THE GRADUATE INSTITUTE | GENEVA

INSTITUT DE HAUTES ETUDES
INTERNATIONALES ET DU DÉVELOPPEMENT

GRADUATE INSTITUTE OF INTERNATIONAL
AND DEVELOPMENT STUDIES

Graduate Institute of International and Development Studies
Working Paper No: 02/2012

Country Portfolios with Heterogeneous Pledgeability

Tommaso Trani
Graduate Institute of International Studies

Abstract

In a two-country portfolio model with leverage constraints, I focus on private assets in order to understand how their behaviour can justify an expected excess return as well as the flight-to-safety observed in the data. The specific goal is to study how much these phenomena are explained by the fact that investors cannot always borrow the same amount of resources pledging domestic assets as pledging foreign collateral. Modeling the leverage constraints accordingly, I propose a methodology to deal with this heterogeneous pledgeability and solve for country portfolios. The central feature of this approach is that any idiosyncratic shocks generate an expected excess returns which compensate the current effects of the shock on the relative riskiness of local versus foreign collateral. The resulting portfolio solution shows that, in equilibrium, investors care for this risk and renounce to a part of the expected excess return - favouring current borrowing. The main consequences are: the home equity bias is smaller than in a model where assets are homogeneously pledgeable; the ex post dynamics of the relative premium paid on collateralized assets contribute to the cross-border transmission of shocks. Given these dynamics, idiosyncratic shocks to the pledgeability of local assets affect the value of external claims and liabilities of the country hit by the shock in such a way that its net foreign assets match those observed in the data during times of flight-to-safety.

Country Portfolios with Heterogeneous Pledgeability[1]

Tommaso Trani[2]
The Graduate Institute | Geneva

Preliminary draft: August 2011
This draft: February 12, 2012

Abstract

In a two-country portfolio model with leverage constraints, I focus on *private* assets in order to understand how their behaviour can justify an expected excess return as well as the flight-to-safety observed in the data. The specific goal is to study how much these phenomena are explained by the fact that investors cannot always borrow the same amount of resources pledging domestic assets as pledging foreign collateral. Modeling the leverage constraints accordingly, I propose a methodology to deal with this heterogeneous pledgeability and solve for country portfolios. The central feature of this approach is that any idiosyncratic shocks generate an expected excess returns which compensate the current effects of the shock on the relative riskiness of local versus foreign collateral. The resulting portfolio solution shows that, in equilibrium, investors care for this risk and renounce to a part of the expected excess return - favouring current borrowing. The main consequences are: the home equity bias is smaller than in a model where assets are homogeneously pledgeable; the ex post dynamics of the relative premium paid on collateralized assets contribute to the cross-border transmission of shocks. Given these dynamics, idiosyncratic shocks to the pledgeability of local assets affect the value of external claims and liabilities of the country hit by the shock in such a way that its net foreign assets match those observed in the data during times of flight-to-safety.

JEL classification: E44, F32, F41, G11, G15

Keywords: portfolio choice, riskiness of pledged collateral, return differentials, macroeconomic interdependence

[1] The title of the preliminary version of this work was: "Heterogeneous Pledgeability: Implications for Premia and International Portfolios".

[2] I wish to thank Cédric Tille for encouraging and advising me in pursuing this project. I am grateful to Rahul Mukherjee for very helpful comments and to Christopher A. Gohrband and Kristy L. Howell for generously sharing their data. Clearly, all errors remain exclusively mine.

Contact: tommaso.trani@graduateinstitute.ch

1 Introduction

I examine the portfolio choice of leveraged investors when each internationally traded asset can be individually pledged as collateral. In this case, investors evaluate how much they can borrow pledging local assets and how much pledging foreign assets. So, the endogenous portfolio problem does not only boil down to comparing asset retuns across countries, but also to considering that domestic assets may serve better as collateral than foreign assets or the other way around.

This consideration of the pledgeability of assets is of primary importance for borrowers that hold internationally diversified portfolios and face leverage constraints. Simply said, these constraints are not so tight for assets that represent good collateral, and the demand for these assets is not necessarily dominated by return-seeking purposes. Safe collateral is not expected to yield a high return in the future. Here, this property of pledgeable *private* assets affects international investors' portfolio choice problem and allows for an endogenous *expected excess return*. Through this variable, I can analyze some features of the *flight-to-safety* effects observed in the data.

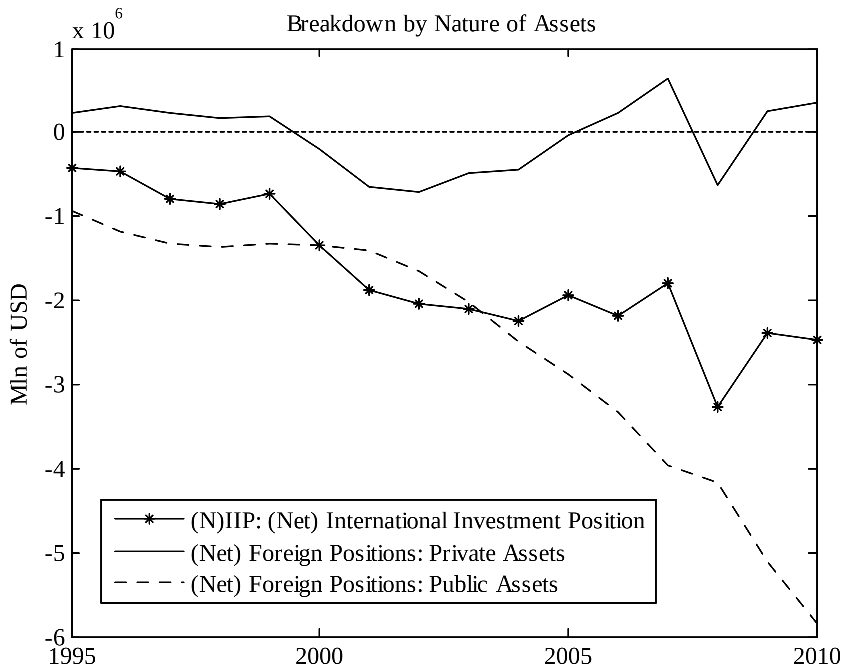
I build a two-country model with leverage constraints, which depend on the specific pledgeability of every collateral asset present in international portfolios. What I inted for pledgeability is simply the amount of savings that investors can borrow against each collateral asset (i.e., the specific debt-to-asset ratio). So, as Aiyagari and Gertler (1999) suggested, in a model with multiple assets the total debt of a leveraged investor is the sum of the collateralized loans she receives. While this aspect is generally considered in financial studies, the macroeconomic literature has not devoted much attention to it.

It is natural to think about the availability of multiple pledgeable assets in international economics. However, the corresponding portfolio choice problem cannot be straightforwardly solved using the recently developed methods (e.g., Devereux and Sutherland, 2011a; Tille and Van Wincoop, 2010). These approaches are not meant to treat the influence exerted by contractual limits (such as debt-to-asset ratios) on the pay off of traded (and pledgeable) financial instruments. This is instead what I attempt to do here, building on the principal results obtained by the previous literature; in this sense, my contribution is methodological.

I adopt a solution strategy which takes advantage of the fact that the riskiness of collateral assets changes with the state of the economy. This aspect is modeled through state-dependent debt-to-asset ratios. This means that, after a negative idiosyncratic shock, the local assets of the country hit by the shock become less pledgeable than the assets from the foreign country. In turn, this has *short-run* effects on the relative pay off on local versus foreign assets. But in the steady state equilibrium, collateral assets from various countries are pledgeable in the same way. This assumption suits quite well the properties of private assets as opposed to government securities, which explains why here I exclusively consider the former type of financial instruments. And the question is relevant for understanding the behaviour of the net foreign assets of a country, especially in the aftermath of big shocks.

One of the most puzzling regularities in the data is that the U.S. earn a positive income balance on their net foreign assets, although their net international investment position (NIIP) is persistently negative. And the last financial crisis has not corrected this puzzle. Between 2006 and 2010 the net income receipts on U.S. assets increased from around \$ 54.7 bln to about \$ 174.5 bln, with just a minor flection between 2008 and 2009, while the U.S. external deficit continued to widen.

FIGURE 1. Net Foreign Assets in the U.S.



Source: International Investment Position, U.S. Department of Commerce, Bureau of Economic Analysis.

These regularities are at the centre of the debate on global imbalances. On one hand, the apparent contrast between net external positions and net external income is explained in terms of a *persistent* expected excess return on U.S. holdings of foreign assets versus foreign holdings of U.S. assets[3]. On the other hand, the expected excess return that favours the U.S. is said to be consequence of the leading role that their assets have in international markets as well as of the development of their financial sector[4]. Yet, there is also who cast doubts on the persistence of the expected excess return on U.S. assets: there are statistical issues that bias the estimates upward, and in some sense the income balance puzzle is justified by the composition of the external claims and liabilities of the U.S[5].

[3] See, for instance, Gourinchas and Rey (2007) and Lane and Milesi-Ferretti (2005).

[4] Caballero et al. (2008) and Mendoza et al. (2009).

[5] See Curcuru, Dvorak and Warnock (2008) and Curcuru, Thomas and Warnock (2012).

Clearly, the debate on global imbalances highlights long-run consequences of the economic difference between integrated countries. And these effects are notably related to one particular asset class: Treasury debt securities. The point here is that also the behaviour of private assets is especially important. As Figure 1 shows, while the downward trend in the U.S. NIIP is due to net external positions in government-related assets (dashed line), the fluctuations around this trend are mainly due to the behaviour of private assets (continous line). The match was particularly striking in 2008-2009. The revised statistics for this period show that debt securities and equities issued by private U.S. residents were not immune from flight-to-safety, and their reaction to the crisis is actually at the basis of an increase in home bias in assets (Gohrband and Howell, 2009). In other terms, the 2008 fall in the NIIP follows from the fact that the impact of the shock on the demand for foreign private assets by U.S. residents was bigger than the impact on the demand for U.S. private assets by foreigners.

Therefore, the potential of focusing on asset classes different from Treasury securities (and FDI flows) and on effects that die away as time goes by is to understand why and how the external investment position of a country fluctuates. And attempting to find an explanation for these fluctuations in the features of private securities that are used as collateral is justified by current practice in financial contracts as well as by the key role that these types of securities have played in the recent crisis.

