Space Is the Place

The Laws of Form *and Social Systems*

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The single most striking characteristic of George Spencer-Brown's *Laws of Form* is the variety of misunderstandings concerning its reception.[1] Its basic idea is actually quite easy: "form" or "something" is identical to the difference it makes (with anything else) and (thus) eventually different from itself. All "something" or "form" or "being" is explained as the residual of a more fundamental level of *operations* (namely, the construction of difference), including the "calculus of indications" explaining the very *Laws of Form*. Due to its constructivist nature, the calculus has enjoyed admiration from a variety of people, some of whom are regarded of major importance in their respective scientific disciplines. After a meeting with Spencer-Brown in 1965, the philosopher and logician Bertrand Russell congratulated the young and unknown mathematician for the power and simplicity of this calculus with its extraordinary notation. In 1969, shortly after the publication of *LoF*'s first edition, the father of neocybernetics, Heinz von Foerster, enthusiastically described it as a book that "should be in the hands of all young people."[2] In the cybernetic tradition, by the way, *LoF*'s resonance is undiminished. The international journal *Cybernetics and Human Knowing* published a Charles Sanders Peirce and George Spencer-Brown double issue in 2001; there exist two extensive Web sites with *LoF* material and new Spencer-Brown mathematical work (see "Spencer-Brown–related sources" in the notes below); and a revised English edition of *LoF* is forthcoming. One would conclude that *LoF* is very much alive indeed. But as noted above, appraisal for the calculus is certainly not univocal. There exist (some very advanced) criticisms of the calculus. Some authors regard it as misconstrued from its very beginning: for Cull and Frank, the *Laws of Form* is no more than the *Flaws of Form*. The greater bulk of disapproving comments is, however, less than a spelled-out, intricate argument. In general, it aims at the status of *LoF* within the mathematical tradition and rejects it as a mere variant of Boolean algebra, simply using

a new notation. *Nil novum sub sole*, so to speak. Whatever be the case, *LoF*'s thinking, especially where it concerns its far-reaching constructivist implications, has clearly not yet been well established. Spencer-Brown's (promising) claims notwithstanding, the context of his work, its notation, and its exotic vocabulary need a great deal of clarification.

For that very reason, the adoption of the calculus in contemporary sociological theory cannot be an obvious course. And yet some sociologists—most notably sociologists working in the systems-theoretical tradition of Niklas Luhmann—work with it "as if it were not only common knowledge, but as if one had fully grasped the transformation of the deep ontological structure it induces."[3] This is most certainly a reason for surprise or doubt. What does it mean when concepts (as "forms") are consistently formulated as a distinction? In an example, what is the sense in defining systems theory as the theory of the difference between system and environment? Next, is Luhmann's use of paradox, another central notion in *LoF*, more than an inflated postmodernist rhetorical device? Why does Luhmann insist on constructing a circular epistemology (that is, sociology as a way of society to picture itself in itself)? And why, above all, does Luhmann claim his theory to be universalist *yet not* solipsist? It is deplorable that the aforementioned reasonable doubts have generated a stream of publications harshly rejecting all Luhmannian theory. Danilo Zolo, for instance, has denounced the theory as a very complicated version of circular reasoning. Gerhard Wagner, on the other hand, specifically attacks Luhmann's epistemological grounding in *LoF*. Those differentialist claims, so Wagner argues, are no more than the foundationalist or essentialist thinking to which Luhmann himself claims to react.[4]

In the following, I intend to tackle exactly these problems. I will do so by systematically discussing the construction and argumentation of *LoF* (1) as I believe it holds the key to itself and to the sociological claims of Niklas Luhmann, but also (2) because such analysis has been conspicuously absent in the existent literature. A great deal of attention will be paid to problems associated with the circular construction of the calculus. At all times one should be aware of the difficulties or impossibilities of presenting circularity in a circular way; at least the medium of the book or oral presentation demands that we proceed linearly—that is, from the first to the last page respectively, or from the opening to the concluding remarks. As we shall see, this limitation (or, if one wants, "paradox") contains the solution to a rightful understanding of *LoF* and Luhmannian theory construction: "Reason, or the ratio of all we have already known, is not the same that it shall be when we know more" (William Blake). I will start by showing how *LoF* relates to the mathematical tradition

and how this refutes a great part of existing criticism. Next, after a treatment of the calculus's topological notation, I will briefly show the linear development of the calculus out of the primary arithmetic and primary algebra. Especially the planar foundation of both allows us to understand why the grounding of a theory of (social) systems in Spencer-Brown's calculus can hardly be an obvious course of theory building. Yet the presentation of self-reference in the calculus notation, as Spencer-Brown demonstrates, is possible if and only if we are prepared to change the medium in which we are writing. Self-reference defies presentation in plane space yet can be presented in topologically more intricate versions of space. As we will see, the latter corrodes our most profound ontological presuppositions radically; on a higher level, it is also responsible for sharp differences between the systems theories of Talcott Parsons and Niklas Luhmann. Last but not least, *LoF*'s altered treatment of space (and its relationship with time) allows an exploration of the peculiar epistemologies of both Luhmann and Spencer-Brown.

Starting Point: A Nonnumerical Arithmetic

Before the discussion can commence, I should draw the reader's attention to the extraordinary economy with which Spencer-Brown equipped the calculus. This is *not* a common logical calculus founded on postulates. It is not even a logical calculus. *LoF* must be studied as a book of mathematics, an "arithmetic whose geometry as yet has no numerical measure." No numerical measure, indeed! What does that mean? As Spencer-Brown rightfully underscores, we find ourselves here in the primitive a priori dimension of all notation: two-dimensional space.[5] What we will be doing here, to put it bluntly, is drawing figures in sand or on a piece of paper. Interestingly, this is what constitutes the challenge for a twentieth- or twenty-first-century public: the level of investigation is as deep as being "beyond the point of simplicity where language ceases to act normally as a currency for communication."[6] Our investigations will have to take place at a prediscursive level, where "something" comes into being so to speak.[7] This is very different from the level of number, and certainly logic, which is not so much concerned with the world but with the rather limited domain of our (human) *cognitive relationship with* the world. "Logic is not, and has never been, a fundamental discipline," Spencer-Brown therefore argues.[8] And for the same reason, postulates cannot exist here. Spencer-Brown departs from a very basic experience of dealing with the world, with "things," "stuff." This in itself makes the calculus a true rarity in the history of mathematical thought. I concur with von Kibéd and Matzka that Wittgenstein's *Logisch-philosophische*

Abhandlung (the *Tractatus*), and in several respects Charles Sanders Peirce's work, are the kind of inquiry closest to the one presented in *LoF*.[9] In view of its rigorous confinement to the very fundamentals of logical systems, *LoF* is most acutely referred to as a protologic: a research into ordinary arithmetic, rather than ordinary algebra, or "an inquiry into the pre-discursive laws emerging with the most elementary position of 'something.' These laws must be situated at a level preceding the level of expression grasped by classical logic."[10] Thus, *LoF* anticipates and wards off the major part of its critics at its most elementary level. This is important: when Kuroki Gen, a harsh critic of *LoF*, describes the work as a reformulation of propositional logic or Boolean algebra, he is at least neglecting the calculus's construction and is possibly ignorant of the meaning of its very starting point.[11] We will encounter the consequences of this misunderstanding below.

