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GLOBAL POPULATION GROWTH, TECHNOLOGY, AND MALTHUSIAN CONSTRAINTS: A QUANTITATIVE GROWTH THEORETIC PERSPECTIVE

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Global Population Growth, Technology, and Malthusian Constraints: A Quantitative Growth Theoretic Perspective*

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Abstract

We study the interactions between global population, technological progress, per capita income, the demand for food, and agricultural land expansion over the period 1960 to 2100. We formulate a two-sector Schumpeterian growth model with a Barro-Becker representation of endogenous fertility. A manufacturing sector provides a consumption good and an agricultural sector provides food to sustain contemporaneous population. Total land area available for agricultural production is finite, and the marginal cost of agricultural land conversion is increasing with the amount of land already converted, creating a potential constraint to population growth. Using 1960 to 2010 data on world population, GDP, total factor productivity growth and crop land area, we structurally estimate the parameters determining the cost of fertility, technological progress and land conversion. The model closely fits observed trajectories, and we employ the model to make projections from 2010 to 2100. Our results suggest a population slightly below 10 billion by 2050, further growing to 12 billion by 2100. As population and per capita income grow, the demand for agricultural output increases by almost 70% in 2050 relative to 2010. However, agricultural land area stabilizes by 2050 at roughly 10 percent above the 2010 level: growth in agricultural output mainly relies on technological progress and capital accumulation.

Keywords: Economic growth; Population projections; Technological progress; Endogenous fertility; Endogenous innovations; Land conversion; Food security

JEL Classification numbers: O11, O13, J11, C53, C61, Q15, Q24.

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1 Introduction

World population has doubled over the last fifty years and quadrupled over the past century (United Nations, 1999). During this period and in most parts of the world, productivity gains in agriculture have confounded Malthusian predictions that population growth would outstrip food supply. Population and income have determined the demand for food, and thus agricultural production, rather than food availability determining population. However, the amount of land that can be brought into the agricultural system is physically finite, and the predominant view is that population will reach a steady state in the next century (United Nations, 2013). Our aim in this paper is to understand how the evolution of population and land interact with technological progress and income growth that occurred in recent history and that can be expected to continue over the coming century.

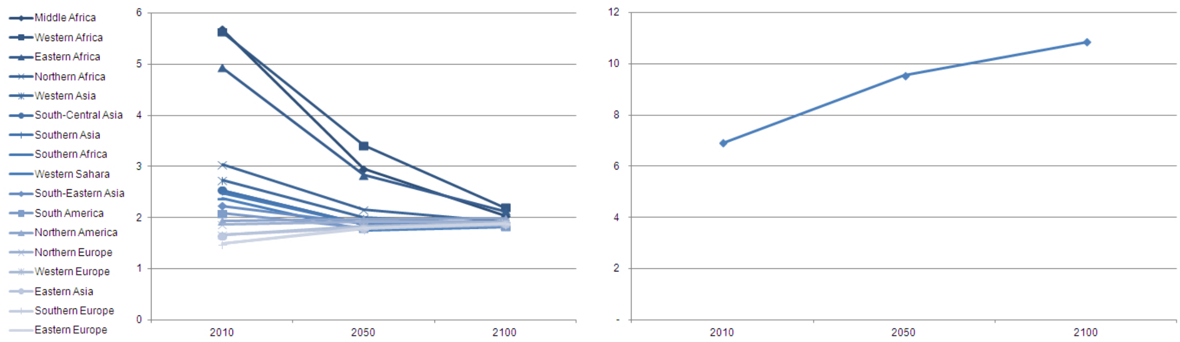
Despite the importance of understanding global population change, and how it interacts with per capita income growth, food production, and technology, few economists have contributed to the debate about population projections. Instead, the *de facto* standard source of demographic projections is the United Nations' series of *World Population Prospects*, updated every two years. The latest version projects a global population, on a medium scenario, of 9.6 billion in 2050 and 10.9 billion in 2100, by which time the population growth rate is close to zero (United Nations, 2012).[1] The crucial assumption of the medium scenario, displayed in Figure 1, is that all countries around the world converge to a replacement fertility rate of 2.1 over the next 100 years, irrespective of their starting point (see Lutz and Samir, 2010, for a discussion of the approach).

Because of the long time horizon considered, UN projections are highly sensitive to the assumed trajectory for fertility. The UN's two other scenarios, the low and high fertility variants forecast population size in 2100 to between 6.8 and 16.6 billion respectively. The difference between these two scenarios is driven entirely by varying the convergence fertility rate by increments of half a child above and below the medium assumption (giving target rates of 1.6

[1] The UN uses a so-called 'cohort-component projection method', i.e. it works from the basic demographic identity that the number of people in a country at a particular moment in time is equal to the number of people at the last moment in time, plus the number of births, minus the number of deaths, plus net migration, all of this done for different age groups. This requires assumptions about fertility, mortality and international migration rates, which are exogenously determined.

Figure 1: United Nations projections 2010 – 2100 (United Nations, 2013)

(a) Regional fertility (b) World population



and 2.6 children per woman for the low and high variants respectively). Small variations in the fertility trajectories for countries in Asia and Africa in particular account for most of the variance in population projections.[2] These are precisely the regions for which uncertainty about the evolution of fertility is large, and empirical evidence in developing countries suggests no clear pattern of convergence towards a low fertiliy regime (Strulik and Vollmer, 2013).

The purpose of this paper is to contribute a new, macro-economic approach to global population forecasting. We formulate a behavioral model linking child rearing decisions to per capita income and availability of food, making the path for fertility an outcome rather than an assumption. More specifically, we develop a model of endogenous growth in which households have preferences over own consumption, the number of children they have and the utility of their children, in the tradition of Barro and Becker (1989). Child rearing is time intensive, and fertility competes with other labor-market activities. In order to capture the well-documented complementarity between human capital and the level of technology (Goldin and Katz, 1998), we posit a positive relationship between the cost of fertility and technological progress. Thus technological progress implies a higher human capital requirement, so that population increments need more education and are thus more costly. As in Galor and Weil (2000), the opportunity cost of fertility increases over time, implying a gradual transition to low fertility, and a decline in population growth.

[2] Using the UN's cohort-component method, imposing the 'high' fertility scenario in these regions alone, so that they converge to a fertility rate of 2.6 rather than 2.1, implies a global population of around 16 billion by 2100.

Aside from the time required for child rearing and education, the other key constrain to population growth is food availability. We make agricultural output a necessary condition to sustain population, and assume the demand for food increases with both the size of the population and per capita income, the latter capturing changes in diet as affluence rises (Subramanian and Deaton, 1996, e.g.). An agricultural sector, which meets the demand for food, requires land as an input, and agricultural land has to be converted from a stock of natural land. Therefore, as population and income grow, the demand for food increases, raising the share of land in agricultural use. In the model, land is a potential constraint to growth in agricultural output. First, productivity of marginal lands declines as conversion occurs, so that the marginal cost of land conversion increases over time. Second, the total amount of land is physically finite, potentially generating a Malthusian constraint to long run population development.

Whether land conversion acts as a constraint to population growth mainly depends on technological progress. We model the process of knowledge accumulation in the Schumpeterian framework of Aghion and Howitt (1992), where the growth rate of total factor productivity (TFP) increases with labor hired for R&D activities. A well known drawback of such a representation of technological progress is the population scale effect (see Jones, 1995a).[3] Following Chu et al. (2013), we 'neutralize' the scale effect by making the growth rate of TFP a function of the *share* of labor allocated to R&D. This implies that long run growth can occur without the need for population to grow.

Because our main contribution is to make population projections using growth theory as a framework, we do not report analytical results. Rather, we formulate a quantitative version of a standard growth model and take it to the data.[4] Most of the parameters of the model are either imposed or calibrated from external sources. However, the parameters determining the marginal cost of population, labor productivity in R&D and labor productivity in land clearing

[3] The population scale effect implies that productivity growth is proportional to population growth, which is at odds with empirical evidence (Jones, 1995b; Laincz and Peretto, 2006). This is particularly important in a setting with endogenous population, as it would imply that population would be a fundamental driver of long run technology and income growth.

[4] Of course, projections will depend on the particular model structure we assume. While this is a first attempt to use growth theory for the purpose of doing quantitative population projections, this approach has long been used for monetary policy or climate policy for example. For one, the fact that we structurally estimate the model over 50 years of data implies a great deal of robustness in the trajectories we derive, as illustrated by our sensitivity analysis. For another, as a complement to the traditional approach to population projections, focusing the discussion on the structure of the model rather than assumptions about fertility is a virtue of our approach.

are structurally estimated with simulation methods. We use the 1960 – 2010 data on world population, GDP, TFP growth and crop land area to define a minimum distance estimator comparing observed trajectories and those simulated from the model.

Our model closely replicates observed data, and we then employ the model to make projections until 2100. Our quantitative results suggest a population of 9.85 billion by 2050, further growing to 12 billion by 2100. These numbers are above current UN projections (United Nations, 2013), and they lie on the upper limit of the 95 percent confidence interval implied by the probabilistic projections reported in Lutz and Samir (2010).[5] Although population *growth* declines over time, it remains positive over the period we consider, and population does not reach a steady state. We further find that land conversion for agriculture stops by 2050 at around 1.77 billion hectares, a 10 percent increase relative to 2010, which is about the same magnitude as projections by the Food and Agriculture Organization (Alexandratos and Bruinsma, 2012).[6] A direct implication of our work is that the total amount of natural land is never exhausted, even though our projections are rather conservative in terms of technological progress (TFP growth in both sectors is below one percent per year and declining from 2010 onwards).