The core mechanism characterizing the portfolio solution strategy developed here works as follows. Upon a shock in one country, the collateral assets of that country allow to borrow less than the collateral from other countries. This means that in the future the poor collateral assets are expected to pay more than the good ones, and this is true until the model converges back to the stationary equilibrium. In this way, the current heterogeneity in the pleadgeability of internationally traded securities must be compensated in the future by a temporary "asymmetry" in the relative rate of return on those assets, giving rise to an endogenous expected excess return. These two contrasting effects characterize the relative equity premium on local versus foreign assets. It is accounting for these characteristics that I can compute the equilibrium portfolios and study the conditional dynamics around them, which give rise to *first order* valuation effects.

There is empirical support for this mechanism. Analyzing the 2007-2009 crisis, Broner et al. (2011) find that U.S. external assets and liabilities reacted asymmetrically, conforming to a more general post-crisis phenomenon and responding to other stimuli than a mere productivity shock. In addition, Table 1. not only confirms that external private assets are not persistently safe[6], but shows also that the flight-to-safety and heightened home bias that followed the crisis are largely justified by valuations effects; hence, my choice of leaving out for now changes in flows. The model here can only capture the valuation effects related to movements in asset prices; there is no role for the exhange rate.

[6] The effect of the shock on equity claims and liabilities, which are issued exclusively by private by firms, shows up clearly. In contrast, the external positions in long-term bonds are characterized by a compensation between corporate bonds and Treasury securities. In particular, the decline in private debt sold by U.S. corporates to foreign residents was compensated by the steady purchase of U.S. Treasuries.

Table 1. U.S. External Claims and Liabilities by Type of Assets (Bln USD)

Year	Initial Position	(1) Flows	(2) Valuation Changes			Total Change	Final Position
			Prices	Exchange-Rate	Other	(1)+(2)	
Foreign Long-Term Debt in U.S. Country Portfolio							
2006	1,011.6	227.8	-10.7	13.5	33.3	264.0	1,275.5
2007	*1,275.5*	*218.7*	*8.1*	27.9	56.8	311.6	1,587.1
2008	*1,587.1*	*-158.9*	*-123.0*	-51.0	-16.9	-349.8	1,237.3
2009	*1,237.3*	144.9	84.1	27.3	0.0	256.3	1,493.6
Foreign Equities in U.S. Country Portfolio							
2006	3,317.7	137.3	689.5	184.5	0.0	1,011.3	4,329.0
2007	*4,329.0*	*147.8*	*386.0*	385.3	0.0	919.0	5,248.0
2008	*5,248.0*	*-39.0*	*-1,962.4*	-498.2	0.0	-2,499.6	2,748.4
2009	*2,748.4*	63.3	937.3	228.4	0.0	1,229.0	3,977.4
U.S. Long-Term Debt in Foreign Countries Portfolios							
2006	4,421.8	953.7	-28.7	25.4	32.7	983.1	5,404.9
2007	*5,404.9*	*713.9*	*90.0*	53.0	7.6	864.5	6,269.4
2008	*6,269.4*	*110.0*	*-46.1*	-67.0	-36.7	-40.0	6,229.4
2009	*6,229.4*	325.0	11.8	56.3	-8.5	384.6	6,614.0
U.S. Equities in Foreign Countries Portfolios							
2006	2,304.0	145.5	330.7	0.0	11.7	487.9	2,791.9
2007	*2,791.9*	*275.6*	*130.5*	0.0	33.6	439.8	3,231.7
2008	*3,231.7*	*126.4*	*-1,262.7*	0.0	37.2	-1,099.1	2,132.6
2009	*2,132.6*	160.5	537.1	0.0	0.0	697.6	2,830.2

Source: Taken from Gohrband and Howell (2009).

The three main results generated by the inverse relation between current pledgeability and expected rates of returns are as follows. First, since the pledgeability of assets from a given country is positively correlated with the domestic economic cycle, agents are less enthusiatic of earning an equity premium on local assets vis-à-vis foreign assets than they are in models where collateral values do not affect the portfolio choice problem. This reduced desirability of local assets attenuates the home equity bias characterizing the equilibrium.

Second, after an idiosyncratic shock in one country, the collateral assets of that country become relatively more or less pledgeable than the assets from other countries, depending the shock being positive or negative. The excess return is then expected to react in the opposite direction. It is this reaction of the expected excess return that governs the dynamics of the equity premiums on internationally traded assets. The consequence is that the behaviour of the equity premiums is affected by the cross-border transmission of idiosyncratic shocks in a way which is absent from recent work on borrowing limits as international transmission channels.

Finally, defining it as a shock to the pledgeability of an asset, a negative financial shock can generate a negative reaction in the net foreign assets of the country which is hit by the shock. In contrast, given the structure of the model, a productivity shock is not as powerful, despite the home equity bias. The crucial aspect is however that the model sheds some lights on how the financial shocks should be modeled in international macroeconomic models with endogenous portfolios and borrowing limits. The financial shocks of a model with homogeneously pledgeable assets are already capable of reproducing the observed dynamics of the net foreign assets. However, this result is due to the implicit assumption that only the residents of the country receiving the shock lose access to credit; foreigners do not, and the composition of international portfolios does not matter. In contrast, the model built here predicts that the financial shock affects all agents, conditional on their portfolios, through the dynamics of equity premiums.

This paper continues with a brief literature review (section 2) and a brief description of the model (section 3). In section 4, I analyze the equity premiums of pledgeable traded assets. Section 5 is then entirely devoted to the approach that I develop to solve the related portfolio choice problem. The application of this method is in Section 6, and Section 7 concludes. There is an appendix at the bottom of the paper and a separate one for further details.

2 Literature

This paper is clearly connected with the literature on the determination of international portfolios in open economy macro-models: Devereux and Saito (2006), Devereux and Sutherland (2011a), Evans and Hnatkovska (2011), Pavlova and Rigobon (2010) and Tille and Van Wincoop (2010). In particular, I draw on the major features of the second and the latter works. My contribution is to solve for steady state portfolios when the value of traded assets is wider than the corresponding rates of return; here this happens because of collateralized loan contracts.

I conduct my analysis within a two-country model with a stylized financial accelerator generated by leverage constraints. In this sense, the paper is connected with Devereux and Yetman (2010), Devereux and Sutherland (2011b) and Trani (2012). The new feature of the analysis here concerns the leverage constraints which are binding in both countries. I introduce equity-specific debt-to-asset ratios, so that the form of the leverage constraints can help understand the asset side of investors' balance sheet. This differs from Trani (2012), where the focus is on the liability side. Specifically, here I build on the few comments made by Ayiagari and Gertler (1999) on how multiple margin requirements can work in macroeconomic models with credit limits.

Therefore, I model the debt-to-asset ratios in accordance with the financial literature. Beneficial insights are from Gorton (2009) and some of my considerations on the equity premiums resemble those of Garleanu and Pedersen (2012), who show how differences between margins lead to differences between asset prices. As far as the financial shocks are concerned, I draw on Jermann and Quadrini (2012).

Because of my interest in flight-to-safety, the model incorporates and analyzes the empirical facts emphasized by papers such as Bertaut and Pounder (2009), Broner et a. (2011) and Gohrband and Howell (2009). Note that, in support of the short-term expected excess return generated by the model at hand, one of the findings of Gohrband and Howell (2009) is that recent statistical evidence hardly shows a persistent net gain on U.S. external equity claims and corporate bonds, while the opposite is true for FDI flows.

Like me, also Cao and Gete (2011) and Blengini (2011) have recently analyzed the international dimension of flight-to-safety from a theoretical perspective. But both works differ from mine. I consider private collateral assets and the variation in their riskiness to ultimately capture short-run valuation effects such as those shown in Table 1. Cao and Gete (2011) analyze the demand for on U.S. collateral assets, but they focus on Treasuries and the long-term safety that characterizes them. Blengini (2011) uses an international portfolio model such as mine, but her goal is to show how uncertainty shocks can generate portfolio rebalancing within different types of asset classes, none of them being pledgeable.

3 Model: a Brief Sketch

The basic structure of the model is similar to the framework developed by Devereux and Yetman (2010). In this section, I solely present the main building blocks for just one economy. The corresponding equilibrium conditions are in the appendix, and those for the foreign country are alike.

Consider a symmetric two-country, one-good and two-agent model, where the only *potential* source of asymmetry is the pledgeability of collateral assets. In both the home, H, and foreign, F, country, perfectly competitive firms produce a homogeneous traded good. This production is financed by investors, who purchase the tradable equity claims issued by the domestic firms against their productive capital.

Investors represent one group of households, with size n, the other being the group of savers, which have size $1 - n$. Investors are impatient consumers, who need to augment their internal resources attracting funds from savers. Savers are willing to lend - on a unique (and perfectly integrated) bond market - because they are patient and total investors' borrowing is secured by collateral (the equity holdings). The specification of the corresponding leverage constraint is the novelty I introduce here. Finally, savers produce a part of the economy-wide output, but this part is entirely consumed within their group of households.

3.1 Investors

Being leveraged and international traders, investors face the following problem:

$$\max_{\left\{c_t^I, b_t^I, k_{it}^I\right\}_{i=H,F}} \quad E_0 \sum_{t=0}^{\infty} \eta_t^I \frac{\left(c_t^I\right)^{1-\sigma}}{1-\sigma}$$

subject to budget and collateral constraints

$$c_t^I - q_t^b b_t^I + q_{Ht}^e k_{Ht}^I + q_{Ft}^e k_{Ft}^I = w_t - b_{t-1}^I + \left(q_{Ht}^e + d_{Ht}\right) k_{Ht-1}^I + \left(q_{Ft}^e + d_{Ft}\right) k_{Ft-1}^I \quad (1)$$

$$b_t^I \leq \kappa_{Ht} q_{Ht}^e k_{Ht}^I + \kappa_{Ft} q_{Ft}^e k_{Ft}^I \quad (2)$$

where $\eta_{t+1}^h = \beta\left(C_t^h\right)\eta_t^h$, with $h = I, S$ denoting the households' group of investors and savers, respectively. Agent h's endogenous discount factor depends on the average consumption in her household group, C_t^h: $\beta\left(C_t^h\right) = \zeta^h\left(1 + C_t^h\right)^{-\phi}$. Budget and leverage constraints (1)-(2) read as follows. Investors borrow from savers, selling an international bond b_t^I at price q_t^b. This is a riskless debt security so the loan rate is $R_{t+1} = 1/q_t^b$. However, investors cannot commit to the full repayment of their debts (equation (2)), so they must pay a *shadow* loan premium, which is defined below. The reason for attracting savings is to boost the investment in home and foreign equities, k_{Ht}^I, k_{Ft}^I. The price and dividend of equity i, with $i = H, F$, are respectively denoted as q_{it}^e and d_{it}, so $r_{it} = \left(q_{it}^e + d_{it}\right)/q_{it-1}^e$ is its rate of return.