This being said, we can proceed to what Spencer-Brown grants us in order to commence calculating (chapter 1 in *LoF*). And that is not much. Spencer-Brown is very cautious not to break with the objective of starting from the very beginning. He simply delivers a definition of form ("Distinction is perfect continence") and two axioms contained in the definition:[12] (1) The law of calling refers to the descriptive aspect of distinctions. Once (a delineated) something has been given a name (call), recalling it does not alter it—"The value of a call made again is the value of the call." (2) The law of crossing concerns the injunctive or more clearly operational aspect of distinctions. Here a difference "does make a difference." One can only be in the form, or not—"The value of a crossing made again is not the value of the crossing."

Yet its mathematical economy notwithstanding, let us not be mistaken about the *LoF*'s intentions: "The theme of this book is that a universe comes into being when a space is severed or taken apart. The skin of a living organism cuts off an outside from an inside. So does the circumference of a circle in a plane. By tracing the way we represent such a severance, we can begin to reconstruct, with an accuracy and coverage that appears almost uncanny, the basic forms underlying linguistic, mathematical, physical and biological science, and can begin to see how the familiar laws of our own experience follow inexorably from the original act of severance."[13] As will be clear, this passage contains *LoF*'s undeniably universalistic (and thus circular) aspirations: starting out from an original act of distinguishing, *LoF* intends to describe its consequences for (1) the possibility of the world ("things" as form), (2) the possibility of developing a (cognitive) relationship with the world of things (knowledge or "cognitive categories" as form), and (3) eventually, the possibility of describing the possibility of discovering these possibilities (the laws of form as the precondition of

all form, or the universe as "what would appear if it could").[14] The last concerns the pure circularity of the calculus: the Form as an explanation of itself. It is this part as well that led Heinz von Foerster to link *LoF* with Wittgenstein's "problem of the world"—that is, the fact that the world we know is constructed in order to see itself, while that appears to be a logical impossibility.[15]

Forms Taken out of the Form

But it is too soon to discuss the link with Wittgenstein. Let us commence calculating. As should be expected from this basic inquiry, the calculus—which starts in chapter 2, subsequent to the outlining of the conception of the primal form—begins with a command of surprising naïveté: "Draw a distinction!"[16] "Draw a line," "make a distinction," is the primal injunction. As such, Luhmann would say, one performs the operation of "observation." One de-lineates something and *simultaneously* indicates one of the sides separated by the distinction.

In order to express the conception of the "form" through a formal notation, Spencer-Brown employs the ⌐, the "mark of distinction," a topological notation. At this stage of the calculus, the mark ⌐ represents a "cross" (descriptive) that also ought to be taken to mean "cross!" (injunctive). The mark is, in Peirce's sense, a *portmanteau-symbol*: it combines both the aspects of plain denotation and an injunctive or instructive meaning to cross the distinction and indicate one of the separated sides.[17] "Let any token be intended as an instruction to cross the boundary of the first distinction. Let the crossing be from the state indicated on the inside of the token. Let the crossing be to the state indicated by the token."[18] This is no more than stating the obvious. When we draw a distinction (for example, a circle), then the distinction cannot be neglected; it has affected the space in which it is written, and we are, as such, "in" the form. The first distinction literally is a first judgment, an *Ur-teil*, which determines everything coming after it. Once the distinction has been drawn, a "universe" is there, and the gates to return to a state of nothingness are closed; that world is the mere "nameless origin of heaven and earth," the phenomenology of which is lost.[19] The notation ⌐ (alternatively ( ) or <<>>) thereby expresses that topological *asymmetry* as well. Simultaneously with the drawing of a distinction, one of the sides is indicated. The concave side of the mark thereby represents the "inside" of the form or the "marked state." The other side is the outside; it is a nameless residual, an unmarked leftover, from which the marked side is delineated. We must stress here that the drawing of the distinction and the indication of one of the separated sides are two simultaneous aspects of one operation. In Spencer-Brown's terms: "We take as given the idea of distinction and the

idea of indication, and that we cannot make an indication without drawing a distinction."[20] It is equally impossible to draw a distinction without making an indication. Why else would one draw the distinction at all? The indication is the motive of the distinction. For the time being, we can thus conclude that form, indication, and distinction are *implied* within each other, not to say identical. For it is clear that every observation implies the drawing of a distinction, and this implies in turn that every form has to be conceived of as a distinction: "Call the form of the first distinction the form."[21] This then allows the reader to follow what could be called a syntax of form (*LoF*, chapter 2): aspects of the form ("name," "content," "value," etc.); basic calculatory possibilities ("the form of cancellation"/"the form of condensation"); and more complex notions—for example, the "unwritten cross."