As a corollary to its ambitions of integrating technological progress and food constraints to evaluate population dynamics, our work necessarily relies upon a number of simplifications. Among these, perhaps the most important is that of a representative agent, which misses out on the age-structure and regional heterogeneity of population around the globe.[7] Another important simplification is implied by the fact that we solve for the social planner representation of the problem. While this makes the formulation of the problem simpler and allows us to exploit efficient solvers for constrained non-linear optimization, it abstracts from externalities that would arise in a decentralized equilibrium (see Romer, 1994, for example). To some extent,

[5] Probabilistic projections by Lutz and Samir (2010) use the same cohort-component method but weight different fertility scenarios at the country level by some probability distributions. Being based on the assumption that all countries converge to replacement fertility, at the median population stops growing by 2050 and remains around 9 billion.

[6] This corresponds to a conversion of natural land towards agriculture of 150 million hectares, roughly the area of Mongolia or three times that of Spain. Because developed countries will likely experienced a decline in agricultural land area (Alexandratos and Bruinsma, 2012), land conversion in developing countries will likely amount to more than that.

[7] See Mierau and Turnovsky (2014) for a more realistic treatment of age-structured population in a general equilibrium growth model. To keep the model tractable however, they have to treat the demographic structure as exogenous. Integrating a richer representation of population heterogeneity into a model with endogenous fertility decisions remains an important research topic.

however, both the age-structure of the population and market imperfections prevailing over the estimation period will be reflected in the parameters that we estimate from observed trajectories. Nevertheless, we advise against a too literal interpretation of our work, which should mainly be seen as an attempt to see population projection through the lens of endogenous growth theory, and thus a complement to existing approaches.

1.1 Related literature

Population projections are mostly produced by demographers working for governmental agencies, and work by economists in this area is scarce. However, this paper relates to at least three stands of research on economic growth. First, there is unified growth theory, which studies economic development and population over the long run. Seminal contributions include Galor and Weil (2000) and Jones (2001). Jones (2003) and Strulik (2005) analyze the joint development of population, technological progress and human capital (see also Tournemaine and Luangaram, 2012, for a recent investigation and comprehensive overview of the literature). Similar to the present paper, Hansen and Prescott (2002) and Strulik and Weisdorf (2008) consider the role of agriculture and manufacturing activities along the development path. The structure of our model, linking technology and economic growth with population and human capital, is close to these papers, although an important aspect of our work is the potential constraint to growth represented by food and land availability.

A share of the literature in unified growth theory treats technological progress as either exogenous or relies on the scale effect as a way to generate the take-off phase of economic development. To circumvent the scale effect, a second group of papers related to ours formulates 'product line' representations of technological progress that match stylized fact on growth (see Dinopoulos and Thompson, 1998; Peretto, 1998; Young, 1998, for seminal contributions).[8] These recent models have been used to develop theories of endogenous population and resource constraints, most notably Peretto and Valente (2011) and Bretschger (2013), and these theoret-

[8] In a product line representation of technological progress, the number of product grows with population, thereby diluting R&D inputs, so that long-run growth doesn't necessarily rely on population growth rate, but rather on the share of labor in the R&D sector. An other strategy to address the scale effect involves postulating a negative relationship between labor productivity in R&D and the existing level of technology, giving rise to "semi-endogenous" growth models (Jones, 1995a). In this setup however, long-run growth is only driven by population growth, which is also at odds with empirical evidence (Ha and Howitt, 2007).

ical contributions are thus close in spirit to our work. Importantly, Chu et al. (2013) show that the qualitative behavior of our Schumpeterian representation of R&D is in line with the product line representation of technological progress, and thus provides a good basis to study growth in recent history.

A final stream of papers has in common with us the use of a quantitative macroeconomic model to study demographic and economic development, mostly (but not exclusively) in the U.S. and in England. These include Mateos-Planas (2002), Bar and Leukhina (2010), Jones and Schoonbroodt (2010), and Ashraf et al. (2013). Models developed in these papers are close to those developed in unified growth theory, demonstrating that this kind of model captures essential features of the demographic transition that occurred in developed countries. However, none of these papers draw implications for future population and economic development. One notable exception is the work reported in Strulik and Weisdorf (2008; 2014) which uses simple quantitative growth model with the scale effect to look at the evolution of *growth rates* for the U.S. economy from 1200 to 2200.

The remainder of the paper is structured as follows. The structure of the model is laid out in Section 2 and Section 3 describes the ensuing optimization problem and estimation strategy. Section 4 reports projections with the model. Sensitivity analysis is provided in Section 5, and we discuss some broader implication of our results in Section 6. Some concluding comments are provided in Section 7.

2 The economy

We study an economy extending over an infinite horizon and treat time as discrete. The economy comprises two production sectors. The first sector, “manufacturing”, produces a homogeneous aggregate good which can be consumed or invested to build up a stock of capital. The second sector, “agriculture”, produces food with the sole purpose of sustaining current population. Manufacturing output thus represents the traditional activity in a one-sector growth model, while agriculture supports the evolution of population and contributes to welfare only indirectly. Manufacturing and agriculture use labor and capital as primary inputs, but agriculture also requires land. Agricultural land is itself *produced* by applying labor to a finite stock of natural land.

We consider the planner's problem of allocating labor and capital to maximize the utility of a representative dynastic household. There are two core dynamic processes at play in addition to capital accumulation and natural land conversion. First, endogenous fertility decisions by a representative household *à la* Barro and Becker (1989) determine population growth. Second, sectoral technological progress is determined by the allocation of labor to R&D activities in the framework of Aghion and Howitt (1992).

2.1 Production

In agriculture and manufacturing aggregate output is represented by a constant-returns-to-scale production function with endogenous Hicks-neutral technological change.[9] In manufacturing, aggregate output in period t is given by a standard Cobb-Douglas production function:

$$Y_{t,mn} = A_{t,mn} K_{t,mn}^{\vartheta} N_{t,mn}^{1-\vartheta} , \tag{1}$$

where $Y_{t,mn}$ is real manufacturing output at time t, $A_{t,mn}$ is an index of productivity in manufacturing, $K_{t,mn}$ is capital allocated to manufacturing, $N_{t,mn}$ is the workforce allocated to manufacturing, and $\vartheta \in (0,1)$ is a share parameter. Since technical change is Hicks-neutral, the assumption that output is Cobb-Douglas is consistent with long-term empirical evidence (Antràs, 2004).

In agriculture, land services from the stock of converted land, X_t, are included as a factor input, and we posit a two-stage constant elasticity of substitution (CES) functional form (e.g. Kawagoe et al., 1986; Ashraf et al., 2008):

$$Y_{t,ag} = A_{t,ag} \left[(1-\theta_X) \left(K_{t,ag}^{\theta_K} N_{t,ag}^{1-\theta_K} \right)^{\frac{\sigma-1}{\sigma}} + \theta_X X_t^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}} , \tag{2}$$

where $\theta_{X,K} \in (0,1)$, and σ is the elasticity of substitution between a capital-labor composite factor and agricultural land. This specification provides flexibility in specifying how capital and labor can be substituted for land, and it nests the Cobb-Douglas specification as a special case

[9] Assuming technological change is Hicks-neutral, so that improvements to production efficiency do not affect the relative marginal productivity of input factors, considerably simplifies the analysis at the cost of abstracting from a number of interesting issues related to the direction or bias of technical change (see Acemoglu, 2002).

($\sigma = 1$). While a Cobb-Douglas function is often used to characterize aggregate agricultural output (e.g. Mundlak, 2000; Hansen and Prescott, 2002), it is quite optimistic in that, in the limit, land is not required for agricultural production, and long-run empirical evidence reported in Wilde (2013) indeed suggests that $\sigma < 1$.

2.2 Innovations and technological progress

The evolution of sectoral TFP is based on a discrete time version of the Schumpeterian model by Aghion and Howitt (1992). In this framework innovations are drastic, so that a firm holding the patent for the most productive technology temporarily dominates the industry until the arrival of the next innovation. The step size of productivity improvements associated with an innovation is denoted $s > 0$, and we conventionally assume that it is the same in both sectors.[10] Without loss of generality, we assume that there can be at most $I > 0$ innovations over the length of a time period, so that the maximum growth rate of TFP each period is $S = (1 + s)^I$. For each sector $j \in \{mn, ag\}$, the growth rate of TFP is then determined as a share of the maximum feasible TFP growth, which in turn depends on the number of innovations arriving within each time period:[11]

$$A_{t+1,j} = A_{t,j} \cdot (1 + \rho_{t,j} S) \ , \quad j \in \{mn, ag\} \, . \tag{3}$$

where $\rho_{t,j}$ is the arrival *rate* of innovations each period, in other words how many innovations were achieved compared to the maximum number of innovation.

R&D in manufacturing and in agriculture hires labor as the main determinant of innovation:

$$\rho_{t,j} = \overline{\lambda}_{t,j} \cdot N_{t,A_j} \ , \quad j \in \{mn, ag\} \, ,$$

where N_{t,A_j} is labor employed in R&D for sector j and $\overline{\lambda}_{t,j}$ measures labor productivity. An

[10] In general the "size" of an innovation in the Aghion and Howitt (1992) framework is taken to be the step size necessary to procure a right over the proposed innovation. For the purposes of patent law, an innovation must represent a substantial improvement over existing technologies (not a marginal change), which is usually represented as a minimum one-time shift.

