = \frac{(2c(1+(N-2)\gamma) - \gamma(1-\sigma)(N-1))(1-\gamma)}{2\sigma(2+(N-3)\gamma)^2} \quad \Leftrightarrow \quad c \leq \underline{c}, \tag{14}$$

$$\frac{\partial^2 \pi(\overline{\theta}^{\star\star}, \underline{\theta}^{\star\star})}{\partial \underline{\theta}\partial \overline{\theta}} = 0. \tag{15}$$

The Hessian matrix is thus negative semi-definite at $(\overline{\theta}^{\star\star}, \underline{\theta}^{\star\star}) \in (-\sigma, \sigma)$ if and only if $c \geq \overline{c}$ and $c \leq \underline{c}$, which are indeed incompatible since $\overline{c} > \underline{c}$. As a result, the permission set that maximizes industry profit either features a cap, a floor, full delegation or full pooling.

Consider first, $N \leq \overline{N}$ and $\gamma \leq \frac{2}{3}$, so that $\overline{N} \geq 2$. In this region of parameters $\overline{c}^{+} > \overline{c} > \underline{c} > \underline{c}^{-}$. Then, the result follows immediately since: (i) $\overline{\theta}^{\star\star} \leq \sigma$ for $c \geq \overline{c}$ and $\overline{\theta}^{\star\star} \geq -\sigma$ for $c \leq \overline{c}^{+}$; (ii) $\underline{\theta}^{\star\star} \geq -\sigma$ for $c \leq \underline{c}$ and $\underline{\theta}^{\star\star} \leq \sigma$ for $c \geq \underline{c}^{-}$. Second-order conditions are satisfied in all these equilibria and are given by (13) when $\mathcal{P}^{\star\star}$ features a cap and by (14) when $\mathcal{P}^{\star\star}$ features a floor.

Next, suppose that $N > \overline{N}$. In this region of parameters, $\overline{c} > \overline{c}^{+} > \underline{c}^{-} > \underline{c}$. Hence, the maximization problem always features corner solutions. That is: (i) $\overline{\theta}^{\star\star} = \underline{\theta}^{\star\star} = -\sigma$ for $c \geq \overline{c}$, implying that $\overline{p}^{\star\star} = \underline{p}^{\star\star} = \hat{p}(-\sigma)$; (ii) $\overline{\theta}^{\star\star} = \underline{\theta}^{\star\star} = \sigma$ for $c \leq \underline{c}$, implying that $\overline{p}^{\star\star} = \underline{p}^{\star\star} = \hat{p}(\sigma)$; (iii) $\underline{\theta}^{\star\star} = -\sigma$ and $\overline{\theta}^{\star\star} = \sigma$ for $c \in (\underline{c}, \overline{c})$, implying that $\mathcal{P}^{\star\star} = [\hat{p}(-\sigma), \hat{p}(\sigma)]$.

The comparative statics follows from direct differentiation of the equilibrium values. ■

Proof of Corollary 4. The proof follows immediately by the definition of $\underline{c}$ and $\overline{c}$. ■

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