In a next step (chapter 3 and further), the calculus is developed linearly (!) out of the aforementioned syntactical complex—the latter represent the rules of the game, so to speak. It grows into a *primary arithmetic*, which serves, in turn, as the foundation for a *primary algebra* (chapters 4 through 6; the line of the argument does not demand we enter theorems consequences or canons developed here).[22] At several points in the appendices, but also in the preface(s), introduction, and the "note on the mathematical approach," the difference between these is stressed, and Spencer-Brown definitely favors the primary arithmetic over the more commonly investigated level of algebras—that is, Boolean algebra.[23] Arithmetic, he says, is about the constants, the individuality of form, and the individuality of the calculatory relationships the form builds. The algebraist, on the other hand, "is not interested in the individuality of numbers, he is interested in the generality of numbers. He is more interested in the sociology of numbers."[24] The formulation of the arithmetic was thus formative to the development of an algebra (Boolean) in the first place: "So, to find the arithmetic of the algebra of logic, as it is called, is to find the constant of which the algebra is an exposition of the variables—no more, no less. Not just to find the constants, because that would be, in terms of arithmetic of numbers, only to find the number. But to find out how they combine, and how they relate—and that is the arithmetic."[25]

Elena Esposito has speculated that both the primary arithmetic and the primary algebra may be instrumental to a sound understanding of cybernetic *constructivism*, especially where it relates to the difference between first-order and second-order observations.[26] It all boils down to appreciating the arithmetic and the algebra as autonomous parts of the calculus that correspond to existent (because observable) systemic levels, the level of elements and the level of systemic organization respectively. (1) The arithmetic, as Esposito argues,

represents the "formalization of autopoiesis." Here everything is about the execution of rules relating to constants ("specific objects"). It is a first-order level of observation, a level where there can be awareness only of the existence of form. (2) The algebra, which could be constructed through the "theorems of connection," is a different matter. This truly is a *calculus taken out of the calculus*, a level where the forms developed in the arithmetic are object of other forms at a higher level (in turn bound by the rules of the arithmetic!). It is about arrangements (and hence indication *and distinguishing*) of arithmetic forms (*distinctions*). In a sense, it is the systemic level. Esposito explains: "The algebra formalizes a specific type of autopoiesis (and thus requires the validity of all arithmetic formulae): the autopoiesis of a system, the operations of which are observations. Yet, it remains a fact that its operations (including the operations of observation) can only be observed by an external observer. This contains an openness which implies and eventually requires the integration of first order observations with observations of higher orders."[27]

Yet however tempting Esposito's metaphorical use and/or explanation of the different levels may be (and I am certainly not saying Esposito's remarks are entirely wrong!), it is mathematically unsound: autopoiesis is an issue in neither the primary arithmetic nor the primary algebra. What we are doing here is still drawing distinctions in the plane, discovering how they relate, how they may cancel each other out.

Intervention 1: *Laws of Form* and Social Systems

The reader will understand that this leads to further doubts: how is the notion of "form" in the calculus of indications to be linked to a theory of social systems? By no means should the calculus of indications be understood as a "brand" of systems theory. I repeat: the calculus is best viewed as a *protologic*; it was primarily written in reaction to some assumptions held in logic. And being a *proto*-logic, it was not even stamped as a logical calculus, but as a *mathematical inquiry*. Another difference, however, between the Spencer-Brownian calculus and Luhmann's systems theory is that the former mainly represents finite forms (as the calculus demands, they exist as a finite number of crosses), whereas social systems *by definition* hold out the prospect of *infinity*. They have never been set in motion "at a certain point in time," as such a point would presuppose an earlier communication to which they could connect; and *vice versa*, they don't break off at a certain point, as such a point would hold out the prospect of continuation. In Spencer-Brown's calculus, potentially infinite forms are mentioned only where the calculus has been taken "so far as to forget it."[28] An

attempted coupling of *LoF* and the theory of social systems should therefore come as a surprise.

Let us take a look at how an apparent contradiction contains the key to its solution. First of all, how do we define a system? A system exists when there is something capable of identifying a specific operation as belonging to itself—that is, when there is something capable of discriminating that operation from operations that do not belong to itself (with attention to the sheer tautology of systemic operations). A system then uses the products of preceding systemic operations for the performance of new and different operations, again identified as belonging to the system, not to its environment. Systems thus necessarily carry an image of what they are not, although in a truly ambiguous way. This operational mode is designated as self-reference. Self-reference expresses the unity the system creates for itself. It indicates that a system refers to itself in all its operations: "There are systems that can develop a relationship with themselves and can distinguish this relationship from relationships with the environment."[29]

This conceptualization of self-reference in the terms of the *LoF* does not seem a self-explanatory course. After all, it would come to mean that forms, distinctions (for example, system/environment), develop a relationship with themselves, although self-contact is implied as being an impossibility in the definition of the "primal form." Once more, I stress that the operation(s) of distinguishing and indicating should be studied as a single operation. And once more, I must emphasize that the indication is the one and only motive of the distinction. Taken together, does this not mean that the distinction is employed simply in order to be forgotten in the indication?[30] Consequently, if the whole range of the distinction is in itself the residual of an observation, how can self-reference possibly be realized? Is not self-reference an impossibility, as it implies the distinction's capability of referring—that is, indicating—itself in itself and employing earlier indications for the production of new indications in the (same) form? Is self-reference not excluded in the very definition of the primal form? In brief: is self-reference not inhibited because of the fact that distinctions, differences, "make a difference"?

Reentry of the Form into the Form

And yet: is this really true? Do all distinctions make a difference? In some obviously neglected but crucial passages of the "appendix" to the calculus, Spencer-Brown reminds us of the use of covert conventions in mathematics: we have agreed to some rules without being consciously aware of the fact we

did so.[31] In the beginning of the calculus, and for that sake in this essay, the reader has, for instance, assumed that the distinctions were drawn in a *plane*: a piece of paper, for instance, or the surface of the earth. As we know, distinctions drawn in a plane do indeed build a distinction. But the use of a different mathematical medium makes things a lot more complicated, to the point that seemingly obvious facts are, in fact, not self-evident at all. If, at the outset, we had confined ourselves to writing on a torus (a "doughnut"), for instance, the distinction *would not have constituted a distinction.*[32]

Clearly, our unconscious choice to write in a plane, on a piece of paper—*that* has made the *real* difference. If we are only willing to work with a different medium, with a different conception of the spaces in which the distinctions are drawn, it may very well produce a wholly different arithmetic, algebra, and logic. Such willingness would, moreover, not be without a cause. As we explained above, we do in fact assume the existence of forms (that is, systems) that are thoroughly self-referential, that thus demand a different topological treatment (as they defy representation in the limiting terms of a plane surface or even Euclidean space). During his career as a civil engineer for British Railways, George Spencer-Brown and his mysterious brother, D. J. Spencer-Brown, had already developed special-purpose computer circuits that exhibited all characteristics of self-referential expressions, the prohibition of their use in conventional logic and mathematics notwithstanding.[33] For Spencer-Brown, the question is thus a purely mathematical one. His interests lie with showing the *validity* of imaginary values (for example, $\sqrt{-1}$), the *use* of which has been common in, for example, electromagnetic theory. As they can be used meaningfully for the solution of equations that cannot be solved otherwise, we must accept "imaginary" as a "third" category independent from (1) true (tautology: $x = x$) and (2) untrue (contradiction: $x = -x$). For Luhmann, the problem is to describe self-referentially operating social systems, consisting of operations that take their own results as a base for further operations. These are forms that "in-form" themselves.