[11] The arrival of innovations is a stochastic process, and we implicitly made use of the law of large number to smooth out the random nature of growth over the discrete time intervals. Our representation is qualitatively equivalent, but somewhat simpler, to the continuous time version of the model where the arrival rate of innovations is described by a Poisson process.

important issue with this representation of technological progress has to do with the "scale effect", or an implied positive relationship between population size and technology growth, which contradicts empirical evidence (e.g. Jones, 1995b; Laincz and Peretto, 2006). In the present paper we work with the scale-invariant version of Aghion and Howitt (1992) proposed by Chu et al. (2013), where $\overline{\lambda}_{t,j}$ is specified as a decreasing function of the scale of the economy. In particular, we define $\overline{\lambda}_{t,j} = \lambda_j N_{t,A_j}^{\mu_j - 1} / N_t^{\mu_j}$, where $\lambda_j > 0$ is a productivity parameter and $\mu_j \in (0,1)$ is an elasticity. Including population N_t in the denominator, so that innovation depends on the share of labor allocated to R&D, neutralizes the scale effect and is in line with more recent representations of technological change (see Dinopoulos and Thompson, 1998; Peretto, 1998; Young, 1998, for example). Furthermore, our representation of R&D implies decreasing returns to labor in R&D through the parameter μ_j, which captures the duplication of ideas among researchers (Jones and Williams, 2000).

2.3 Land

As a primary factor, land used for agriculture has to be converted from a finite stock of land $\overline{X}$. Converting land from the available stock requires labor, therefore there is a cost in bringing new land into the agricultural system. Once converted, agricultural land gradually depreciates back to the stock of natural land with a linear depreciation rule. Thus the allocation of labor to convert land determines the amount of land available for agriculture each period, and over time the stock of land used in agriculture develops as:

$$X_{t+1} = X_t(1 - \delta_X) + \psi \cdot N_{t,X}^{\varepsilon}, \quad X_0 \text{ given}, \quad X_t \leq \overline{X}, \tag{4}$$

where $\psi > 0$ measures labor productivity in land clearing activities, $\varepsilon \in (0,1)$ is an elasticity, $N_{t,X}$ is labor allocated to land clearing activities, and the depreciation rate δ_X measures how fast converted land reverts back to natural land.

One important aspect of equation (4) is that decreasing returns to labor in land clearing activities implies that the accumulation dynamics are non-linear. In other words, as the amount of land used in agriculture increases, labor requirements to avoid it depreciating back to its natural state increase more than proportionally. In turn, the marginal cost of adding new land to agricultural production increases with the existing stock of agricultural land. Intuitively, this captures the fact that the most productive agricultural plots are converted first, whereas marginal land still available at a later stage of land conversion are less productive. Labor can be used to bring these marginal plots into agricultural production, although the costs of such endeavors increases as the total land area is exhausted.

2.4 Preferences and population dynamics

A representative household is assumed to have preferences over its own consumption of the manufacturing good c_t, the number of children it has n_t, and the utility its children will experience in the future $U_{i,t+1}$. We use the class of preference by Barro and Becker (1989) defined recursively as $U_t = u(c_t) + b(n_t) \sum_{i=0}^{n_t} U_{i,t+1}$, where $u(\cdot)$ is the per-period utility function, and the function $b(\cdot)$ specifies preferences for fertility. Following Alvarez (1999), we define a dynasty as the set of households with a common ancestor, so that the size of the dynasty coincides with total population N_t, and use the equivalent dynastic representation for household preferences. Using constant elasticity functions $u(c_t) = \frac{c_t^{1-\gamma}-1}{1-\gamma}$, where $1/\gamma$ is the intertemporal elasticity of substitution, and $b(n_t) = \beta n_t^{\eta}$, where $\beta < 1$ is the discount factor and η is an elasticity determining how the utility of parents changes with n_t, it is straightforward to show that sequential substitution yields the utility of the dynasty head in terms of aggregate quantities:

$$U_0 = \sum_{t=0}^{\infty} \beta^t N_t^{\eta} \frac{(C_t/N_t)^{1-\gamma} - 1}{1-\gamma} , \tag{5}$$

where $C_t = c_t N_t$ is aggregate consumption in t.

As Alvarez (1999) notes, the dynastic head formulation is useful because it makes the problem technically simpler. First, the objective is defined in terms of aggregate quantities and the horizon of the dynastic head coincides with that of a social planner. Because household preferences are defined recursively, the sequence of decisions by households in period 0 will be the same as that decided by households in period t. Second, parametric restrictions ensuring overall concavity of the objective and in turn existence and uniqueness of the solution are easy to impose. For $\gamma > 1$, which is consistent with empirical evidence on the intertemporal elasticity of substitution, concavity of Equation (5) in (C_t, N_t) requires $\eta \in (0, 1)$. This implies that, depending on η, preferences of the dynastic head have both classical and average utilitarian objectives as limiting cases.[12]

Aggregate consumption is derived from the manufacturing sector. Given the social planner representation, manufacturing output can either be consumed by households or invested into a stock of capital:

$$Y_{t,mn} \;=\; C_t + I_t \,, \tag{6}$$

where C_t and I_t are aggregate consumption and investment respectively. The accumulation of capital is then given by:

$$K_{t+1} \;=\; K_t(1-\delta_K) + I_t \,, \quad K_0 \text{ given} \,, \tag{7}$$

where δ_K is a per period depreciation rate. In this formulation investment decisions mirror those of a one-sector economy (see Ngai and Pissarides, 2007, for a similar treatment of savings in a multi-sector growth model).

In each period, fertility n_t determines the change in population together with mortality d_t:

$$N_{t+1} \;=\; N_t + n_t - d_t \,, \quad N_0 \text{ given} \,. \tag{8}$$

We make the simplifying assumption that population equals the total labor force, so that n_t and d_t represent an increment and decrement to the stock of effective labor units, respectively. The mortality rate is assumed to be constant, so that $d_t = \delta_N$. As in Jones and Schoonbroodt (2010), δ_N essentially captures the expected working lifetime $1/\delta_N$.

The cost of fertility consists of both child rearing time and human capital embodied in workers. We again exploit the social planner representation which allows to jointly treat formal and informal time spent raising and educating children as a single activity. We thus write the production of effective labor units as a function of labor $N_{t,N}$ allocated to increasing the stock of

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[12] See Baudin (2010) for a discussion of the relationship between dynastic preferences and different classes of social welfare functions.

effective labor units:

$$n_t = \overline{\chi}_t \cdot N_{t,N},$$

where $\overline{\chi}_t$ is an inverse measure of the time cost of producing effective labor units. As in the standard model of households' fertility choices (Becker, 1960; Barro and Becker, 1989), child rearing competes with other labor-market activities. In addition, we characterize the complementarity between technology and skills of the labor force (Goldin and Katz, 1998) by postulating an increasing relationship between the time cost of child rearing and the level of technology (Galor and Weil, 2000): $\overline{\chi}_t = \chi N_{t,N}^{\zeta-1}/A_t^{\omega}$, where $\chi > 0$ is a productivity parameter, and $\zeta \in (0,1)$ is an elasticity representing scarce factors required in child rearing,[13] A_t is an index of technology and $\omega > 0$ measures how the cost of children increases with the level of technology. This implies that, as the stock of knowledge in the economy grows, additions to the stock of effective labor units become increasingly costly because of the additional human capital requirements. In other words, technological progress requires an increase in the "quality" of labor, increasing the opportunity cost of fertility and inducing a transition to a low fertility regime.

Population dynamics are further constrained by food availability, as measured by agricultural output. Specifically, the agricultural sector sustains contemporaneous population in proportion to the per capita demand for food, denoted $\overline{f_t}$. Therefore the market for agricultural output acts as a constraint on population $Y_t^{ag} = N_t\overline{f_t}$, and food availability will affect social welfare through its impact on population. Per capita demand for food determines the quantity of food required for maintaining an individual in a given society, and captures both physiological requirements (e.g. minimum per capita caloric intake) and the positive relationship between the demand for food and per capita income, reflecting changing diet. The relationship between food expenditures and per capita income, however, is not linear, and we specify the demand for food as a concave function of per capita income: $\overline{f} = \xi \cdot \left(\frac{Y_{t,mn}}{N_t}\right)^{\kappa}$, where ξ is a scale parameter and $\kappa > 0$

[13] More specifically, ζ captures the fact that the costs of child rearing over a period of time increases more than linearly with the number of children (see Barro and Sala-i Martin, 2004, p.412 and Bretschger, 2013).

is the income elasticity of food consumption.[14]

3 Optimal control problem and empirical strategy

The aim of this section is to use the model to replicate recent history for the main quantities of interest, as a basis for long-run projections of population and land use. We first discuss the intuition behind the optimal control problem we consider. We then describe the numerical optimization problem and how we take the model to the data, fitting simulated trajectories for GDP, TFP growth, world population and crop land to observed trajectories from 1960 to 2010.

3.1 Intuition for the optimal control problem

We consider a social planner choosing paths for C_t, $K_{t,j}$ and $N_{t,j}$ by maximizing the utility of the dynastic head (5) subject to technological constraints (1), (2), (3), (4), (6), (7), (8) and market clearing conditions for capital and labor:[15]

$$K_t \;=\; K_{t,mn} + K_{t,ag}\,, \quad N_t \;=\; N_{t,mn} + N_{t,ag} + N_{t,A_{mn}} + N_{t,A_{ag}} + N_{t,N} + N_{t,X}\,.$$

As noted previously, the social planner formulation simplifies a number of aspects of the problem, although it assumes away externalities associated with the R&D activities. Moreover, since our aim is to use the model to make quantitative projections, and given the number of variables – nine control and five state – it considerably simplifies the numerical solution method.