Equation (2) and its foreign counterpart are the key relations of the present analysis. Specified in that way, the leverage constraint allows the representative investor to borrow b_t^I summing up the loans received pledging home collateral (equal to κ_{Ht} times their holdings of home equities) and those received against foreign collateral (equal to κ_{Ft} times their holdings of foreign equities). This is akin to what happens with margin requirements in presence of multiple assets, which have been widely used in finance and was briefly mentioned by Aiyagari and Gertler (1999).

Therefore, collateral constraints such as (2) are not standard in the macroeconomic literature. The main feature of that specification is that debt-to-asset ratios are asset-specific and time-varying. Previous studies are instead characterized by an (implicit) homogeneity in the pledgeability of collateral assets. For example, Devereux and Yetman (2010) assume that home and foreign investors (denoted by a "star") are, respectively subject to

$$b_t^I \leq \kappa\left(q_{Ht}^e k_{Ht}^I + q_{Ft}^e k_{Ft}^I\right) \quad ; \quad b_t^{*I} \leq \kappa\left(q_{Ht}^e k_{Ht}^{*I} + q_{Ft}^e k_{Ft}^{*I}\right) \quad (3.a)$$

where κ is constant and the same for home and foreign equities. An alternative specification where debt-to-asset ratios are anyway equivalent across assets is the following:

$$b_t^I \leq \kappa_t\left(q_{Ht}^e k_{Ht}^I + q_{Ft}^e k_{Ft}^I\right) \quad ; \quad b_t^{*I} \leq \kappa_t^*\left(q_{Ht}^e k_{Ht}^{*I} + q_{Ft}^e k_{Ft}^{*I}\right) \quad (3.b)$$

where κ_t and κ_t^* vary over time and are agent-specific[7].

In general, the limits imposed on collateralized debt reflect both each borrower's credit risk and the riskiness of the collateral assets she pledges (Gorton, 2009). Equation (3.b) combines these two sources of risk in a unique measure of counterparty risk, and (3.a) represents a simplification of this case. In contrast, the form of the collateral constraints used here (equation (2)) allows to separate the riskiness of collateral assets - captured by κ_{Ht}, κ_{Ft} - from the borrowers' credit risk - which is left out of the analysis. Table 2 classifies these cases in order to fix ideas for the comparisons below.

TABLE 2. Assumptions on the Pledgeability of Each Collateral Asset

	Homogeneous (Pledgeability) Case		Heterogeneous (Pledgeability) Case
	eq. (3.a)	eq. (3.b)	eq. (2)
Home Borrowers	κ	$\kappa = \kappa_{Ht} = \kappa_{Ft}$	$\kappa_{Ht} \neq \kappa_{Ft}$
Foreign Borrowers	κ	$\kappa^* = \kappa_{Ht}^* = \kappa_{Ft}^*$	$\kappa_{Ht} \neq \kappa_{Ft}$

3.2 Savers

Savers are willing to supply short-term funding in the integrated debt market because they discount the future less heavily than investors: $\beta\left(C_t^S\right) > \beta\left(C_t^I\right)$. Their dynamic programming problem is as follows:

$$\max_{\left\{c_t^S, B^S, k_{Ht}^S\right\}} \quad E_0 \sum_{t=0}^{\infty} \beta\left(C_t^S\right) \frac{\left(c_t^S\right)^{1-\sigma}}{1-\sigma}$$

subject to the budget constraint

$$c_t^S + q_{Ht}^e \left(k_{Ht}^S - k_{Ht-1}^S\right) - q_t^b b_t^S = w_t + z\left(k_{Ht-1}^S\right)^{\nu} - b_{t-1}^S \tag{4}$$

Savers do not take positions across the borders, yet they absorb a part of the stock of capital available in the economy, k_{Ht}^S, to produce in the "backyard" sector at decreasing returns to scale: $z\left(k_{Ht-1}^S\right)^{\nu}$, with $\nu < 1$.

3.3 Production

Competitive firms in both countries produce a homogeneous traded good using a standard Cobb-Douglas technology. So

$$Y_{Ht} = A_t \left(K_{Ht-1}\right)^{\alpha} 1^{1-\alpha}$$

[7] See, for example, Devereux and Sutherland (2011b), where the agent-specific debt-to-asset ratios are completely exogeneous, and Trani (2012), where similar ratios vary endogenously.

where $l = 1$ is the normalized amount of labour hours, A_t is productivity, α is the capital share and K_H is the stock of capital purchased by firms. This demand for capital is financed issuing equities, which are traded across borders. It follows that $K_{Ht} = n\chi_{Ht}$, where χ_{Ht} is the per-capita stock of shares, and this is owned across borders: $\chi_{Ht} = k_{Ht}^I + k_{Ht}^{*I}$.

3.4 Competitive Equilibrium

The clearing conditions for the good, bond, and capital markets are, respectively, as follows:

$$\begin{aligned} n\left(c_t^I + c_t^{*I}\right) + (1-n)\left(c_t^S + c_t^{*S}\right) &= Y_{Ht} + Y_{Ft} + (1-n)\left[z\left(k_{Ht-1}^P\right)^\nu + z\left(k_{Ft-1}^{*P}\right)^\nu\right] && (5) \\ n\left(b_t^I + b_t^{*I}\right) + (1-n)\left(b_t^S + b_t^{*S}\right) &= 0 && (6) \\ n\chi_{it} + (1-n)\,k_{it}^S &= 1 \text{ for } i = H, F && (7) \end{aligned}$$

For $t = 0, ..., \infty$, the competitive equilibrium is a vector of allocations (c_t^I, c_t^{*I}, c_t^S, c_t^{*S}, b_t^I, b_t^{*I}, k_{Ht}^I, k_{Ht}^{*I}, k_{Ht}^S, k_{Ft}^I, k_{Ft}^{*I}, k_{Ft}^S) and a vector of prices (q_t^b, q_{Ht}^e, q_{Ft}^e, w_t, w_t^*, d_{Ht}, d_{Ft}) such that: (a) the representative investor maximizes her lifetime utility subject to the budget and collateral constraints; (b) the representative saver maximizes her lifetime utility subject to the budget constraint; (c) firms purchase capital in order to maximize profits; (d) all markets clear.

The equilibrium conditions for the home country are indicated in appendix A.1. These equations - together with the similar ones valid for the foreign economy - constitute the "non-portfolio equations".

4 Pledgeability and Equity Premiums

Maximizing investors' utility under (1)-(2), their demand for savings, home and foreign equity claims are respectively as follows:

$$\begin{aligned} \lambda_t^I - \mu_t &= \beta\left(c_t^I\right) E_t \lambda_{t+1}^I R_{t+1} && (8) \\ \lambda_t^I - \mu_t \kappa_{Ht} &= \beta\left(c_t^I\right) E_t \lambda_{t+1}^I r_{Ht+1} && (9) \\ \lambda_t^I - \mu_t \kappa_{Ft} &= \beta\left(c_t^I\right) E_t \lambda_{t+1}^I r_{Ft+1} && (10) \end{aligned}$$

where $\lambda_t^h = \left(c_t^h\right)^{-\sigma}$, with $h = I, S$, and μ_t is home investors' marginal value of borrowing under collateral. Equations (9)-(10) show clearly that differences in the pledgeability of home versus foreign assets can have consequences on the desirability of home equities vis-à-vis foreign equities, even though μ_t is equivalent across collateral assets. Reforging Aiyagari and Gertler's (1999) argument, I combine (9) and (10), eliminating μ_t, and obtain

$$\frac{\kappa_{Ht}}{\kappa_{Ft}} = \frac{1 - E_t \Lambda_{t,t+1}^I r_{Ht+1}}{1 - E_t \Lambda_{t,t+1}^I r_{Ft+1}} \qquad (11)$$

where $\Lambda^h_{t,t+1} = \beta\left(c^h_t\right) E_t\lambda^h_{t+1}/\lambda^h_t$ is household h's stochastic discount factor. Assume for instance that $\kappa_{Ht}/\kappa_{Ft} > 1$: in t home equities are better collateral than foreign equities. As a consequence, investors need to be compensated to invest in foreign assets rather than home equities: $r_{Ht+1} < r_{Ft+1}$. Intuitively, assets which have a high (and stable) market value are good hedges against changes in credit market conditions, that is, the margin calls which are not explained by changes in borrowers' wealth. This hedging properties constitute an important motive for demanding such assets, so investors do not expect them to earn high returns.

The fact that one equity may be preferred as collateral to other equities is reflected on the relative premiums that investors can accrue on these types of assets. To fix ideas, I use (8)-(10) and (A.3) in the appendix to define the loan rate, the loan premium (i.e., *guarantee premium*, GP) and the equity premium (EP), respectively, as follows:

$$\begin{aligned} R_{t+1} &\equiv \left(E_t\Lambda^S_{t,t+1}\right)^{-1} && (12)\\ GP_t &\equiv \frac{E_t\left(\Lambda^S_{t,t+1} - \Lambda^I_{t,t+1}\right)}{E\Lambda^S_{t,t+1}E_t\Lambda^I_{t,t+1}} = \frac{\mu_t}{\beta\left(c^I_t\right)E_t\lambda^I_{t+1}} && (13)\\ EP_{it} &\equiv E_t\left(r_{it+1} - R_{t+1}\right) = \varrho_{it} + \underbrace{GP_t m_{it}}_{\text{default insurance}} \quad \forall i && (14)\end{aligned}$$

where

$$\varrho_{it} = -\frac{cov_t\left(\Lambda^I_{t,t+1}, r_{it+1}\right)}{E_t\Lambda^I_{t,t+1}}\text{: consumption insurance} \quad ; \quad m_{it} = 1 - \kappa_{it}\text{: haircut on collateral } i, \forall i$$

Savers earn a riskless return on their loans to investors (equation (12)). As a consequence, investors - who are impatient - must pay a loan premium (equation (13)), given by the shadow cost of borrowing against collateral (in terms of the marginal utility of future consumption). By leverage, this loan premium modifies the EP investors expect to earn on assets pledged as collateral (equation (14)). The EP pays not only for the risk of a future reduction in consumption caused by a negative state, ϱ_{it}, but also for the current insurance against default that investors must provide lenders with, $GP_t m_{it}$.