In the mathematics of *LoF* (chapter 11), the following solution for our problem is proposed: let us conceive of "space" in a different, operational, way—that is, space as a relation between elements.[34] If we are able to abandon the idea of space as a Euclidean "container" (that is, space as something "in which" things are positioned), it is indeed possible to conceive of self-reference as forms turning up in their own form.[35] Back to the calculus: Spencer-Brown insists that we must therefore allow some way of *contact* between the separated sides of the distinction written in the plane surface. In order to show the self-reference of a form/distinction, the distinction must, quite literally, be *undermined.* Let us

therefore dig holes, tunnels, under the surface in which the distinction appears and "corrupt" (from Latin *cor-rumpere*, to break together) the cross.[36] That space is a torus. If considered operationally, distinctions written on a torus can subvert (turn under) their boundaries, travel through the torus, and reenter the space they distinguish, turning up in their own forms, thus capable of developing some kind of contact with themselves.[37]

Clearly, such self-referential form cannot be decided (from Latin *de-cedere*, "to cut off") in the plane. The marked state cannot be clearly distinguished from the unmarked state anymore, leading to "indeterminacy." The form is neither marked nor unmarked. It is an imaginary value, flipping between marked and unmarked, thanks to the employment of *time*.[38] However, this does not preclude its existence: "The value [of self-referential forms] being indeterminate in space, may be called imaginary in relation with the form. Nevertheless . . . it is real in relation with time and can, in relation with itself, become determinate in space, and thus real in the form."[39] Self-referentially operating systems should thus be understood as the *operational difference* between themselves and their environment, a difference that is made through some sort of self-referential oscillating between the two sides of the distinction (that is, system and environment). By means of self-reference, the environment "out there" can be observed as being drawn topologically into the "inside" of the system (compare the inside and outside of a Möbius ring). This is the meaning of reentry: the two sides of the distinction are reinserted into one of its parts.

Spencer-Brown repeats the notational ramifications of such subversion: "In a simple subverted expression of this kind, neither of the . . . parts are, strictly speaking, crosses, since they represent, in a sense, the same boundary. It is convenient, nevertheless, to refer to them separately, and for this purpose we call each separate . . . part of any expression a marker. Thus a cross is a marker, but a marker need not be a cross."[40] The distinction could thus also be said to have been alienated from its original intent or motive (that is, indicating one separated side), and this by value of being processed within the form (system), in order to safeguard the difference between itself and other distinctions *over time*. The aforementioned notational arrangement does, however, have intriguing consequences for the form's being. The excursion through the tunnel of the torus and the consequent time employed to return into itself make the self-referential form peculiarly look as shifting between what it is indicating ("cross!") and what it uses to make indications ("marker"). The self-referential form is *flippety*: "I am the link between myself and observing myself" (Heinz von Foerster). In the parlance of ontology (cf. infra): the self-referential form is both identical to and different from itself.[41]

Laws of Form as Ontology

The mathematical visualization of self-reference in mind, it may be instructive to reconsider an important critique on Luhmann's notion of "system." Interestingly, some authors (most prominently represented by Gerhard Wagner) seemingly assume that Luhmann has proposed to simply replace a thing's or system's *identity* by means of a difference—namely, the difference between system and environment—just like that. Obviously, such a shift has not been the case. Rather, what is lacking *in the critique* is due attention to the growth of the calculus's injunctions in the direction of the "form" of self-reference. Whereas Luhmann himself has, on several occasions, referred to systems theory as an invitation to draw a distinction between system and environment, that distinction is an obvious advance on the topological ramifications prominent only in the concluding chapters of the calculus. Clearly, Wagner mistakenly views the difference between system and environment as the *immediate* offspring of the primal construction, "Draw a distinction!"; his argument has not "followed" up to the coda, let alone self-reference having been "understood." He is still *in the plane.* Consequently, he is myopic concerning the operational aspect of self-reference. Hence, the difference between system and environment cannot be evaluated in its full complexity. As a matter of fact, it must ultimately come to be seen as a rather trivial reiteration of foundationalism: "The fact is that, by the way in which Luhmann understands foundational difference, he practically commits his position to identity."[42] In such a mistaken view system and environment can easily and erroneously come to be conceived of in terms of a polar opposition, quite similar to Hegel's notion of negativity.[43] As my remarks have hopefully made clear, the elaborate notion of a system as a self-referential form can be realized only in the more advanced chapters of *LoF*—namely, chapters 11 and 12—in which there is made mention of the *reentry.*

The system's self-reference can thus only be defined as the act of self-reference, as self-referential performance. And this, as we have seen, demands a quite intricate topological arrangement. The system must reflect, in the formal sense, itself and its environment as a corollary of itself *in all of its operations.* It can secure the connectivity of its operations only by establishing itself as an imaginary value and by employing the time of the tunnel to develop a relation with itself. In short, imaginary space is the only topological possibility for a system to be systemic. For that reason, the difference between the system and the environment cannot be an essentialist difference, let alone a new version of foundationalist thought: "It does not cut all of reality into two parts: here system, there environment. Its either/or is not an absolute; it pertains only in

relation to the system, though objectively. It is correlative to the operation of observation, which introduces this distinction (as well as others) into reality."[44] This operational aspect apparently provides the clue to the numerous misunderstandings about Luhmann's ontological premises. And time after time, the peculiar position of the environment has been at the core of the problem. Yet if you are aware of the topological qualities of the torus, you can easily understand that position. The environment is not so much *out there* as *in there*: it simply emerges out of the reentry of the distinction into itself. The environment is *constructed* by the system; it exists only with the form of the system—that is, if there is a boundary that can be employed in order to reenter the system's own inner space.[45]