The dynastic head values both aggregate consumption and population. Aggregate consumption derives from the manufacturing sector, and thus relies upon the allocation of capital and labor to the manufacturing sector, on the allocation of labor to manufacturing R&D, and on the saving rate. The latter determines how manufacturing output is shared between investment and

[14] Formally, households do not respond to changes in the relative price of food and non-food products, but have a fixed demand for food that is proportional to their wealth. In a decentralized setting, this would generate an externality. However since we solve for the central planner problem the demand for food is fully internalized by the planner.

[15] These market clearing equations implies perfect factor mobility across sectors. In a deterministic setting where there are no unexpected shocks to factor prices, this assumption has no qualitative implications for our results. In the presence of uncertainty, perfect factor mobility will imply that the transitional dynamics following the unexpected events will appear less costly than in a situation where capital and/or labor are at least partially sector specific.

consumption.

More interestingly, the impact of population on welfare depends on the elasticity of substitution between fertility and the welfare of children. We consider the intuitive case where these are complements, so that better prospects for children are associated with higher fertility, *all other things being equal*. Given the dynastic representation of preferences, this implies that consumption and population are complements. Complementarity between consumption and population is the traditional assumption made by Barro and Becker (1989), although assuming instead that consumption and fertility are substitutes has been put forward by Jones and Schoonbroodt (2010) as a way to explain declining fertility during the U.S. demographic transition. Our preference is to retain the complementarity assumption, which in our view makes intuitive sense, and to induce the decline in fertility through its cost, which we now discuss.

There are two forces determining the cost of fertility over time. First, children are time consuming. In our model there are decreasing returns to labor in the 'production' of children, implying that the marginal cost of children increases with the number of children born in a given period. This implies a form of congestion (see Bretschger, 2013). In addition, the technological advancement of the economy increases human capital requirements. This increases the marginal cost of children, because more time is required for their education, generating a transition from the quantity to the quality of children. Taken together, the marginal cost of population increment will increase over time, slowing down population growth.

The second cost of population is food requirements, and the demand for food increases (at a decreasing rate) with per capita income. Thus growth in population and per capita income demand growth in agricultural output. This can be achieved either through technological progress, or by allocating primary factors that are labor, capital and land, to agriculture. Importantly, land is ultimately fixed, either because it is constrained by physical availability or because its conversion cost increases with the area already converted. Thus over time, the cost of agricultural output will increase, adding a further break to population growth.

3.2 Numerical solution concept

The optimization problem is an infinite horizon optimal control problem, and we use mathematical programming techniques to solve for optimal trajectories.[16] In particular, we employ a solver for constrained non-linear optimization problems, which directly mimics the welfare maximization program: the algorithm searches for a local maximum of the concave objective function (the discounted sum of utility), starting from a candidate solution and improving the objective subject to maintaining feasibility as defined by the constraints of the problem.[17]

A potential shortcoming of direct optimization methods, as compared to dynamic programming for example, is that they cannot explicitly accommodate an infinite horizon.[18] However, as long as $\beta < 1$ only a finite number of terms matter for the solution, and we consider a finite horizon problem truncated to the first T periods. The truncation may induce differences between the solution to the infinite horizon problem and its finite horizon counterpart because the shadow values of stock variables are optimally zero in the terminal time period, whereas they will be so asymptotically if the planning horizon is infinite. Since we are interested in trajectories over the period from 1960 to 2100, we check that the solution over the first $T' = 200$ periods are not affected by the choice of T, finding that $T = 300$ is sufficient to make computed trajectories over the first $T' = 200$ periods independent of T. Similarly, re-initializing the model in $T' = 200$ and solving the problem onwards, we remain on the same optimal path with a precision of $1e-5$ for all the variables in the model. Given the truncation over 300 periods, the model solves in a matter of seconds.

3.3 Empirical strategy

Having defined the numerical optimization problem, our empirical strategy proceeds in three steps. First, a number of parameters are determined exogenously. Second, we calibrate some

[16] The main alternative class of numerical solution methods is dynamic programming (see Judd, 1998). This approach however is subject to the curse of dimensionality with respect to the number of continuous state variables, which is clearly a problem in our setup.

[17] The program is implemented in GAMS and solved with KNITRO (Byrd et al., 1999, 2006), which alternates between interior-point and active-set methods.

[18] By definition, the objective is a sum with an infinite number of terms, and the set of constraints includes an infinite number of elements, which is incompatible with finite computer memory. Obviously, a recursive formulation avoid this problem.

Table 1: List of parameters of the model and associated numerical values

Imposed parameters			
ϑ	Share of capital in manufacturing	0.3	
θ_K	Share of capital in capital-labor composite for agriculture	0.3	
θ_X	Share of land in agriculture	0.25	
σ	Elasticity of substitution between land and the capital-labor composite	0.6	
δ_K	Yearly rate of capital depreciation	0.1	
S	Maximum increase in TFP each year	0.05	
$\lambda_{mn,ag}$	Labor productivity parameter in R&D	1	
γ	Inverse of the intertemporal elasticity of substitution	2	
η	Elasticity of altruism towards future members of the dynasty	0.001	
κ	Income elasticity of food demand	0.25	
β	Discount factor	0.99	
Initial values for the stock variables and calibrated parameters			
N_0	Initial value for population	3.03	
X_0	Initial the stock of converted land	1.35	
$A_{0,mn}$	Initial value for TFP in manufacturing	4.7	
$A_{0,ag}$	Initial value for TFP in agriculture	1.3	
K_0	Initial value for capital stock	20.5	
δ_N	Exogenous mortality rate	0.022	
δ_X	Rate of natural land reconversion	0.02	
ξ	Food consumption for unitary income	0.4	
Estimated parameters (range of estimates for relaxed goodness-of-fit objective in parenthesis)			
μ_{mn}	Elasticity of labor in manufacturing R&D	0.581	(0.509 – 0.585)
μ_{ag}	Elasticity of labor in agricultural R&D	0.537	(0.468 – 0.545)
χ	Labor productivity parameter in child rearing	0.153	(0.146 – 0.154)
ζ	Elasticity of labor in child rearing	0.427	(0.416 – 0.448)
ω	Elasticity of labor productivity in child rearing w.r.t. technology	0.089	(0.082 – 0.106)
ψ	Labor productivity in land conversion	0.079	(0.078 – 0.083)
ε	Elasticity of labor in land-conversion	0.251	(0.238 – 0.262)

of the parameters to match observed quantities, mainly to initialize the model based on 1960 data. Third, we estimate the remaining parameters with simulation methods. We now discuss each step in turn. The full set of parameters determining the quantitative trajectories are listed in Table 1.

3.3.1 Choice of imposed parameters

Starting with production technology, we need to select values for share parameters ϑ, θ_K and θ_X, and for the elasticity of substitution σ. In manufacturing, the Cobb-Douglas assumption implies that the output factor shares (or cost components of GDP) are constant over time, and we use a standard value of 0.3 for the share of capital (see for example Gollin, 2002). In agriculture the CES functional form implies that the factor shares are not constant, and we choose θ_X to approximate a value for the share of land in global agricultural output of 0.25 in 1960. While there are no data on the global land factor share, the land factor share has been shown to be negatively correlated with income (Caselli and Feyrer, 2007), and 25 percent is in line with recent data for developing countries reported in Fuglie (2008).[19] For the capital-labor composite, we follow Ashraf et al. (2008) and also use a standard value of 0.3 for the share of capital. Taken together, these estimates of the output value shares in agriculture are also in agreement with factor shares for developing countries reported in Hertel et al. (2012).[20]

As mentioned previously, the long-run elasticity of substitution between land and the capital-labor composite input is expected to be less than one, which is confirmed by empirical evidence reported in Wilde (2013). Using long-run data on land and other inputs in pre-industrial England, he finds robust evidence $\sigma \simeq 0.6$. While external validity of these estimates may be an issue (in particular in developing countries with rapidly growing population), it reflects long-run substitution possibilities that are consistent with our CES functional form assumption (2). While we consider $\sigma = 0.6$ to be the best estimate available, in the sensitivity analysis we derive trajectories assuming $\sigma = 0.2$ and $\sigma = 1$.

Among the remaining 'imposed' parameters, the yearly rate of capital depreciation δ_K is set to 0.1 (see Schündeln, 2013, for a survey and evidence for developing countries), and maximum TFP growth per year S is set to 5 percent. The latter number is consistent with evidence about yearly country-level TFP growth rates from Fuglie (2012) which do not exceed 3.5 percent. The labor productivity parameter in R&D, $\lambda_{mn,ag}$, is not separately identified from S, and we set it to 1 without affecting our results.

The next set of imposed parameters determines preferences over consumption and fertility. First, the income elasticity of food demand is 0.25, which is consistent with evidence across countries and over time reported in Subramanian and Deaton (1996), Beatty and LaFrance

[19] Because evidence about the value share varies across sources, we do not attempt to directly calibrate the parameter of the CES function for agriculture.

[20] For 2007, the factor shares for the global agricultural sector reported in Hertel et al. (2012) are 0.15 for land, 0.47 for labor, and 0.37 for capital. However, shares for developing countries are probably a better estimates of the value shares prevailing in the mid-20$^{\text{th}}$ century.