Due to this default insurance component, the relative profitability of a given equity goes beyond its expected rate of return relative to that of other assets. Given (14), the EP on home equity claims relative to foreign equity claims is

$$EP_{Ht} - EP_{Ft} = \varrho_{Ht} - \varrho_{Ft} - GP_t\left(\kappa_{Ht} - \kappa_{Ft}\right) \tag{15}$$

It is easier to interpret this relation in terms of the same example above, so consider again the case $\kappa_{Ht}/\kappa_{Ft} > 1$. Equation (15) suggests that there are no arbitrage opportunities only if the reward for the relative safety of home equities, $GP_t\left(\kappa_{Ht} - \kappa_{Ft}\right)$, is matched by a relative consumption insurance premium, $\varrho_{Ht} - \varrho_{Ft}$, of an opposite sign.

The choice between home and foreign equities is clearly going to be affected by this default insurance component of premiums (and its relationship with the consumption insurance component). The solution for equilibrium portfolio is therefore more complex than in the homogeneous pledgeability cases listed in Table 2. In fact, when assets are homogeneously pledgeable as collateral, each borrower earn a specific equity premium independently of the type of collateral:

$$EP_{it} = \varrho_{it} + GP_t (1-\kappa) \;\forall i \quad ; \quad EP^*_{it} = \varrho_{it} + GP^*_t (1-\kappa) \;\forall i \tag{16.a}$$

$$EP_{it} = \varrho_{it} + GP_t (1-\kappa_t) \;\forall i \quad ; \quad EP^*_{it} = \varrho_{it} + GP^*_t (1-\kappa^*_t) \;\forall i \tag{16.b}$$

so, for example from the viewpoint of home residents, the relative equity premium is not (15) but simply

$$EP_{Ht} - EP_{Ft} = \varrho_{Ht} - \varrho_{Ft} \tag{17}$$

Equation (17) is more standard for macroeconomic models (especially, those concerning the open economy): in this case, the "spread" between EP_{Ht} and EP_{Ft} has to do uniquely with the corresponding rates of return of the two types of equity claims. Then, given a model with multiple but homogeneously pledgeable assets, one can clearly excpect the representative investor to define her optimal demand for these assets in the same way as if they were not used as colateral.

Garleanu and Pedersen (2011) carry out an analysis which has some similarities with the arguments in this section. Garleanu and Pedersen show how, other things being equal, differences in haircuts justify differences in asset prices. Since I focus on endogenous country portfolios, I implicitly capture some of their relevant results with the comparison between home and foreign equities. What my model simply adds is that one can capture the effects of heterogeneous haircuts even without altering the implications of RE pricing for general macroeconomic equilibrium. According to (11) and (15), the heterogeneous pledgeability between assets translates into different rates of returns, but the two gaps tend to compensate each other toward a worldwide equalization of equity prices.

5 Portfolio Choice

Here, I describe a viable approach to solve for international portfolios when heterogeneous debt-to-asset ratios affect the demand for assets (equations (9)-(10) and their foreign counterparts). To describe the method I propose and apply, I address the following three questions. First, what is the *order* of the heterogeneous pledgeability between assets? In this perspective, I shall compare a "zero order heterogeneity", affecting the long-run (deterministic) equilibrium of the global economy, with a "first order heterogeneity", affecting the short-run. Second, for each of these two cases, what are the properties of the equilibrium portfolio shares and the corresponding asset returns? Finally, can we recast the solution for portfolios in terms of the methodologies already present in the literature?

The two main solution methods are Devereux and Sutherland (2011a) and Tille and Van Wincoop (2010) - respectively, DS and TvW in what follows. I develop my argument using a mixed approach, which benefits from the main findings and strategies of both works. In fact, DS and TvW propose two approaches that are essentially equivalent and characterized by interpretable key passages.

The notation I shall use is inspired by TvW. In approximating the model, I highlight the *components* of both equations and variables, distinguishing *by orders of the Taylor expansion*. This allows me to show how the portfolio solution for the homogeneous pledgeability case and that for (the two instances of) the heterogeneous pledgeability case rely on different order components of model variables, keeping track of the linear elements of order bigger than one.

In general, any variable can be written as the sum of its order components: for example, $Z_t = Z_t(0) + Z_t(1) + Z_t(2) + \ldots$ The zero order component is the steady state value of Z_t, while the other order components show up in the approximations around the steady state. In turn, the degree of approximation determines what order component of the equations is useful to solve the model: for example, solving a linearized model requires computing the first order components of its equilibrium conditions. The main point is that the order of the solution of a model with endogenous portfolios is not necessarily the same as the order of model variables that allows to capture risk - and, thus, to determine the portfolio shares. This is the principle emphasized by DS for the properties of a portfolio solution, and in this regard I follow them.

I refer again to TvW when I ultimately compute the equilibrium portfolios.

Some details are in the appendix herewith, while a description of the solution for model variables and the derivation of the main results below are in the separate appendix.

5.1 Portfolio Equations

5.1.1 Long-Run Heterogeneity: an Issue

Consider first a model in which collateral assets are always homogeneously pledgeable, as in (3.a)-(3.b). For each of these two alternatives, the first order conditions for equity i, with $i = H, F$, are

$$\lambda_t^I - \mu_t \kappa = \beta\left(c_t^I\right) E_t \lambda_{t+1}^I r_{it+1} \tag{18.a}$$
$$\lambda_t^I - \mu_t \kappa_t = \beta\left(c_t^I\right) E_t \lambda_{t+1}^I r_{it+1} \tag{18.b}$$

The portfolio Euler equations corresponding to *any* of these first order conditions - and their foreign counterparts - are:

$$E_t \lambda_{t+1}^I \left(r_{Ht+1} - r_{Ft+1}\right) = 0 \quad ; \quad E_t \lambda_{t+1}^{*I} \left(r_{Ht+1} - r_{Ft+1}\right) = 0 \tag{19}$$

which are the same portfolio choice conditions one would obtain in a model with non-binding leverage constraints. These conditions imply $r_H(0) = r_F(0) = r(0)$, meaning that asset returns are symmetric across borders. I can therefore apply a method such as the one developed by DS in its plain form.

Taking a second order approximations of equations (18) and combining them twice, I obtain

$$0 = E_t \left(\lambda_{t+1}^I (1) - \lambda_{t+1}^{*I} (1)\right) r_{xt+1} (1) \tag{20}$$

$$E_t r_{xt+1} (1) = 0 + T (2) \tag{21}$$

where $r_{xt} (j) = r_{Ht} (j) - r_{Ft} (j)$ is the j-th order component of the excess return between home and foreign assets, with $j > 0$, and $T (j)$ involves j-th order terms, with $j > 1$. The expression for $T (2)$ on the right hand side of (21) is detailed in appendix A.2. Equation (20) shows the first key property of the recent solution methods for international portfolios - **Property 1**. The equilibrium (or zero order) portfolios depend on risk, which affects the second order component of portfolio equations (18) and, thus, cannot be simply retrieved from the steady state of the model. But since the steady state is symmetric, only one term of the second order expansion of (19) shows up in the portfolio choice condition (20). This second order term is simply the product between the first order component of the excess return on equities and the discrepancy between home and foreign investors' marginal utility of future consumption. The condition for asset returns corresponding to the equilibrium portfolios satisfying (20) is given by (21) - **Property 2**. At the level of approximation suitable for determining the zero order component of country portfolios (i.e., the first order behaviour of model variables), the expected excess return on home versus foreign assets is *zero*. Indeed, $E_t r_{xt+1} (1)$ is only affected by second order and quadratic terms. Hence, as DS conclude, the first order solution of an open economy model involving financial trade can be computed by replacing $r_{xt} (1)$ with a wealth shock, which is expected to be null.

Consider now the heterogeneously pledgeable assets case. Given (9)-(10) and their foreign counterparts, the portfolio Euler equations change as follows:

$$E_t \lambda_{t+1}^I (r_{Ht+1} - r_{Ft+1}) + M_t (\kappa_{Ht} - \kappa_{Ft}) = 0 \quad ; \quad E_t \lambda_{t+1}^{*I} (r_{Ht+1} - r_{Ft+1}) + M_t^* (\kappa_{Ht} - \kappa_{Ft}) = 0 \tag{22}$$

where, for convenience, I have expressed each agent's shadow cost of borrowing in terms of her endogenous rate of time-preference: $M_t = \mu_t / \beta \left(c_t^I\right)$, $M_t^* = \mu_t^* / \beta \left(c_t^{*I}\right)$. It is then easy to see that an appropriate solution for portfolios cannot solely account for the rates of return on home and foreign equities. The problem with such a method is that it cannot satisfy Property 2 above. For example, in the perspective of home investors, equation (22) implies that

$$E_t \lambda_{t+1}^I (r_{Ht+1} - r_{Ft+1}) = -M_t (\kappa_{Ht} - \kappa_{Ft})$$

so the excess return on home versus foreign equities is expected to be "asymmetric".

Suppose that this sort of asymmetry affects all the order components of model equations, that is, that there is a difference between rates of return even in the nearly-stochastic steady state of the model. In such a case,

$$r_x (0) = -GP (0) (\kappa_H (0) - \kappa_F (0)) = (EP_H (0) - EP_F (0)) \tag{23}$$

where $GP(0) = M(0)/\lambda^I(0) > 0$ is the zero order component of the guarantee premium. Let $\kappa_x(j) = \kappa_H(j) - \kappa_F(j)$ be the j-th order component of the relative pledgeability of home versus foreign equities, with $j > 0$. Then, from equation (23), the inverse relation between r_x and κ_x characterizing above the relative equity premium (equation (15)) shows up immediately: $\kappa_x(0) \gtrless 0 \Rightarrow r_x(0) \lessgtr 0$.