This form-centered conceptualization has clearly parted with more established distinctions such as *subject* versus *object*, *man* versus *world*, or *self* versus *difference*. In *LoF* and systems theory, everything, all emergent reality, is discussed as the corollary of a construed difference. So what are the ramifications of self-reference for our "ontology of the world"?[46] Central to our discussion here is the connection between *operationalism* and form. Systemic operations, we have stressed, presuppose self-contact, and, vice versa, self-contact implies systemic "in-formation." Clearly, the form is on its own; it is a self-sufficient and self-engendering reality. Actually, the definition of the primal form, stated at the very outset of the calculus of indications, had already made this clear: "Distinction is perfect continence."[47] But at this stage of the calculus, what we have already known is not the same as what we know now (cf. the citation of William Blake in the introduction). At this point we realize that "form" is the symbol of the world or the universe; *all* form is part and product of a self-engendering self-referential whole,[48] of which even the first form must be embedded in a further form (most fundamentally the difference between drawing a distinction and deciding not to do so).[49] This formal introversion (from Latin *intro-vertere*, turning inside), this very self-reference, refutes essentialism. After all, while we may take it that the universe undoubtedly is itself—that is, indistinct from itself—we must accept the fact that, as self-reference, it is indeed false, or distinct, to itself: "We may take it that the world undoubtedly is itself (i.e. is indistinct from itself), but, in any attempt to see itself as an object, it must, equally undoubtedly, act so as to make itself distinct from, and therefore false to, itself. In this condition it will always partially elude itself."[50] The world is not "what there is."[51] Yet this foundationalist crisis à la Kurt Gödel's theorem of incompleteness should not be seen as a reason for despair. Self-referential paradox, meaning indeterminacy, must be construed as the price systems and the world pay for the possibility of operations, activity, and systemic evolution.

For contemporary systems theory, paradox is not seen as an accident to be avoided, but rather as the creative presupposition of the whole construction. Paradox "is not the fatal end, the definitive failure of all ontological constitution. On the contrary . . . it is the starting point of a history, a movement of system-constitution, full of risks and bifurcations. Paradoxes do not make things impossible, but rather possible."[52]

The French philosopher Jean Clam has therefore speculated that it may be analytically fruitful to employ a difference between *apophantic* and *ergetic* paradoxes, paradoxes occurring in logical expressions (in the plane!) and paradoxes imminent in operations or systemic space respectively. In the apophantic sense, paradoxes do indeed block observation: they do defy determinacy and may therefore be judged destructive or corrupt according to the foundationalist tradition. But the excursion through the tunnel has shown that there is a merit in subversion: what apparently blocked cognition has become an operational loophole, a compelling chance for *system genesis*. Systems must operate in order to achieve the fictitious unity that could not be achieved by the ontologically more elegant way of self-identity and integrity. Systems must operate in order to bypass situations of a structural standstill. And semantically, systems must operate in order to cover up the devastating consequences of manifest inconsistency and contingency: deparadoxicalization. "Operating is always the introduction of a component that avoids the stand-still, because it broadens the space of possibilities. The operating of systems is nothing else than this handling of components which create more possibilities and condensing them into a self-continent but not-finalizable ergetic whole."[53]

Intervention 2: Luhmann versus Parsons

In retrospect, one will agree that the ever-growing prominence of paradox in Luhmann's thinking has changed the concept of social system in ways so fundamental that a sociologist such as Talcott Parsons could not have imagined. At no point, we must concur, can a system be described in terms of invariant structural characteristics. Confronted with the utter impossibility of unity and consistency, in favor of indeterminacy and contingency, systems emerge as mere sequences of ongoing operations. They are no more than a momentary derivative of passing operations, characterized by a self-reinforcing restlessness. Admittedly, Luhmann has never been a committed structuralist. In *Social Systems*, he rejected structuralism on the grounds that "structuralists have never been able to show how a structure can produce an event."[54] His theory of social systems has therefore subordinated structure to function and has shifted the

focus from structure to event, the network of which produces the unity of the system, in the event only. But through time, the concept of autopoiesis, which expresses the self-production of the network, has undergone some major changes as well. Whereas the notion had originally been defined in close reference to the way it was designed by Humberto Maturana, Luhmann forsook this definition almost completely in the 1990s. Finally, the notion would be fully rewritten in the terminology of the calculus of indications: "Autopoiesis is thus not to be conceived as the production of a peculiar 'Gestalt.' Crucial is rather the formation of a difference between system and environment."[55] Autopoiesis, the reader will understand, is nothing but the form of a system's basal unrest, the abbreviated expression of the system's concern with getting around its nonidentity. The strong self-referential, and hence reflexive, bias of the notion shows what that means. Enclosing itself in itself—that is, enclosed in itself—the form incessantly crosses its own internal boundary, thereby adding to its level of complexity, but never able to become identical to itself.[56]

This latter point may be helpful in understanding Luhmann's particular brand of functionalist methodology. Already back in 1970, he had criticized Parsonian functionalism on the grounds of assuming a semi-identity of *function* and *causality*.[57] Clearly, the notion of causality, implying necessity and absoluteness, is at odds with a theory that converges around contingency—in politics, in law, in science, in intimacy, in art—in brief, modernity![58] Luhmann therefore returns to Kingsley Davis's critique of functional method and manages to turn this critique inside out: the rejection of functionalism is—in a typically functionalist guise—employed as a solution for some conceptual deficiencies of the functionalist method. First, the relationship between function and causality is asymmetrized: causality must be classified as one exceptional instance of function. Second, functionalism is outlined as a method for *comparing* the potential of systemic arrangements aimed at the maintenance of the system's unity, rather than for indicating the "systematic" relationships between function and achievement. Luhmannian functionalism is a *functionalism of difference* and as such is more than a mere rhetorical upgrading or fine-tuning of a well-known functionalist repertoire. After all, the quest for historically contingent and factually variable functional equivalencies effectively avoids the structural determination of theoretical judgment.[59] With respect to content, attention shifts from the functional arrangement to what could be called the *construction of problems*. And again we encounter self-reference here. Rejecting a social system's (structural) permanence and subscribing to the idea of systems as forms that react to their self-generated complexity, functionalist method also entails adequate attention to the way in which systemic problem solutions (functional

arrangements) "trans-form," expand the system's operational status, and thus *reenter* the system's space at a new level. At that level, the problem to which the functional arrangement initially reacted may have disappeared or may be encountered in a different, possibly more accrued, manner. Such are the methodological consequences of self-conditioning self-reference.