(2005), and Logan (2009). Second, the elasticity of intertemporal substitution is set to 0.5 in line with estimates by Guvenen (2006). In the model, this corresponds to selecting $\gamma = 2$. Given the constraint on η to maintain concavity in the objective function, we initially set it to 0.01 so that our objective is effectively in line with Classical Utilitarianism. Intuitively, this implies that altruism towards the welfare of children remains constant as the number of children increases. Correspondingly, we also assume a high degree of altruism by setting the discount factor to 0.99, which corresponds to a pure rate of time preferences of 1 percent per year. This will tend to produce relatively high population projections, and we report further results for the case where altruism declines with n_t, in particular $\eta = 0.5$, and for a discount factor of 0.97.

3.3.2 Initial values and external calibration

Starting values for the state variables are calibrated to observed quantities in 1960. Initial population N_0 is set to an estimate of the world population in 1960 of 3.03 billion (United Nations, 1999). Initial crop land area X_0 is 1.348 billion hectares (Goldewijk, 2001). For the remaining state variables, sectoral TFP $A_{0,ag}, A_{0,mn}$ and the stock of capital K_0, there are no available estimates, and we target three moments.[21] First, we use an estimate of world GDP in 1960 of 9.8 trillion 1990 international dollars (Maddison, 1995; Bolt and van Zanden, 2013). Second, we assume that the share of agriculture in total GDP in 1960 is 15%, which corresponds to the GDP share of agriculture reported in Echevarria (1997). Third, we assume that the marginal product of capital ($MPK_{t,j}$) in 1960 is 15 percent. While this may appear relatively high, it is not implausible for developing economies (see Caselli and Feyrer, 2007). Solving for the targeted moments as a system of equations gives initial values of 4.7 and 1.3 for TFP in manufacturing and agriculture respectively, and a stock of capital of 20.5.

The exogenous mortality rate δ_N is calibrated by assuming an average adult working life (after infant mortality) of 45 years (United Nations, 2013) which implies $\delta_N = 0.022$. We vary that assumption in the sensitivity analysis, using instead $\delta_N = 0.015$, or a 65 year working life. We further set $\delta_X = 0.02$ which corresponds to period of regeneration of natural land of 50 years.

The parameter measuring food consumption for unitary income (ξ) is calibrated such that

[21] This calibration strategy is adapted from Nordhaus (2008) to a two-sector setting.

the demand for food in 1960 represents about 15% of world GDP, which is consistent with the calibration targets for initial TFP and capital. This implies $\xi = 0.4$.

3.3.3 Estimation of the remaining parameters

The seven remaining parameters $\{\mu^{mn,ag}, \chi, \zeta, \omega, \psi, \varepsilon\}$ are conceptually more difficult to tie down using external sources, and we therefore estimate them using simulation-based structural methods. The moments we target are taken from observed trajectories over the period 1960 to 2010 of world GDP (Maddison, 1995; Bolt and van Zanden, 2013), world population (United Nations, 1999, 2013), crop land area (Goldewijk, 2001; Alexandratos and Bruinsma, 2012) and sectoral TFP (Martin and Mitra, 2001; Fuglie, 2012).[22] In the model these correspond respectively to $Y_{t,mn} + Y_{t,ag}$, N_t, X_t, $A_{t,mn}$ and $A_{t,ag}$. We target one data point for each 5-year interval, yielding 11 data points for the targeted quantity (55 points in total), and use these to formulate a minimum distance estimator.

For each parameter to be estimated from the data, we start by specifying bounds of a uniform distribution. For elasticity parameters, these bounds are 0.1 and 0.9 and for the labor productivity parameters we use 0.03 and 0.3. We then solve the model for 10,000 randomly drawn vectors of parameters and evaluate the error between the simulated trajectories and those observed. Specifically, we compute squared deviations between the solution of the model and the observed data points, where the error across different variables is scaled to be comparable, i.e. we select the vector of parameters that minimizes:

$$\sum_k \left[\sum_\tau (Z^*_{k,\tau} - Z_{k,\tau})^2 / \sum_\tau Z_{k,\tau} \right] , \tag{9}$$

where $Z_{k,\tau}$ denotes the observed quantity k at time τ and $Z^*_{k,\tau}$ is the corresponding value simulated from the model. Having identified a narrower range of parameters for which the model approximates observed data relatively well, we reduce the range of values considered for each parameter and draw another 10,000 vectors to solve the model. This procedure gradually con-

[22] Data on TFP is derived from TFP growth estimates and are thus subject to some uncertainty. Nevertheless, a robust finding of the literature is that the growth rate of TFP economy-wide and in agriculture is on average around 1.5-2% per year. To remain conservative about the pace of future technological progress, assume it declines from 1.5 percent between 1960 and 1980 to 1.2 percent from 1980 to 2000, and then stays at 1 percent over the last decade.

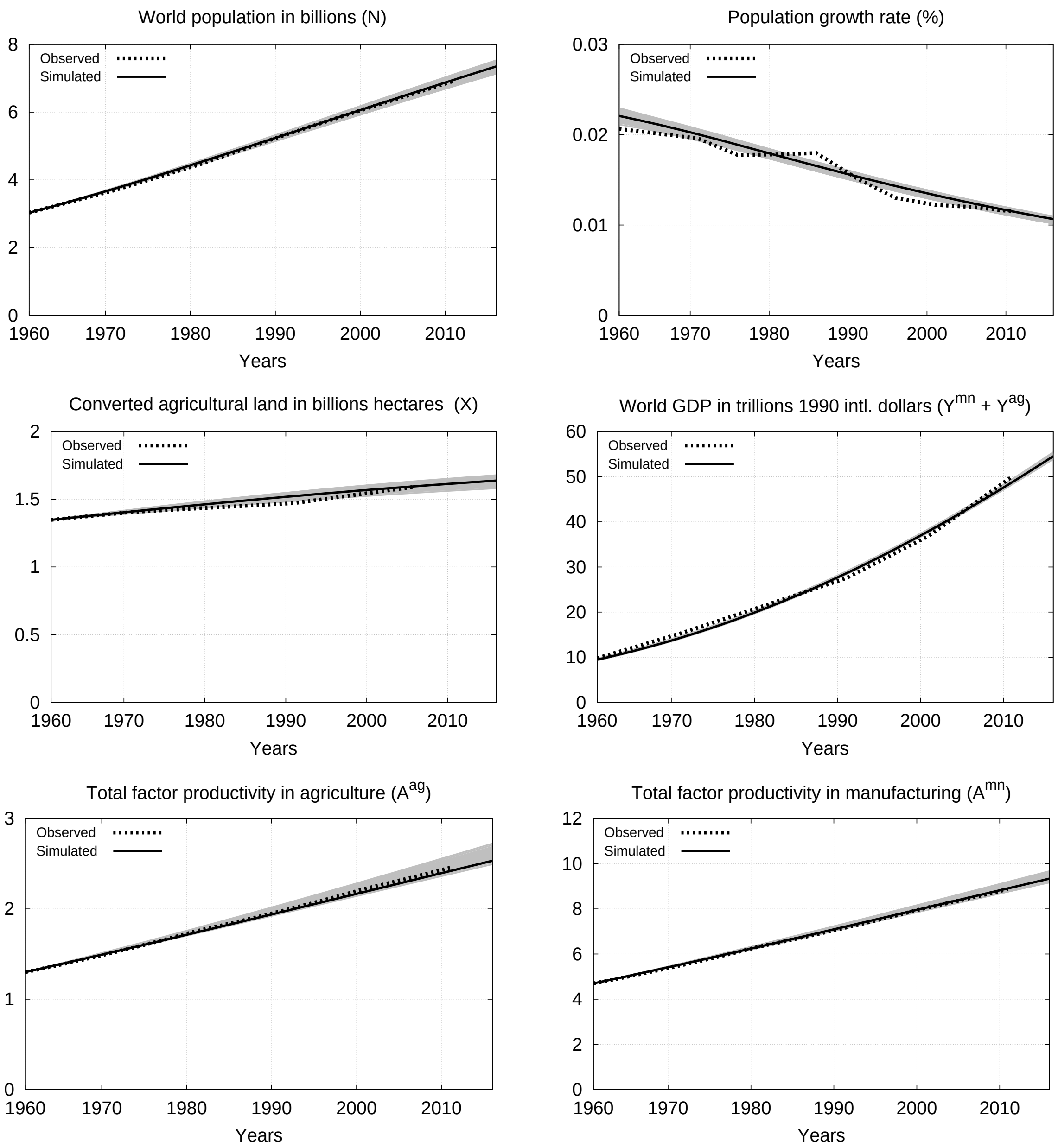
verges and the vector of parameters that minimizes goodness-of-fit objective (9) are reported in Table 1.[23]

The resulting simulated trajectories are reported in Figure 2 and compared to observed ones (data supporting these Figures are reported in Appendix A). The model closely fits observed trajectories, with a relative squared error of 3.52 percent across all variables. The size of the error is mainly driven by the error on output (3.3 percent), followed by land (0.1 percent) and population (0.03 percent). Figure 2 also reports runs for which the goodness-of-fit objective is relaxed by 10% relative to the best fit achieved. In other words, the shaded area reports the set of simulated trajectories with an error of 3.9 percent. Figure 2 also reports the growth rate of population, which is not directly targeted by the estimation procedure, showing that the simulated trajectory closely fits the observed dynamics of population growth.