Result 1. If H and F assets are not equally pledgeable as collateral already in the steady state of the model, an excess return of the same order arises. This excess return (and it inverse relations with the difference between debt-to-asset ratios) does not allow the model to satisfy the two main properties of the solution methods for international portfolios.

Taking a second-order approximation of equations (22) and combining them as previously done, I get

$$\begin{aligned}
0 \quad = \quad & 2r_x(0)\,E_t\left(\lambda^I_{t+1}(1) - \lambda^{*I}_{t+1}(1)\right) + GP(0)\,\kappa_x(0)\,(M_t(1) - M^*_t(1)) \\
& +2r_x(0)\,E_t\left[\lambda^I_{t+1}(2) - \lambda^{*I}_{t+1}(2) + \frac{1}{2}\left(\lambda^I_{t+1}(1)^2 - \lambda^{*I}_{t+1}(1)^2\right)\right] \\
& +E_t\left(\lambda^I_{t+1}(1) - \lambda^{*I}_{t+1}(1)\right)(r_H(0)\,r_{Ht}(1) - r_F(0)\,r_{Ft}(1)) \\
& +GP(0)\,\kappa_x(0)\left[M_t(2) - M^*_t(2) + \frac{1}{2}\left(M_t(1)^2 - M^*_t(1)^2\right)\right] \\
& +GP(0)\,(M_t(1) - M^*_t(1))\,(\kappa_H(0)\,\kappa_{Ht}(1) - \kappa_F(0)\,\kappa_{Ft}(1)) \qquad (24)
\end{aligned}$$

and

$$\begin{aligned}
E_t\left(r_H(0)\,r_{Ht+1}(1) - r_F(0)\,r_{Ft+1}(1)\right) \quad = \quad & -GP(0)\,(\kappa_H(0)\,\kappa_{Ht}(1) - \kappa_F(0)\,\kappa_{Ft}(1)) \\
& -r_x(0)\,E_t\left(\lambda^I_{t+1}(1) + \lambda^{*I}_{t+1}(1)\right) \\
& -\frac{1}{2}GP(0)\,\kappa_x(0)\,(M_t(1) + M^*_t(1)) + T(2) \qquad (25)
\end{aligned}$$

where the expression for $T(2)$ is again in appendix A.2. Intuitively, since $r_x(0) \neq 0$, the first order component of the expected excess return does not only depend on second order terms, $T(2)$, but also on first order terms (i.e., all the other terms on the right hand side of (25)). In other words, differently from what (21) suggests, now the first order component of the excess return is not expected to be null. To compute portfolios, this excess return must be discounted with the discrepancy between home and foreign investors' expected marginal utility of future consumption - and the difference in the pledgeability of collateral assets must be evaluated in line with the marginal utility of leveraged borrowing of both investors. It follows that (24) cannot be computed simply knowing the first order component of model variables, as it was instead the case for (20). Even simplifying (24) further, one would need to know either $M_t(2), M^*_t(2)$ or $\lambda^I_{t+1}(2), \lambda^{*I}_{t+1}(2)$. ■

The *main issue* is not so much accounting for the first order component of the expected excess return (actually, this is part of my solution strategy below) but the implications of this first order expected excess return on the portfolio choice condition (25). To capture risk at the second order, one would need to solve a quadratic approximation of the model, before knowing the zero order component of country portfolios. This means for sure that the second order approximation of the model would not only be affected by the equilibrium value of ω_t, but also by its dynamics around this value (Devereux and Sutherland, 2010; TvW). Maybe, the unique approach to follow in this case is a sort of iterative guess and verify procedure: 1) guess the equilibrium portfolios; 2) obtain both the first- and second-order solutions of the model, conditional on the guess; 3) verify that both the guess and the implied (elements of) portfolio dynamics satisfy (24) and (25).

The reason for this complexity is the zero order difference in pledgeability between home and foreign collateral assets, $\kappa_x(0) \neq 0$. In turn, this difference must originate from an assumption made in building the model. One could, for example, use the two-country model to study a situation in which already the long-run equilibrium shows that assets from the home country serve as collateral better than the assets from the foreign country. But given the evidence presented in the introduction and the financial studies on the determination of haircuts, such an assumption is neither strictly necessary nor appropriate for private assets. The relative riskiness of collateral assets is subject to shocks. Next, I assume $\kappa_x(0) = 0$ and obtain the heterogeneous pledgeability of assets as an endogenous consequence of risk. This allows me to simplify the portfolio choice problem and to solve it in line with DS and TvW.

5.1.2 Short-Run Heterogeneity: a Solution

Drawing on the finance literature, when savers lend to leveraged agents, the debt-to-asset ratio attached to each collateral asset depends on the fundamental value of the asset itself *plus* other informational frictions (e.g., informed versus uninformed savers, optimistic versus pessimistic beliefs, etc.). Leaving this informational frictions out of the model, I just focus on the dependency of κ_{Ht}, κ_{Ft} on fundamentals: q_{Ht}, q_{Ft} (i.e., d_{Ht}, d_{Ft}), respectively. In turn, home and foreign equity prices change with the state of the corresponding economy.

Assumption. The debt-to-asset ratios are state-dependent but conform to the symmetry characterizing the steady state of the model. Formally, let the i-th debt-to-asset ratio be

$$\kappa_{it} = f\left(Y_{it} - Y(0)\ ,\ \varepsilon^{i}_{\kappa t}\right) \quad \text{for} \quad i = H, F \tag{26}$$

so that

$$\kappa_H(0) = \kappa_F(0) = \kappa(0) \quad \Leftrightarrow \quad \kappa_x(0) = 0$$

In (26), the changes in fundamentals are captured by the fluctuations of the output produced by firms around its steady state value. I have chosen output instead, for instance, productivity because the capital share α is very important in the equilibrium between sectors (Devereux and Yetman, 2010; Trani, 2012): firms, on one side, savers' backyard production, on the other. And clearly q_{Ht}, q_{Ft} must satisfy the consumption Euler equations of both owners of the capital used by firms (investors) and savers[8].

Result 2. Under symmetric but state-dependent debt-to-asset ratios, both Property 1 and Property 2 of the portfolio solution methods are satisfied. Differences in the pledgeability of home versus foreign assets affect the first order component of model variables, generating a first order "asymmetry" between rates of return. The no-arbitrage condition between the two assets is satisfied in terms of an "overall excess yield".

Under the above assumption, now the combination between portfolio Euler equations (22) yields:

$$0 = E_t \left(\lambda^I_{t+1}(1) - \lambda^{*I}_{t+1}(1)\right) r_{xt+1}(1) + \frac{GP(0)\,\kappa(0)}{r(0)} \left(M_t(1) - M^*_t(1)\right) \kappa_{xt}(1) \tag{27}$$

$$E_t r_{xt+1}(1) = -\frac{GP(0)\,\kappa(0)}{r(0)} \kappa_{xt}(1) + T(2) \tag{28}$$

because, given (15), the zero order component of the debt-to-asset ratio implies

$$GP(0)\,\kappa(0) \Rightarrow EP_H(0) = EP_F(0) = EP(0)$$

Coherently, the term $GP(0)\,\kappa(0)/r(0)$ in (27)-(28) is a sort of relative weight between the zero order component of the symmetric equity premium and the symmetric rate of return. In fact, investors purchase equities both to earn capital income and to borrow at leverage. Being the condition that determines the zero order portfolios, equation (27) satisfies Property 1 above. As a result, the first order component of model variables is sufficient to compute the second order component of the combined portfolio Eulers.

In light of (28), let (27) be equivalently written as follows:

$$\begin{aligned} & E_t \left(\lambda^I_{t+1}(1) - \lambda^{*I}_{t+1}(1)\right) \Upsilon_{t+1}(1) \\ = & -\frac{GP(0)\,\kappa(0)}{r(0)} \left[\left(M_t(1) - M^*_t(1)\right) - E_t \left(\lambda^I_{t+1}(1) - \lambda^{*I}_{t+1}(1)\right)\right] \kappa_{xt}(1) \end{aligned} \tag{27'}$$

where I define

$$\Upsilon_{t+1}(1) = r_{xt+1}(1) + \frac{GP(0)\,\kappa(0)}{r(0)} \kappa_{xt}(1)$$

as the (first order component of the) *"overall excess yield"* on home equities versus foreign equities. Solving (13') for the expected value of this overall excess yield, I obtain:

$$\begin{aligned} E_t \Upsilon_{t+1}(1) = & -\frac{cov_t\left(\lambda^I_{t+1}(1) - \lambda^{*I}_{t+1}(1)\ ,\ \Upsilon_{t+1}(1)\right)}{E_t\left(\lambda^I_{t+1}(1) - \lambda^{*I}_{t+1}(1)\right)} \\ & -\frac{GP(0)\,\kappa(0)}{r(0)} \left[\frac{\left(M_t(1) - M^*_t(1)\right)}{E_t\left(\lambda^I_{t+1}(1) - \lambda^{*I}_{t+1}(1)\right)} - 1\right] \kappa_{xt}(1) \end{aligned} \tag{29}$$

[8] See, for instance, the pricing equations for home equities in appendix A.1.

where the terms on the right hand side are first order terms - the second order numerators are divided by first order denominators. Note that these first order terms must compensate each other out. In fact, given the definition of Υ_{t+1}, equation (28) implies that the overall excess yield is zero up to a first order,

$$E_t \Upsilon_{t+1}(1) = 0$$

which substituted in (29) yields

$$\frac{cov_t\left(\lambda_{t+1}^{I}(1) - \lambda_{t+1}^{*I}(1)\ ,\ \Upsilon_{t+1}(1)\right)}{E_t\left(\lambda_{t+1}^{I}(1) - \lambda_{t+1}^{*I}(1)\right)} = \frac{GP(0)\,\kappa(0)}{r(0)}\left[1 - \frac{\left(M_t(1) - M_t^*(1)\right)}{E_t\left(\lambda_{t+1}^{I}(1) - \lambda_{t+1}^{*I}(1)\right)}\right]\kappa_{xt}(1)$$

Put it differently, through equation (28) Property 2 (i.e., no-arbitrage for the portfolio share satifying (27)) applies in terms of $\Upsilon_{t+1}(1)$ instead of $r_{xt+1}(1)$. And if (28) defines the former as a random variable which is *zero* in expectation, it also implies that the latter must be interpreted as a *non-zero-mean* wealth shock:

$$E_t r_{xt+1}(1) = E_t \Upsilon_{t+1}(1) \underbrace{- \frac{GP(0)\,\kappa(0)}{r(0)}\kappa_{xt}(1)}_{\text{first order mean}} \tag{30}$$

■

To sum up, the assumption that the pledgeability of assets is symmetric but state-dependent allows to solve the portfolio choice problem in a similar way as in DS and TvW. The difference here is that - in the first order solution of the model - the expected excess return is not null because the approach accounts for both the return seeking behaviour of investors and their willingness to borrow against collateral (equation (30)). It is putting both motives together that no equity claim can be expected to be more profitable than the other (equation (28)). This confirms (at a first order approximation) what equity premiums in section 4 show: e.g., if on date t home assets are safer collateral than foreign assets, investors expect to earn a lower rate of return on the former than on the latter in $t+1$.