Laws of Form as an Autological Construct

Finally, I promised a clarification of the metaposition of the notion of form far beyond the scope of its heuristical application. After all, after indicating the possibility of self-referential forms ("reentry," chapter 11), *LoF* offers a perspective on the position *of the calculus* ("Re-entry," chapter 12). The calculus, as a part of the universe, must be one possible form, distinguishing the forms it has been describing as forms making a difference. The very calculus of indications has been a "tunnel" through which Spencer-Brown and the reader have traveled to arrive at the form of the first distinction, which is now seen as legitimized, justified by all canons, theorems, demonstrations, and proofs that followed it. The "first distinction" was deliberate and historically *contingent.* Yet all that followed was its *necessary* consequence: "The whole account of our deliberations is an account of how [the first distinction] may appear, in the light of the various states of mind which we put upon ourselves."[60] *For that very reason*, the clarification of the laws governing this universe must be considered a trivial matter: "Coming across it thus again, in the light of what we had to do to render it acceptable, we see that our journey was, in its preconception, unnecessary, although its formal course, once we had set upon it, was inevitable."[61] The paradoxical combination of *contingency* (of the first distinction) and *necessity* (of its consequences) demonstrates in what fundamental respect the epistemology of *LoF* differs from classical epistemologies.[62] The calculus of indications, ultimately a function of itself, has established itself as an imaginary value. It can be continued endlessly, as Spencer-Brown does not fail to indicate.[63] On the one hand, its inclination toward the imaginary makes the calculus correlate with what it seeks to describe: like reality or "world," the calculus is "form" that seeks to get a hold of itself but does not manage to do so. On the other hand, its constructivism obviously implies the loss of a privileged position of (scientific) knowledge. At this point it is clear how *LoF* relates to Wittgenstein's problem of the world (cf. supra). Self-reference has come 360 degrees. It is not merely at the root of any possible universe. It also dominates and determines observations of the universe and eventually observations of the laws of form governing both. This should affect scientific observations and scientific method considerably. It

implies a shift from a world of things to a world of observations. This is not just a world of the real: "There is a tendency, especially today, to regard existence as the source of reality, and thus a central concept. But as soon as it is formally examined, existence is seen to be highly peripheral, and as such, especially corrupt (in the formal sense) and vulnerable."[64] It is rather a world of the possible and an observer's intention to draw distinctions. Our understanding of the world thus cannot reside in some form of discovery of its present appearance (out there, beyond observation) *but comes from remembering the conventions agreed to in order to bring it about.* The task of the mathematician, whose interest lies with notational elegance and density, may hence lie with bringing the world back to its conventions and abandoning all surplus arrangements. As is well known, Spencer-Brown's conclusions eventually border on the mystical: "To experience the world clearly, we must abandon existence to truth, truth to indication, indication to form, and form to void."[65]

For Niklas Luhmann, however, when presenting a theory of social systems, the challenge is different. His task lies not with abandoning, but rather with expanding. Clearly indebted to *LoF*, Luhmann adopted the notion of form *and the corollary notion of medium.* He has typically used a theory of the latter to identify different types of medium in the social sphere and speculate about their respective topologies, their transformative capabilities, their role in societal evolution, and so on. It is revealing that form/medium came to permeate the whole of Luhmann's theory of society, eventually stretching beyond the original main distinction of system and environment, bringing about obvious problems of theory construction.[66] Epistemologically, furthermore, Spencer-Brown's mathematical conclusions on reentry are expressed as the autology of the distinction between form and medium: form/medium is a distinction, thus form.[67] It, too, must reflect the *triviality* of its *necessity. As a self-referentially organized theory, Luhmann's systems theory represents its own boundary, and its limitations.* The acclaimed universality of the theory can therefore never entail solipsism. If properly observed, the laws of form relate (relativize) the theory's universality to the notation employed, thereby outlining the distinction as its own limitation. Conscious of its social formulation in the social sphere of society, the theory of social systems is simply one possible way of presenting society in society (*die Gesellschaft der Gesellschaft*). It is only one possible form in the all-encompassing medium of meaning. This leads to an interesting question. If the medium of meaning is indeed the ultimate medium of psychic and social systems—that is, if meaning is "the medium of itself"—then what is its "form," the distinction through which it can be expressed? I perceive only one answer: the medium of meaning must be identical to the difference between form and

medium and the reentry of that distinction into itself. Its consequent indecidability is the symbol of our dealing with the world. It expresses the fact that all our attempts to get a hold of the world are doomed to frustration.[68] Meaning as our *phenomenology* of this world can only be partial, as the difference between form/medium can only be actualized as a form. In mathematical terms: meaning is a lambda-domain occupied by communications that, by acting on themselves (= being a function of themselves), produce new communications in the same domain that can in turn act on themselves and further expand the domain.

It will be clear to the reader that such far-going occupation with self-reference must change our view of Spencer-Brown's "form" and Niklas Luhmann's system/environment and form/medium. Their function lies most certainly not with the description of the "objects" in their respective domains and their respective "qualities" as qualities that are eternally true (that is, observer-independent). Rather, the self-referential construction of the universe and especially the medium of meaning demands the construction of theoretical notions capable of reflecting themselves as an object (= communication) in their domains, expanding the domain's horizon beyond their own capability of observing that expansion. Seen in the terminology of topology, form and form/medium are self-locators or fixed points; they are the sole "points" on the map of mathematics and social theory that coincide with the corresponding point in the terrain their disciplines are trying to map. Such points contain their own explanation (that is, their allo-reference and self-reference coincide). They are the pinnacle of self-reference in domains that are self-referentially built. Therefore, both *LoF* and the theory of social systems are not only in the metaphorical sense a formulation of Quine's paradox. When applied to themselves, they "yield a falsehood" (absolute because contingent). Yet, therefore, just as Quine's paradox, they can be absolute theories, which are also theories of themselves. Ironically, the latter also constitutes their absolute weakness, I feel. When Niklas Luhmann, for instance, describes the epistemological premises of his gigantic theory of society as an invitation to rethink existing social theory and to formulate theories that have themselves compared to his project, this must be seen as a (rhetorical?) illustration of his epistemological self-confidence, no more and no less. After all, it is in the nature of meta-theories not to tolerate epistemologies of a different brand. Exactly their meta-nature blocks the possibility of going beyond them—self-reference is infinity in finite guise, as Kauffman also knows.[69] It should be clear that different theories and different epistemologies will have to put to themselves the requirement of contingency and autology in order to qualify as candidates for comparison after all. Whether this paradox has detrimental consequences is a question that must be left open

here. It is only to hope that, as William Blake has said, "Reason, or the ratio of all we have already known, is not the same that it shall be when we know more."

Notes

Spencer-Brown–related Internet sources:

Laws of Form Web page maintained by Dick Shoup: http://www.lawsofform.org.

Laws of Form Web page maintained by Thomas Wolf: http://www.laws-of-form.net.

スペンサー=ブラウンなんていらない (We don't need Spencer-Brown): Japanese page maintained by Sakai Taito 酒井泰斗; includes links to critical remarks by Kuroki Gen 黒木玄: http://thought.ne.jp/luhmann/sb/Osb.html.