While the estimated parameters provide a very good fit to recent history, their plausibility is difficult to evaluate. For example both parameters driving land conversion are relatively low, which corresponds to the very slow development of agricultural land area as compared to agricultural output. These parameters integrate other forces determining the allocation of land to alternative uses, but it is difficult to relate these to micro observations. One exception is the elasticity of labor in R&D activities (μ_j), which is discussed in the literature. However, there is disagreement on what this parameter should be. In particular, Jones and Williams (2000) argue that it is around 0.75, while Chu et al. (2013) use a value of 0.2. These two papers however rely on thought experiments to justify their choices. According to our results, a doubling of the share of labor allocated to R&D would increase TFP growth by around 50%.

[23] As for other simulation-based estimation procedures involving highly non-linear models, the uniqueness of the solution cannot be formally proved (see Gourieroux and Monfort, 1996). Our experience with the model suggests however that the solution is unique, with no significantly different vector of parameters providing a comparable goodness-of-fit objective. In other words, estimates reported in Table 1 provide a global solution to the estimation objective. This is due to the fact that we target a large number of data points for several variables, and that changing one parameter will impact trajectories across all variables in the model, which makes the selection criteria for parameters very demanding.

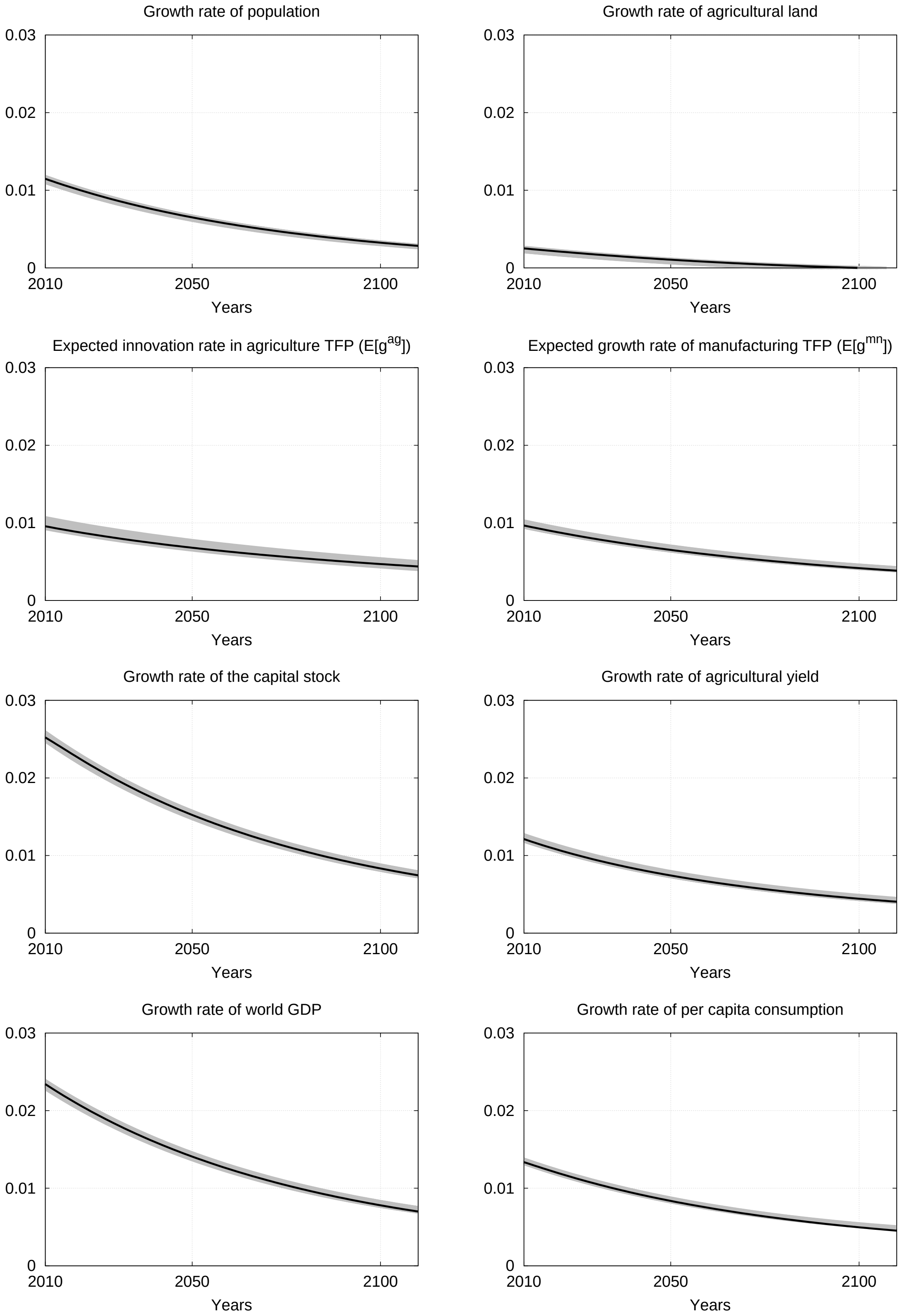
Figure 2: Estimation of the model 1960 – 2010



4 Quantitative analysis: Global projections from the model

Figure 3 displays projections for the growth rate of key variables from 2010 to 2100. The main feature of these paths is that they all decline towards a balanced growth trajectory where population, land and capital reach a steady state. Agricultural land area is the first state variable to reach a steady state as its growth rate becomes negligible by 2050. Population growth on

Figure 3: Estimated model: Growth rate of selected variables



the other hand remains significantly above zero over the whole century, reaching 0.3 percent by 2100. Thus while the population growth rate closely matches what was observed over the last fifty years, the model is far from predicting a complete collapse of population growth over the coming fifty years. Nevertheless population growth continues to decline after that, being around 0.1 percent in 2150.

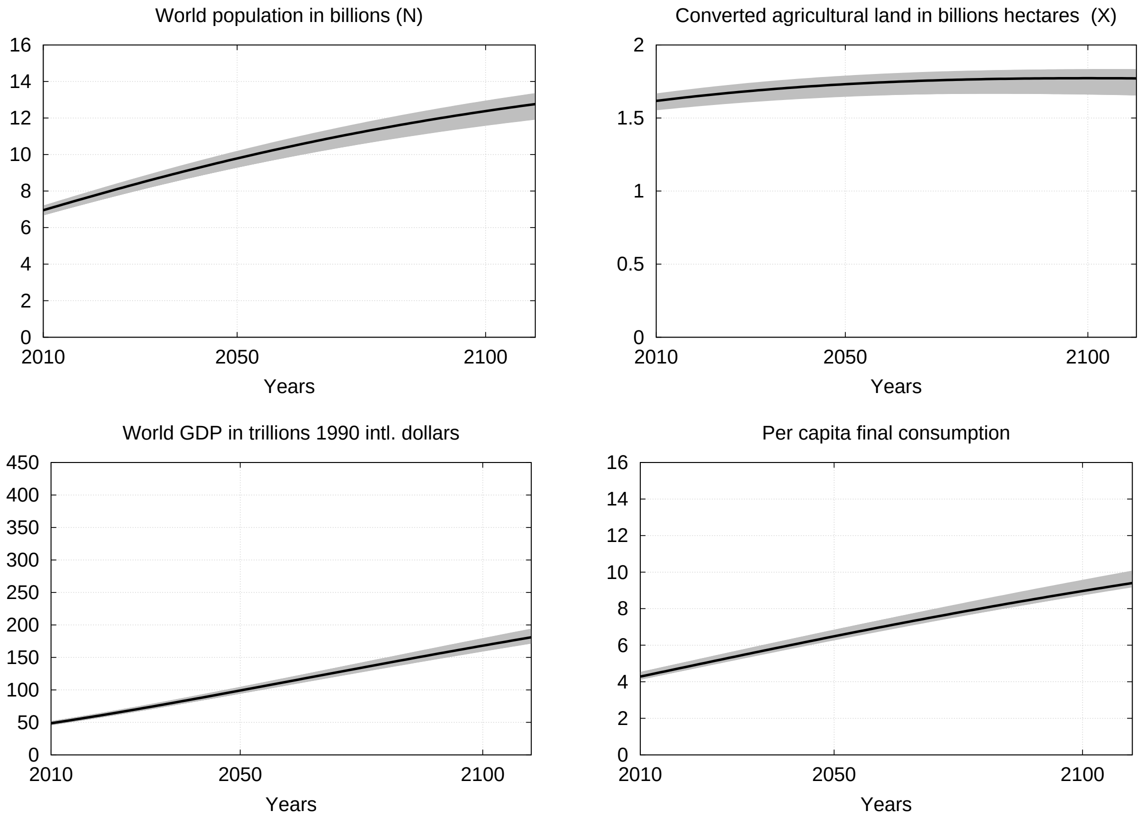
Implications of the model for the *level* of population is reported in Figure 4 (see also Appendix A). World population is around 9.85 billion by 2050 which is broadly consistent with UN projections (United Nations, 2013). While the UN's projections essentially imply zero population growth from 2050 onward, our simulations suggest a positive population growth rate over the century and a population of more than 12 billion by 2100. This figure is lower than high-end UN population projections, but it lies towards the upper bound of the probabilistic forecasts by Lutz and Samir (2010).

Interestingly, the shaded band for the population growth rate, which represents a range of alternative pathways for vectors of parameters with a slightly lower fit, shrinks over time. This demonstrates that the estimation of the parameters does not affect the long-run growth rate of population, whereas different transition paths imply a range of possible population levels between 11 and 13 billion by 2100.