5.2 Non-Portfolio Equations

In order to observe the effect of portfolios on the non-portfolio equations, one should rewrite the model suitably, defining new variables. Let then the net foreign assets of the home country and the relevant portfolio share, respectively, be $NFA_t = q_{Ft}^{e} k_{Ft}^{I} - q_{Ht}^{e}\left(\chi_{Ht} - k_{Ht}^{I}\right)$ and $\omega_t = q_{Ht}^{e}\left(k_{Ht}^{I} - \chi_{Ht}\right)$. Being the negative of foreign investors' holdings of home equity, ω_t means that the ownership of the total stock of equity claims issued by firms in the home country is internationally diversified if $\omega_t < 0$.

Result 3. In a model with heterogeneously pledgeable asset, portfolios do not only affect investors' budget but also their leverage. It follows that the tightness of the borrowing limits depends on portfolio choice. In contrast, in the homogeneous pledgeability case, the effect of portfolios is limited to investors' budget, as it happens with assets which are not pledged as collateral..

Rewriting the budget and leverage constraints for the heterogeneously pledgeable assets case (equations (1)-(2)) in terms of NFA_t and ω_t, I obtain

$$\begin{aligned} c_t^I + NFA_t &= w_t + d_{Ht}\chi_{Ht-1} - q_{Ht}\left(\chi_{Ht} - \chi_{Ht-1}\right) + q_t^b b_t^I - b_{t-1} \\ &\quad + \left(r_{Ht} - r_{Ft}\right)\omega_{t-1} + r_{Ft} NFA_{t-1} \qquad (1') \\ b_t^I &= \left(\kappa_{Ht} - \kappa_{Ft}\right)\omega_t + \kappa_{Ft} NFA_t + \kappa_{Ht} q_{Ht}^e \chi_{Ht} \qquad (2') \end{aligned}$$

where of course worldwide equilibrium implies that $NFA_t + NFA_t^* = 0$[9]. Note that the assumption made in the previous section has implications *also* for portfolio equations. If κ_{Ht}, κ_{Ft} were not symmetric in the steady state, then the first order component of equations (1')-(2') would not only be affected by the zero order component of the portfolio sshare but also by its first order component - another dissimilarity with respect to DS and TvW. Formally, one would have $\omega\left(0\right) r_x\left(1\right) + r_x\left(0\right)\omega\left(1\right)$ for (1') and $\omega\left(0\right)\kappa_x\left(1\right) + \kappa_x\left(0\right)\omega\left(1\right)$ for (2'). Incidentally, the fact that the linearized model would be affected by $\omega\left(1\right)$ confirms that, without the assumption in the previous section, one would need to make a guess for $\omega\left(0\right)$ and approximate around this guess (should this solution strategy work).

Consider now the effect of portfolios on model equations in the homogeneous pledgeability case. Investors' budget constraint remains the same, so it can be re-written as in equation (1'). In contrast, the leverage constraint is as in (3.a)-(3.b). Rewriting the home investors' leverage constraint in terms of NFA_t and ω_t for these two specifications, repectively, I obtain:

$$\begin{aligned} b_t^I &= \kappa NFA_t + \kappa q_{Ht}^e \chi_{Ht} \qquad (3.a') \\ b_t^I &= \kappa_t NFA_t + \kappa_t q_{Ht}^e \chi_{Ht} \qquad (3.b') \end{aligned}$$

It is clear that these two equations differ from (2') because ω_t does not show up in any of them. ■

For an intuitive interpretation of (1') and (2'), recall above results on the expected excess return on equities and on the "overall excess yield". If $\kappa_{Ht} > \kappa_{Ft}$, the investors whose portfolios are tilted toward home equities can currently borrow more than those holding relatively more foreign equities: leverage constraints are tighter for the latter agents. At the same time, equilibrium requires that, from t to $t+1$, the agents that invest heavily in foreign equities must accumulate more foreign assets than the investors who mainly hold home equities. Other things being equal, the impact of portfolios on NFA_{t+1} depends on two factors: one is the size of ω_t; the other is the relationship between $r_{Ht} - r_{Ft}$ and $\kappa_{Ht} - \kappa_{Ft}$ which implicitly chain budget and collateral constraints together.

To compute $\omega\left(0\right)$, I adopt an iterative procedure. One can in principle try to rewrite the linearized model equations in terms of the overall excess yield $\Upsilon_{t+1}\left(1\right)$, combining the budget constraint (1') with the leverage constraint (2') and solving the model in terms of $\Upsilon_{t+1}\left(1\right)$ as a zero-mean wealth shock, fully in accordance with DS. Actually, this was my initial strategy. However, I eventually reverted

[9] NFA_t^* is foreign investors' net foreign assets, which affect their budget and collateral constraints.

to an iterative procedure and refrained from this additional modification of the linearized model for two reasons. First, finding a closed form solution for $\omega(0)$ from (27) is much more complicated (if possible) than finding a closed form solution for $\omega(0)$ from (20). Second, the iterative procedure allows to solve the non-portfolio equations considering (27) in its exact form[10], while combining the first order components of (1') and (2') does not - so requiring an additional assumption on expectations.

Hence, following TvW, I introduce a second order transaction cost τ to capture asymmetric information about foreign assets, which is one of the potential determinants of the home equity bias observed in real data. In fact, the model developed in section 3 cannot generate this home bias endogenously. Then, given an intial value for $\omega_{he}(0)$ (the portfolio share in the heterogeneous pledgeability case), I compute (27) as

$$g\left(\omega_{he}(0)\right) \equiv 2\tau + \mathbf{U}\boldsymbol{\Sigma}\mathbf{L}_1' + \frac{GP(0)\,\kappa(0)}{r(0)}\mathbb{k}_1\boldsymbol{\Sigma}\left(\mathbf{N}_1 - \mathbf{L}_1\right)'$$

and iterate until finding the equilibrium portfolio share: $g\left(\omega_{he}^E(0)\right) = 0$. $\mathbf{U}, \mathbf{L}_1, \mathbb{k}_1, \mathbf{N}_1$ are rows of the first order solution and $\boldsymbol{\Sigma}$ is the variance-covarince matrix of shocks. The details are in appendix A.3.

For comparisons, I also determine the portfolio share that characterizes the equilibrium of a model with homogeneously pledgeable assets, $\omega_{ho}(0)$. With a similar strategy, I compute (20) as

$$g\left(\omega_{ho}(0)\right) = 2\tau + \mathbf{R}\boldsymbol{\Sigma}\mathbf{L}_1'$$

and find the equilibrium portfolio satisfying $g\left(\omega_{ho}^E(0)\right) = 0$. In this case, $\mathbf{R}$ replaces $\mathbf{U}$ and is simply the row of the model solution corresponding to $r_{xt+1}(1)$ - for this formula, see DS.

5.3 Toward a Generalization

The method described in this section is potentially applicable to all the models in which investors are supposed by a contract to take certain actions, that ultimately affect the capital income. Clear examples are all the contracts imposing to a worker or an entrepreneur to comply with an incentive compatibility contraint.

In addition, the solution strategy developed here can capture cases in which some of the assets that are internationally traded give utility value to their holders. Think for instance to a two-country monetary model in which money (deposits) enters into agents' utility function. Accounting for these utility value in the demand for home and foreign assets would lead to first order conditions similar to equations (9)-(10).

[10] According to (27), the random variable Υ_{t+1} is given by the *current* difference between debt-to-asset ratios and the *future* excess rate of return.

6 Numerical Application

6.1 Calibration

I calibrate the model choosing values which are equivalent or close to both those used by Devereux and Yetman (2010) and those that I computed from real data in an earlier work (Trani, 2012).

TABLE 3. Calibration

parameter	description	value
n	number of constrained investors	0.5
ϕ	discount factor parameter	0.022
σ	CRRA	2
α	capital share in income	0.4
z	fixed productivity in backyard production	1
ν	degree of homogeneity in backyard sector	0.1
$R(0)$	gross rate of interest	1.042
$GP(0)$	guarantee premium	0.01
$A(0)$	total factor productivity	1
ρ_A	persistence of productivity shocks	0.81
σ_A	volatility of productivity shocks	0.015
$\rho(\varepsilon_A, \varepsilon_{A^*})$	correlation between productivity shocks across borders	0.3
$\kappa(0)$	debt-to-asset ratio	0.5
e_Y	sensitivity of debt-to-asset ratio to output gap	$\begin{cases} 0.7 & \text{- baseline} \\ 0.3 & \text{- robustness 1} \\ 0.0 & \text{- robustness 2} \end{cases}$
ρ_κ	persistence of shocks to debt-to-asset ratio	0.75
$\sigma_\kappa, \sigma_{\kappa_i}$	volatility of shocks to debt-to-asset ratio	0.02
$\rho\left(\varepsilon_\kappa^H, \varepsilon_\kappa^F\right)$	cross-correlation between shocks to κ_i - heterogeneous case	0.2

Table 1 reports the values chosen for model parameters and for the zero order component of key endogenous variables. I borrow n, ϕ, σ, z, ν from Devereux and Yetman (2010), while the share of capital in production, α, is a bit higher than the one that they use: it equals 0.4. Then, the rate of interest is 4.2 per annum, which is close to the average annualized Libor prevailing in the U.K. and the U.S. This rate is riskless, so investors must pay an (opportunity) risk premium if they want to borrow against their equity holdings. For simplicity, I choose a premium of 100 basis points. Paying this premium and pledging collateral, investors receive a loan whose size is equal to half of the market value of collateral assets. That is, $\kappa(0) = 0.5$[11].