1. I will refer hereafter to the whole of Spencer-Brown's *Laws of Form* as *LoF*; all specific citations, noted as *LoF* with corresponding page numbers, are taken from Spencer-Brown, *Laws of Form*, 1969 (rp. 1994).
2. Von Foerster, "*Laws of Form*." Partial reprints of this review can be found on the back page of several editions of *Laws of Form*; a German translation was included in Baecker, *Kalkül der Form*.
3. Clam, "System's Sole Constituent," 69.
4. See Wagner, "The End of Luhmann's Social Systems Theory"; Zolo, "Function, Meaning, Complexity," and "The Epistemological Status."
5. *LoF*, xi. On mathematics and space, Spencer-Brown has said: "Mathematics is, in fact, about space and relationships. A number comes into mathematics only as a measure of space and/or relationships. And the earliest mathematics is not about number. The most fundamental relationships in mathematics, the most fundamental laws of mathematics, are not numerical. Boolean mathematics is prior to numerical mathematics. Numerical mathematics can be constructed out of Boolean mathematics as a special discipline. Boolean mathematics is more important, using the word in its original sense: what is important is what is imported. The most important is, therefore, the inner, what is most inside. Because that is imported farther. Boolean mathematics is more important than numerical mathematics simply in the technical sense of the word 'important.' It is inner, prior to, numerical mathematics—it is deeper." Spencer-Brown at the American University Masters (AUM) conference, 1973; http://www.lawsofform.org/aum/session1.html.
6. *LoF*, xxviii.
7. Clam, "System's Sole Constituent," 68–69.
8. *LoF*, xi.
9. Varga von Kibéd and Matzka, "Motive und Grundgedanke," 58. A very good illustration of the latter is a series of articles published in a special double issue of *Cybernetics and Human Knowing* (2001), subtitled *Peirce and Spencer-Brown—History and Synergies in Cybersemiotics*. Particularly important when it comes to the direct

relationship between Peirce's notation and the *Laws of Form* is Engstrom, "Precursors to *Laws of Form*."

10. Clam, "System's Sole Constituent," 68–69; compare *LoF*, 96, and Kauffmann: "Self-Reference and Recursive Forms," "The Mathematics of Charles Sanders Peirce," and "On the Cybernetics of Fixed Points."
11. Kuroki constructed an extensive Web site arguing against Spencer-Brown (in Japanese): http://www.math.tohoku.ac.jp/%/7Ekuroki/SB/index.html#abstract.
12. As one sees upon closer examination, both the law of calling and the law of crossing refer to the "closure" or "continence" of the form. On the meaning of continence and the importance of its epistemology, see Varga von Kibéd and Matzka, "Motive und Grundgedanke," 60. Beyond doubt, it contains hints to the form of self-reference, explained in chapters 11 and 12. "A mark or sign intended as an indicator is self-referential" (Kauffmann, "Self-Reference and Recursive Forms," 58).
13. *LoF*, xxix.
14. Ibid., viii.
15. Wittgenstein, *Logisch-philosophische Abhandlung*, § 3.333. In the *Tractatus*'s original:

    > Eine Funktion kann darum nicht ihr eigenes Argument sein, weil das Funktionszeichen bereits das Urbild seines Arguments enthält und es sich nicht selbst enthälten kann. Nehmen wir nämlich an, die Funktion F(fx) könnte ihr eigenes Argument sein, dann gäbe es also einen Satz: <<F(F(fx))>> und in diesem müssen die aüßere Funktion F und die innere Funktion F verschiedene Bedeutungen haben, denn die innere hat die Form φ(fx), die äußere die Form ψ(φ(fx)). Gemeinsam ist den beiden Funktionen nur der Buchstabe <<F>>, der aber allein nichts bezeichnet. Dies wird sofort klar, wenn wir statt <<F(F(u))>> schreiben <<(∃φ): F(φu). φu = Fu>>. (Wittgenstein, *Logisch-philosophische Abhandlung*, 34)

16. *LoF*, 1–3.
17. References to Peirce's mathematics are not accidental here: even in notation, the resemblance of Spencer-Brown's form with Peirce's notation (for example, the "sign of illation") is striking. Excellent references here are Engstrom, "Precursors to *Laws of Form*," and Kauffmann, "The Mathematics of Charles Sanders Peirce."
18. *LoF*, 5.
19. Spencer-Brown uses this obscure passage from the first chapter of the *Tao-te-king* as the opening quote for *LoF*.
20. *LoF*, 1.
21. *LoF*, 3. Obviously one must not necessarily agree with this conception, but if so, one places oneself outside the calculus (and we are clearly not willing to do so here). In the calculus, one is expected to obey the injunctions of mathematical communication and *to obey only them and them only*: "In general, *what is not allowed is forbidden*" (Ibid.; emphasis in original).
22. On how *LoF* may aid our understanding of Boolean algebra, see Meguire, "Discovery Boundary Algebra."