Our model indicates that a significant increase of population over the century is compatible with food production possibilities. Between 1960 and 2010, agricultural output in the model increased by 279 percent, and increases by a further 67 percent between 2010 and 2050. Without being targeted by the estimation, these figures are consistent with an observed two percent annual growth rate in global agricultural output reported in Alexandratos and Bruinsma (2012) for the period 1960 to 2010, and very close to the 72 percent projected by the same authors between 2010 and 2050. After 2050, our model suggests a further increase in agricultural output of 31 percent by 2100, which is associated with a 22 percent growth in population and 32 percent growth in per capita income.

In light of these results, the fact that agricultural land area stabilizes at around 1.77 billion hectares is an important finding. First, this number is slightly higher than land conversion projections by Alexandratos and Bruinsma (2012), and it represents a sizable land area. Moreover, it is important to realize that land conversion will mostly occur in developing countries, while

Figure 4: Estimated model: Projections for selected variables



agricultural area in developed countries has and will decline. Second, TFP growth remains below 1 percent, which is a fairly conservative assumption. The pace of technological progress does not need to be very high to allow for a sustained growth in agricultural output. Third, the halt of land conversion suggests that the elasticity of substitution (σ) is high enough to allow agricultural output to grow from the accumulation of capital (we return to the role of σ in the sensitivity analysis). Indeed, although the share of capital allocated to agriculture declines over time, the stock of capital in agriculture almost doubles between 2010 and 2050.[24] This would mainly represent improvements to irrigation facilities. Both technology improvement and capital accumulation are reflected in the growth rate of agricultural yield (Figure 3), measuring growth in agricultural output per hectare used in agricultural production.

Finally, the growth rate of GDP falls from more than two percent in 2010 to around less than one percent in 2100, which implies that world GDP doubles by 2050 and more than triples by

[24] As expected, both the share and the quantity of labor allocated to agriculture decline over time.

2100. Similarly, per capita consumption more than doubles by 2100 relative to 2010.

5 Sensitivity analysis

We now report the results of sensitivity analysis with respect to a number of assumptions we have made: substitution possibilities in agriculture (σ), the elasticity of altruism towards children with respect to the number of children (η), the discount factor (β) and the expected working lifetime ($1/\delta_N$). For each change in the value of a parameter, it is necessary to re-estimate the vector of parameters to match observed data over the period 1960 – 2010. We plot results for our two main variables of interest, population and agricultural land, against our baseline results discussed above and we report the vector of estimates associated with each sensitivity run in Appendix B.

The parameter σ determines the elasticity of substitution between land and the capital-labor composite input in the agricultural production function. Our baseline case is derived under the assumption that $\sigma = 0.6$, which follows empirical evidence by Wilde (2013). However, evidence with regard to this parameter remains scarce, and it is the main determinant of the demand for agricultural land, and in turn of the ability to produce food and sustain the population.

In order to establish the importance of this assumption for our results, we re-estimate the parameters of the model assuming that $\sigma = 1$, so that agricultural production is Cobb-Douglas, and $\sigma = 0.2$, which provides a lower bound on substitution possibilities in agriculture. The results reported in Figure 5 demonstrate that the choice of σ has a small impact on land conversion and virtually no impact on population. As expected, a high value of σ implies a lower level of land conversion since other factors can be more easily substituted as the marginal cost of land conversion increases. Conversely, a lower σ makes land more important in agriculture, so that the overall amount of converted land is higher than under the baseline path. However, estimating the model over 50 years of data pretty much ties down the trajectory for land use in a robust manner, irrespective of the choice of σ.

The second sensitivity test we conduct targets η, the elasticity of altruism towards future members of the dynasty as their number increases. We consider the case of $\eta = 0.5$, so that the marginal utility of fertility (and population) declines much more rapidly than under our

Figure 5: Sensitivity analysis on substitution possibilities in agriculture

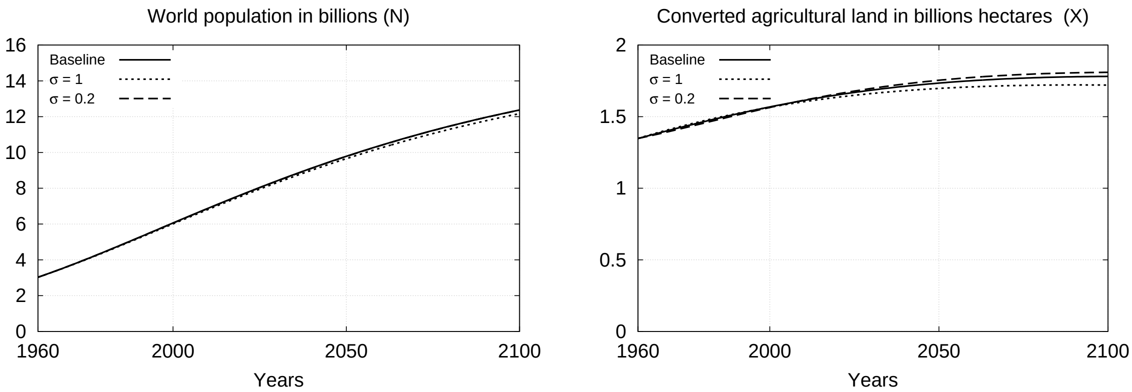
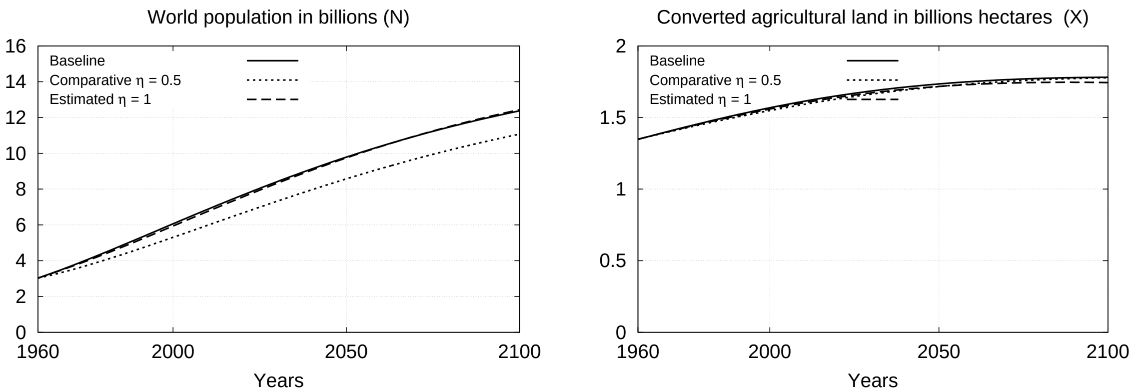


Figure 6: Sensitivity analysis on altruism towards children

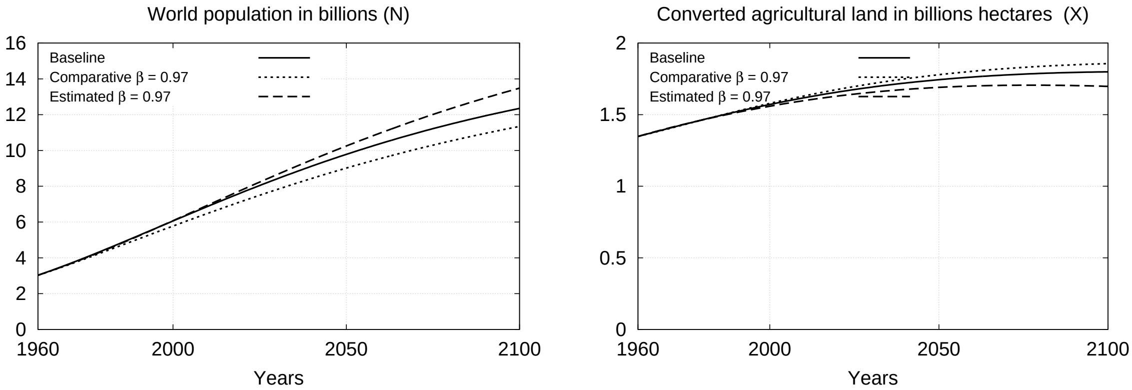


baseline assumption of $\eta = 0.01$.[25] We re-estimate the parameters of the model as described in Section 3.3.3, so that the model fits observed trajectories, and report the resulting trajectories in 6. However, Figure 6 also reports trajectories obtained with $\eta = 0.5$ but keeping the baseline parameter estimates (label "comparative"). In that setting the simulated trajectories will thus not fit observed trajectories over the period 1960 to 2010.

As expected, we find that reducing η while keeping the estimated parameters to their baseline values implies lower population growth. This results from putting less weight on the welfare of future members of the dynasty, so that the dynastic head reallocates resources to increase its own consumption. However, once we re-estimate the model to fit observed trajectories over

[25] Note that in our setting, an average utilitarian objective corresponds to $\eta = 1$, but it implies that the objective function is not globally concave.

Figure 7: Sensitivity analysis for the discount factor



1960 to 2010, the population path is virtually identical to the baseline trajectory. Note that the estimation error is significantly higher for $\eta = 0.5$ (see Appendix B), whereas for other sensitivity runs the estimation error remains very similar to that obtained under the baseline assumptions.