Table 2 shows the parameters that I adopt to characterize the forcing processes present in the model. The forcing variables in the model are productivity and equity-specific debt-to-asset ratios; in the alternative case of homogeneously pledgeable assets, these ratios are replaced by agent-specific debt-to-asset ratios. Respectively:

$$\ln A_t = \rho_A \ln A_{t-1} + \varepsilon_{At} \tag{31}$$

$$\kappa_{it} = \rho_\kappa \kappa_{it-1} + (1 - \rho_\kappa)\left[\kappa(0) + e_Y (Y_{it} - Y(0))\right] + \varepsilon^i_{\kappa t} \text{ for } i = H, F \tag{32}$$

$$\kappa_t = (1 - \rho_\kappa)\kappa(0) + \rho_\kappa \kappa_{t-1} + \varepsilon_{\kappa t} \tag{33}$$

where e_Y is the elasticity of the equity claim issued by country i's producers to fluctuations in country i's output around its long-run level.

Productivity at home and in the foreign country behave as standard AR(1) processes. I assume that the autoregressive parameter, ρ_A, is equal to that of the average Solow Residual in a sample of major OECD countries. The same is true for the standard deviation of productivity shocks, σ_A, whereas I set $\rho(\varepsilon_A, \varepsilon_{A^*}) = 0.3$. As far as the debt-to-asset ratios are concerned, I compare the performance of the model under two alternative assumptions. The heterogeneous pledgeability case is studied using (32). In this case, ρ_κ, the persistence of debt-to-asset ratio shocks, is set equal to 0.75, and their volatility is 0.02. So modifications in margin requirements are less persistent but more volatile than productivity shocks. For expositional purposes, I choose $e_Y = 0.7$ and I make some robustness checks using two lower values. The homogeneous pledgeability case is studied using (33) for both home and foreign investors. In order to make comparisons, I calibrate $\rho_\kappa, \sigma_\kappa$ with the same values used for (32).

6.2 Zero Order Portfolios

The equilibrium (or zero order) portfolios predicted by the model are reported in Table 4. These results refer to a transaction cost τ which can generate about 65 percent home equity bias in the homogeneous pledgeability case: $\tau = 1.770e^{-4}$. This case represents the choice between pledgeable assets which can be solved using the methods by DS and TvW plainly: since each collateral asset allows investors to obtain the same amount of credit, they have no reasons to consider the performance of assets as collateral when they select their portfolios.

[11] See again the calibration in Devereux and Yetman (2010).

TABLE 4. Zero Order Portfolios

		Homogeneous Case		Heterogeneous Case	
		H equity	F equity	H equity	F equity
hs	H Country Savers	0.792	-	0.792	-
hi	H Country Investors	2.055	1.115	1.819	1.350
fs	F Country Savers	-	0.792	-	0.792
fi	F Country Investors	1.115	2.055	1.350	1.819
	portfolio shares	64.8 %	35.2 %	57.4 %	42.6 %

Note: portfolio shares = hi/(hi+fi)=fi/(hi+fi)

The last two columns of Table 4. show that, in a model where the relative pledgeability between assets is a new determinant of portfolio choice, there is a reduced tendency to invest heavily in home equities. Keeping the transaction cost fixed, I find that the portfolio share of home equities plunges from 64.8 percent to 57.4 percent. To interpret this result, think of the labour income risk. Since the return on local equities is positively correlated with the domestic business cycle, agents can insure against shocks to their labour income by holding foreign assets in their portfolios. Indeed, one of the recent results in international portfolio theory is that, in order to endogenously generate home bias, a model must contain a factor that separates capital income from non-capital income.

Similarly, the debt-to-asset ratio attached to the pleadgeable assets issued by the firms of a given country depends on the state of that economy. Given the specification used (equation (32)), savers lend more against home collateral in case a positive shock raises local output above its steady state level, while the opposite happens in case of negative shocks. Intuitively, to contrast a state-contingent reduction in credit, home investors prefer to have relatively more foreign equities in their portfolios than they do in a model with homogeneously pledgeable collateral assets. Put it differently, home investors need to hedge against a *risk of margin calls* on local collateral, which is not triggered by a loss in investors' wealth but by the reaction of financiers to the business cycle of the country where the collateral pledged by investors is from.. The same can be said for investors in the foreign country.

Of course, the importance of the risk of margin calls depends on the sensitivity of the debt-to-asset ratio to the state of the economy, e_Y. Table 4. is obtained setting $e_Y = 0.7$, which is the baseline calibration. In Table 5., I report some robustness checks, asking what happens if e_Y is lower than in the baseline.

Table 5. Portfolio Shares with Heterogeneous Pledgeability: Robustness

	Benchmark Transaction Costs: $\tau = 1.770e^{-4}$		
	Benchmark Sensitivity: $e_Y = 0.7$	Robustness 1: $e_Y = 0.3$	Robustness 2: $e_Y = 0$
H Equity Share	57.4 %	61.1 %	64.2 %
F Equity Share	42.6 %	38.9 %	35.8 %
	Higher τ for Home Bias as in Homogeneous Case		
	$1.973e^{-4}$	$1.863e^{-4}$	$1.784e^{-4}$

Not surprisingly, the message from the last two columns of this table is that the lower the state-dependency of debt-to-asset ratios, the closer we get to the equilibrium portfolio shares found assuming homogeneous pledgeability. In the limit case in which κ_{Ht}, κ_{Ft} are purely exogenous (i.e., for $e_Y = 0$), the local bias in assets is 64.2 percent - just slightly below the local bias predicted by a model with homogeneously pledgeable collateral assets (64.8 percent). However, even this slight difference has a meaning, which shall be clarified in the dynamic analysis below. Only when we account for the heterogeneous pledgeability between collateral assets, there is an international transmission of financial shocks which affect the composition of portfolios - i.e., that is consistent with the empirical evidence in Figure 1. and Table 1 and that somehow features an "asymmetrical" reaction of external claims and liabilities To the shock (Broner et al., 2011).

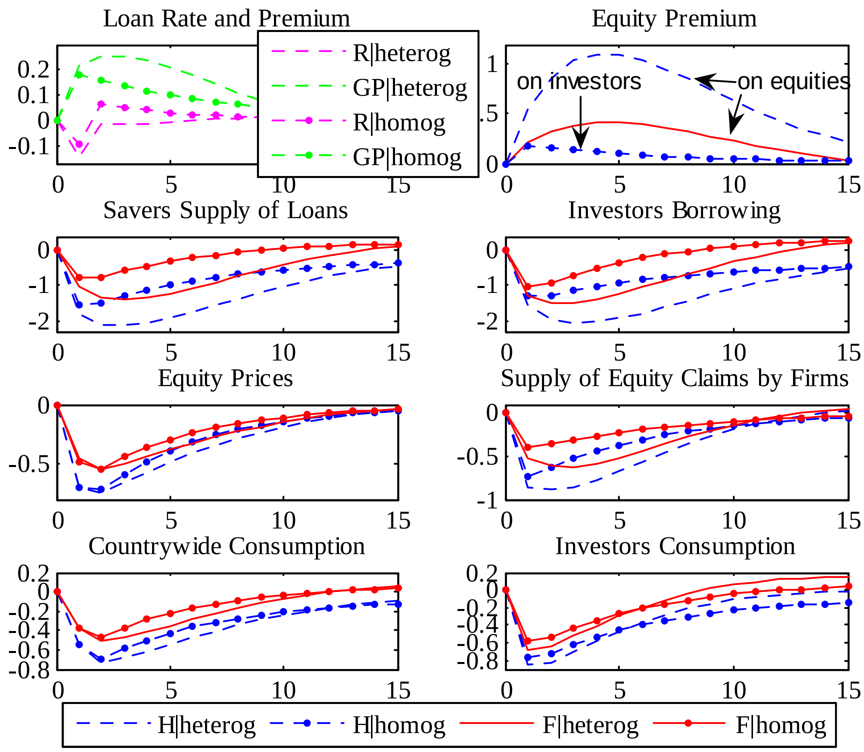
Finally, the bottom row of Table 5. shows how bigger should problems such as asymmetric information about foreign assets (i.e., τ) be in order to generate the same home bias obtained in the homogeneous case.

6.3 Productivity Shocks

Consider a (one unit) decrease in productivity in the home country. Figure 2. shows the reaction of the main variables in both countries and, thus, the transmission of the shock from the home to the foreign economy. The dotted lines represent what happens in the homogeneous pledgeability case, while non-dotted lines refer to heterogeneous pledgeability under the baseline value for e_Y.

Clearly, the main message from Figure 2. the contribution of a model featuring heterogeneous pledgeability is to understand how the risk premiums on home and foreign collateral assets must react for the economies to go back to equilibrium. Other consequences have quantitative nature. Consider again equations (11) and (15), as opposed to (17).

FIGURE 2. Effects of a Shock to Productivity in the Home Country



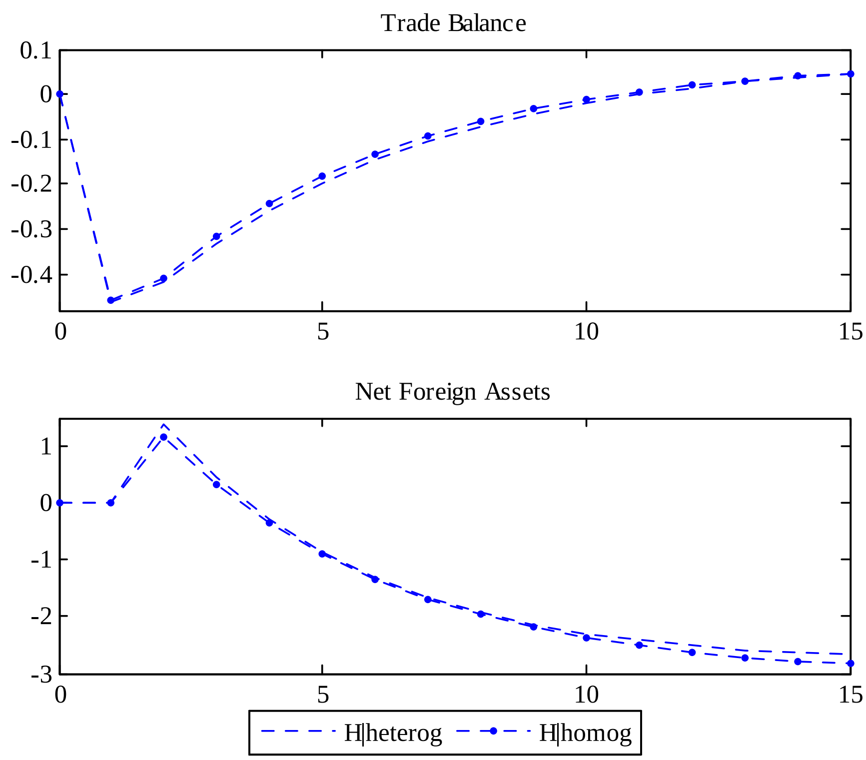
On impact, the price of home equities must fall. Since ex ante both investors hold home equity claims in their portfolios, not only home agents, but also foreign agents suffer from a wealth loss. This drop in wealth cause leverage constraints in all countries to tighten, transmitting the effects of the shock to foreign equities and all the other foreign variables. The cross-border transmission of the shock unfolds in the way described by Devereux and Yetman (2010)[12], assuming homogeneously pledgeable assets.