23. *LoF*, xxi–xxiii, 87ff.
24. Http://www.lawsofform.org/aum/session2.html.
25. Ibid.
26. Esposito, "Ein zweiwertiger nicht-selbstständiger Kalkül," 99–100.
27. Ibid., 104–5.
28. *LoF*, 68. Therefore, the reference to infinite forms emerges as equivalent to the famous phrase of §7 in the *Tractatus*: "What we cannot speak about we must pass over in silence" (Wittgenstein, *Tractatus logico-philosophicus*, 151). It is the part "in" the calculus that embodies reference to the "outside" of the calculus. Consequently, it has an equivalent problematic status: it is the paradox *saying* that there are *things* about which *nothing* can be *said*. Consider, in this regard, Bertrand Russell's remarks on §7of the *Tractatus*: "What causes hesitation is the fact that, after all, Mr. Wittgenstein manages to say a good deal about what cannot be said, thus suggesting to the skeptical reader that possibly there may be some loophole through the hierarchy of languages, or by some other exit" (preface to Wittgenstein, *Tractatus logico-philosophicus*, xxi).
29. Luhmann, *Social Systems*, 31.
30. With attention to the linguistic difficulties of describing the unity and difference of distinction and indication, see Schiltz and Verschraegen, "Spencer-Brown, Luhmann and Autology," 65–70.
31. *LoF*, 85.
32. "The fact that men have for centuries used a plane surface for writing means that, at this point in the text, both author and reader are ready to be conned into the assumption of a plane writing surface without question. But, like any other assumption, it is not unquestionable, and the fact that we can question it here means that we can question it elsewhere. In fact we have found a common but hitherto unspoken assumption underlying what is written in mathematics, notably a plane surface. . . . Moreover, it is now evident that if a different surface is used, what is written on it, although identical in marking, may be not identical in meaning" (*LoF*, 86).
33. *LoF*, 99. For examples of Spencer-Brown's engineering work, see http://www.lawsofform.org/patents/index.html.
34. Junge, "Medien als Selbstreferenzunterbrecher," 127. This is not a particularly "exotic" venture: "Die Bestimmung von Räumen aufgrund der Relationen zwischen bestimmeten Elementen ist nichts ungewöhnliches: Man konstruiert zum Beispiel eine Landkarte auf Grundlage der Entfernungen zwischen den einzelnen Orten. Diese Entfernungen lassen sich aber auf verschiedene Weisen messen: die Entfernung auf der Luftlinie in Kilometern ergibt ander Distanzen als die Kilometerzahl, die man mit dem Auto zu absolvieren hätte. . . . *Wenn man den Raum operational definiert, wird die am ihm geschulte Logik unter Umständen flexibler*" (127–28; my emphasis).
35. From the discussion below, the reader will understand why "turning up" is the accurate term here.
36. *LoF*, 59ff., 100ff. "To corrupt," synonymous to "to destroy the integrity of" (from Latin *in-tangere*, "untouched," undivided).

37. This topological solution seems to refer to a famous dictum: "The medium is the message" (Marshall MacLuhan). And indeed, there exist some attempts to interpret *LoF* in a medium-theoretical way, such as Lehmann, "Das Medium der Form."
38. The reader must realize that time has thus been *created* as a consequence of a type of space—namely, space in which form can relate to itself and, as such, change (change being the measure of time). Time is thus nothing pregiven. Neither is space: "The first state, or space, is measured by a distinction between states. There is no state for a distinction to be made in. If a distinction could be made, then it would create a space. That is why it appears in a distinct world that there is space. Space is only an appearance. It is what would be if there could be a distinction" (http://www.lawsofform.org/aum/session1.html).
39. *LoF*, 61.
40. Ibid., 65.
41. Schiltz and Verschraegen, "Spencer-Brown, Luhmann and Autology," esp. 65ff. Kay Junge explains this as the medium (of the torus) "blocking" direct self-reference ("Medien als Selbstreferenzunterbrecher").
42. Wagner, "The End of Luhmann's Social Systems Theory," 391.
43. Ibid., 397, 399ff.
44. Luhmann, *Social Systems*, 178.
45. This is not to say that there is a complete and perfect equivalence between the system and the environment; rather, there is a fundamental asymmetry, to the advantage of the system. As a result of this basic asymmetry, the environment does not contain distinctions; it is not a piece of information. Distinctions are found only in a system. Systems observe, while there is nothing self-referential or systemic to the environment.
46. Kauffman, "The Mathematics of Charles Sanders Peirce," 137.
47. *LoF*, 1. As Varga von Kibéd and Matzka stress, continence should be interpreted on the basis of its etymological relationship to the Latin "continere," "to hold together." Thus, "distinction is perfect continence" makes clear not only that the origin of the two sides is to be contained in the distinction, but also (and primarily) that the form has no anchoring in any outside reality or foundation; it is the context of itself, it is "closure."
48. It is important to stress similarities between *LoF* and lambda-calculus. A lambda domain is a class of objects that can act on one another to form new objects of the same kind. It thus presupposes self-reference or recursivity. In "The Mathematics of Charles Sanders Peirce," Kauffman provides most obvious clues on the recursivity of the universe and mathematics in a comparison of Spencer-Brown's ideas with the ideas of Charles Sanders Peirce (102ff.).
49. I surmise this is the meaning and function of the mysterious "unwritten cross," already introduced in the calculus's very beginning: "Suppose any s^n [primal space] to be surrounded by an unwritten cross" (*LoF*, 7).
50. Ibid., 105.
51. Quine, "On What There Is."

52. Clam, "Die Grundparadoxie des Rechts und ihre Ausfaltung," 133.
53. Ibid., 135.
54. Luhmann, "The Autopoiesis of Social Systems," 174; see as well Luhmann, *Social Systems*, 377ff.
55. Luhmann, *Die Gesellschaft der Gesellschaft*, 66.
56. Compare Clam, "Die Grundparadoxie des Rechts und ihre Ausfaltung," 136.
57. Luhmann, "Funktion und Kausalität."
58. Luhmann, *Observations on Modernity*, 44–62.
59. Luhmann, *Social Systems*, 52.
60. *LoF*, 68.
61. Ibid., 106.
62. For a comparison with traditional epistemologies, see Christis, "Luhmann's Theory of Knowledge."
63. *LoF*, 68.
64. Ibid., 101.
65. Ibid. This is, I am afraid, one of the main reasons why *LoF* does have a poor reception. Among others, Kuroki Gen perceives the assumedly mystical nature of *LoF* (especially its numerous forewords) and its cult status with the psychedelic generation of the 1960s as a reason to ignore its insights. Beyond doubt, Spencer-Brown is to a large extent responsible for further isolating *LoF* from a broader academic public, and that is deplorable.
66. Brauns, *Form und Medium.*
67. This leads to the question why Spencer-Brown does not employ a similar view as form as the difference between itself and the medium in which it is written. Peter Fuchs has argued that *LoF* does not need the notion of "medium" as its field of inquiry is mathematics, and as such without heuristical aspirations. As my above considerations on *LoF* show, I am not so sure this is the case. Spencer-Brown makes mention of the notion of medium in at least two passages (see Schiltz, "Form and Medium"). Furthermore, he obviously employs the notion of medium in order to be able to show the possibility of self-referential forms (the medium of plane space versus the medium of the torus). Yet I believe he does not pronounce it in view of his utmost attention for notational matters: "Returning, briefly, to the idea of existential precursors, we see that if we accept their form as endogenous to the less primitive structure identified, in present-day science, with reality, we cannot escape the inference that what is commonly regarded as real consists, in its very presence, merely of tokens or expressions" (*LoF*, 104). In principle, *LoF* can suffice with its topological notation, as long as we are aware of its intricacies (for example, the distinction between crosses and markers in chapter 11).
68. Compare *LoF*, 102ff.
69. Kauffman, "The Mathematics of Charles Sanders Peirce," 105.