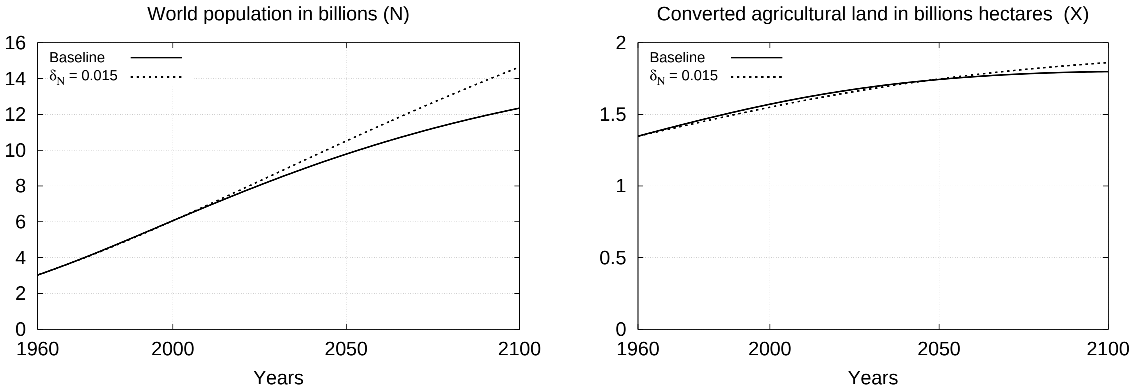
The third parameter we vary is the discount factor. As for η we report trajectories for both the re-estimated model and for the case where we just modify β but keep estimated parameters to their baseline value. Results are reported in Figure 7.

Reducing β alone implies lower population growth because it gives less weight to the welfare of future members of the dynasty, thus reducing the demand for children. In addition, the amount of converted land increases slightly. The reason for this effect is that a lower discount factor implies a lower saving rate, so that there is less capital available for agricultural production, and other input factors have to compensate in order to maintain subsistence of the existing population.

However, by re-estimating the model to fit 1960 – 2010 data under the assumption $\beta = 0.97$, we find that the opposite is true. As compared to the baseline, a lower discount factor implies a higher long run population, while the amount of converted land is lower. The reason is that the estimates for the fertility equation imply higher labor productivity and weaker decreasing returns to labor, and hence a lower marginal cost of fertility both within and across periods. In turn, labor becomes relatively cheap relative to capital and land, whose use is therefore lower. Thus a lower discount factor increases incentives to accumulate population as a substitute to the accumulation of capital and land.

The final sensitivity test concerns our assumption about the death rate δ_N, or equivalently the

Figure 8: Sensitivity analysis on the expected working lifetime



expected working lifetime $1/\delta_N$. We illustrate the effect of this parameter by using a somewhat extreme value of 65 years, corresponding to $\delta_N = 0.015$. Trajectories are reported in Figure 8. As expected this implies a larger long run population, reaching more than 10 billon in 2050 and around 15 billion by 2100. The impact of this parameter is mostly felt in the long run, as it implies that the growth rate of population declines less rapidly over time. This result confirms the importance of the death rate as a key driver to the evolution of population over time, as already demonstrated by Jones and Schoonbroodt (2010) and Strulik and Weisdorf (2014).

6 Discussion

Our approach distinguishes itself from existing population projections in two main ways. First, fertility decisions are endogenous. While our representation of preferences for fertility implies some restrictions and is open to debate, it contrasts with the current practice of assuming some fixed trajectory for fertility. In fact, the rapid decline of population growth towards zero implied by existing projections is an outcome of the assumed convergence of fertility to replacement level. However, there is no formal basis for that assumption except for *developed* countries having converged to a low fertility regime. A rapid decline of fertility in developing countries has strong implications in terms of economic convergence that, in our view, require an underlying theory.

Second, our integrated representation endogenizes the evolution of quantities that are jointly determined with fertility choices, notably technological progress, income per capita and agricultural output. Given the structure of the model, the dynamic relationship between these variables is informed by structurally estimating the model to fit observed trajectories. Our model thus treats the representation of preferences and technology as fixed, with the dynamics being driven exclusively by structural assumptions. This contrasts with existing projections that are carried out in isolation from each other, yet mutually rely on one another. For example, projections for food demand and agricultural land use reported in Alexandratos and Bruinsma (2012) rely on population projections by United Nations (2013), the latter obviously assuming that the projected population can be fed.

Overall, our results confirm the widespread expectation that the long-standing processes of growth in population and land conversion are in decline. This is resulting from the quality-quantity trade-offs, shifting from quantity-based economies with large levels of population growth and associated land conversion toward quality-based economies with investments in technology and education for lower levels of fertility. Importantly, the decline in growth rates is a feature of the data over which the model is estimated. Land is the first quantity to reach a steady state, doing so in the coming half-century. We find, however, that a steady state in land conversion is consistent with a sustained growth in food demand and agricultural output as well as mildly optimistic assumptions about technological progress in the future.

More importantly, the population growth rate declines over time, but it is still positive (and signficiantly so) in 2100. While uncertainty over such a time horizon cannot be overstated, a key finding of our analysis is therefore that population does not reach a steady state in foreseeable future. The decline of population growth is slower compared to those implied by existing population projections from United Nations (2013) and Lutz and Samir (2010). Our finding of sustained population growth over the coming century is plausible because of the amount of inertia in the system, and because better economic prospects will sustain the demand for children despite an increasing cost associated with child rearing and education.[26]

[26] By the same mechanism driving land to a steady state, namely linear depreciation and decreasing return to labor in the accumulation of land, the dynamic equation describing population implies a steady state. However, the data suggests that convergence towards a steady state for population is slow, and the growth rate will drop to zero only in the very long run.

7 Concluding comments

We have studied the implications of using a macroeconomic growth model to make projections of world population and land use over the long run. Our model integrates fertility decisions in a wider framework where technological progress, per capita income, food production and land conversion, are jointly determined. Once the model is fitted to data from recent history, conclusions from the projection are surprisingly robust to different assumptions.

One virtue of our integrated model is that it provides a rich empirical framework to study interactions among key drivers of growth over the long run. In particular, it can be used to evaluate policies affecting different drivers such as the cost of children or constraints to land conversion. In this paper our aim has rather been to study implications of the model with respect to long-term population development. This provides a novel perspective on widely used projections from a small number of sources that use the same assumptions regarding convergence of fertility at replacement level and the halt of population growth over the coming century.

Our results suggest that sustained population growth over the coming century is compatible with an evolution of agricultural output close to what has been observed in the past, mainly on account of technological change and capital accumulation. Thus from that perspective, and to paraphrase Mark Twain, we find the rumors of the imminent demise of global population growth to be exaggerated. This points towards an inconsistency between the structure of our model and the fertility trajectories that are usually assumed for developing countries, and thus calls for a better understanding of linkages between fertility and economic development in countries in Asia and Africa.

Whether the structure we impose to rationalize the last fifty years of data is more appropriate than the assumed path for fertility is up for discussion. However, the fact that existing population projections rely on the same set of assumptions about fertility trajectories, with little empirical basis for such assumption, calls for a different perspective on the problem. Fertility trajectories are inherently difficult to inform, and shifting the debate to the economic structure underlying the dynamic processes at play may be a first step towards a more comprehensive understanding of future population developments.

Appendix A Observed and simulated data

The table below reports both observed and simulated data from 1960 to 2100, by 10-year intervals. Note that agricultural area is not only available for 2005.

Year	Population (billion)		Population growth (%)		Crop land area (billion ha)		GDP (trillions 1990 intl. $)	
	Observed	Simulated	Observed	Simulated	Observed	Simulated	Observed	Simulated
1960	3.03	3.03	0.021	0.022	1.37	1.35	9.8	9.5
1970	3.69	3.74	0.020	0.020	1.41	1.41	15.3	14.3
1980	4.45	4.51	0.018	0.018	1.43	1.47	21.3	20.6
1990	5.32	5.32	0.015	0.015	1.47	1.52	27.5	28.5
2000	6.13	6.14	0.012	0.013			36.9	38.0
2005					1.59	1.60		
2010	6.92	6.95	0.011	0.011		1.62	50.0	48.6
2020		7.74		0.010		1.65		60.5
2030		8.49		0.009		1.69		73.2
2040		9.19		0.007		1.71		86.6
2050		9.85		0.006		1.73		100.5
2060		10.46		0.006		1.75		114.5
2070		11.02		0.005		1.76		128.5
2080		11.53		0.004		1.77		142.4
2090		12.00		0.004		1.77		156.1
2100		12.42		0.003		1.77		169.3

Appendix B Estimates for sensitivity analysis

The table below reports the estimates supporting projections in the sensitivity analysis. The baseline estimates correspond to those reported in Table numbers, while the other refer to the changes in parameters considered for the sensitivity runs. We also report the simulation error as defined by equation (9).

Parameter	Baseline	$\sigma = 0.2$	$\sigma = 1$	$\eta = 0.5$	$\beta = 0.97$	$\delta_N = 0.015$
μ_{mn}	0.581	0.575	0.580	0.751	0.523	0.525
μ_{ag}	0.537	0.549	0.509	0.482	0.383	0.512
χ	0.153	0.155	0.151	0.205	0.155	0.104
ζ	0.427	0.417	0.426	0.399	0.460	0.516
ω	0.089	0.085	0.088	0.161	0.087	0.091
ψ	0.079	0.063	0.083	0.078	0.083	0.077
ε	0.251	0.174	0.256	0.239	0.243	0.186
Estimation error	0.035	0.033	0.035	0.189	0.029	0.045

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