The novel result here is that this transmission is possible because of the (relative) dynamics of equity premiums. The second panel of Figure 2. shows that the shock to home productivity abates the pledgeability of home collateral versus foreign collateral. Therefore, investors need to be compensated to hold an asset against which they can borrow less. That is, the ex post decrease in investors' borrowing (fourth panel) is mostly due to home equities, so EP_{Ht} increases markedly above EP_{Ft} for quite a few periods after the shock. In contrast, when equities are taken as homogeneous collateral assets, the equity premium is agent-specific and follows the dynamics of GP_t, GP_t^* (first panel *plus* equations (16.a)-(16.b) vis-à-vis (14)). I plot just one of these loan premiums as their first order behaviour is equivalent.

[12]See also Devereux and Sutherland (2011b) and Trani (2012).

For all the other variables displayed in Figure 2., the behaviour of haircuts after a shock to home productivity magnifies the shock itself. And Figure 1.A in the appendix shows that this magnification effect is much smaller if haircuts are less responsive to the business cycle (i.e., $e_Y = 0.3$ instead of the baseline value); however, a gap between equity premiums is still needed for the two-country model to converge back to the steady state. The reason for this is that the first order difference between the collateral value of home and that of foreign equities does not affect the post-shock gap between their prices. The first order worsening in the relative pledgeability of home versus foreign assets is compensated by a positive reaction of $EP_{Ht} - EP_{Ft}$ (second panel) so that equity prices can fall in the same way as they would do if collateral assets where homogeneous[13] (fifth panel).

FIGURE 3. International Accounts after Productivity Shocks



Since differences in pledgeability do not cause a noticeable gap between the market values of home versus foreign collateral to open up, under productivity shocks the model fails to reproduce a fall in NFA_t (Figure 3.). This is at odds with the impact of flight-to-safety on the U.S. NIIP during the last crisis (Figure 1.), but it is neither inconceivable nor completely new. First, under macroeconomic shocks, the asymmetric reaction of equity premiums works to satisfy the same type of deleveraging we would observe with homogeneously pledgeable assets. Second, and as noted elsewhere (Trani, 2012),

[13] Cfr. Garleanu and Pedersen (2011).

the framework at hand lacks certain factors and tradable assets that affect asset prices worlwide. Only expanding the model in this sense one can see that the country hit by the shock is subject to a negative valuation on its net external positions.

6.4 On the Origins of Flight-to-Safety: Financial Shocks

In this section, I analyze the effects of a (one unit) decrease in the κ_{Ht}. Following Jermann and Quadrini (2012), I interpret such a shock as a financial shock, amounting to an increase in the margin that lenders apply to collateral from the home country. In other words, the shock can be regarded as an exogenous increase in the riskiness of home collateral, which suddenly turns out to be a "lemon" that lenders initially accepted because of adverse selection[14].

The main result is that, interpreted in this way, financial shocks allow the model to reproduce the observed dynamics of the net foreign assets position, in contrast with what happens after macroeconomic shocks. Observing the responses in Figure 4., the machanism is as follows. The decrease in κ_{Ht} makes home equities less valuable for collateralized borrowing than foreign equities, so EP_{Ht} must increase above EP_{Ft} for the same reasons as above (second panel). Yet, now the "spread" between these two risk premiums is much wider (compare with Figure 2.), reflecting the fact that the relative riskiness between home and foreign collateral is not a mere adaptation to the business cycle. Indeed, Figure 4. is obtained with the baseline value for e_Y, but the same graph can be obtain with any other e_Y (so I omit reporting equivalent impulse response functions).

This notwithstanding, the reaction of the equity premiums is now sufficient to have magnifying effects because the financial shock is "asymmetrically" transmitted from home to foreign investors' borrowing (fourth panel). Other things equal, the fall in κ_{Ht} means that investors can borrow less against home collateral than against foreign collateral. Since capital ownership is internationally diversified, borrowing does not only decrease for home investors, but also for foreign investors. However, ex ante home borrowers hold 65 percent of the domestic tradable stock, so they suffer more. A model with homogeneously pledgeable collateral (e.g., Devereux and Yetman, 2010) cannot capture this instance of the international transmission mechanism.

Moreover, the heterogeneous pledgeability case allows to analyze the ex post behaviour of each asset, while an homogeneous pledgeability model assumption *does not.* To see this consider what would happen in a model such as the latter: the financial shock in the home country would be an exogenous fall of κ_t[15]. The interpretation in this case is that suddenly savers do not want to transact only with leveraged investors from the home country, *regardless of* the assets that they pledge as collateral. There are no consequences for the relative pledgeability of home versus foreign equities, so foreign investors' equity premium hardly move (second panel in Figure 4.) as they can continue to borrow as before the

[14] See Gorton (2009).
[15] See Devereux and Sutherland (2011b).

shock (fourth panel). Ultimately, the effect of the shock to κ_t on all other domestic and foreign variables is quite modest because the loss of confidence is not transmitted across borders.

FIGURE 4. Effects of a Shock to the Pledgeability of H Equity

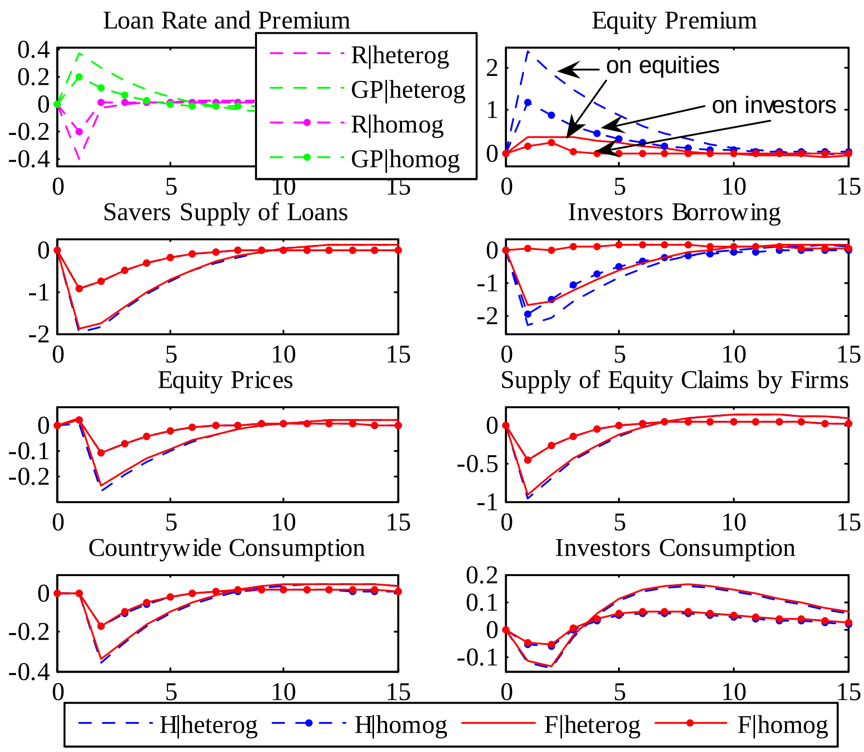
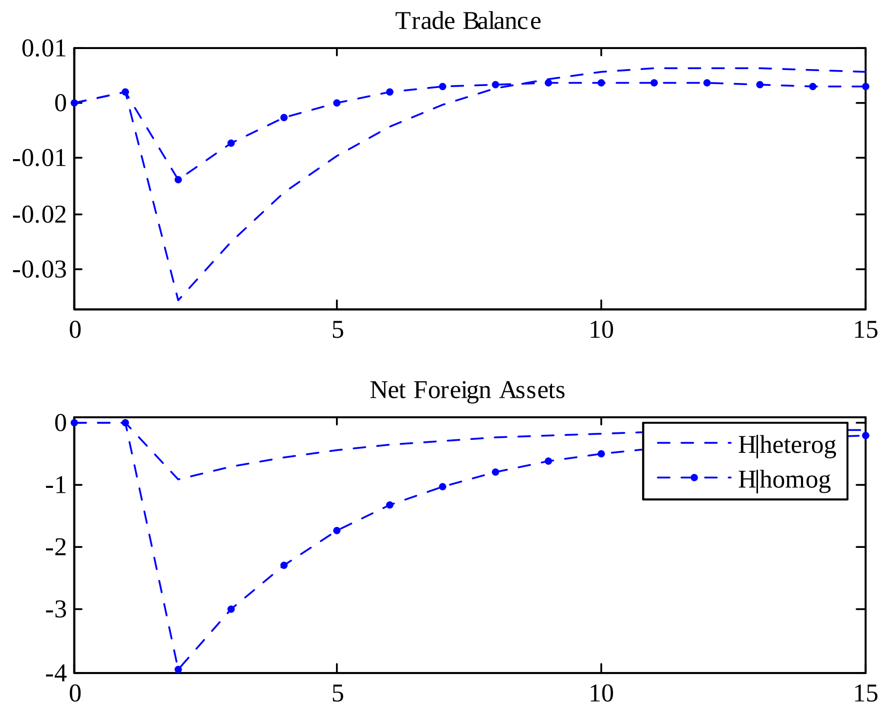


Figure 5. shows that, after a financial shock at home, the model can generate a drop in the net foreign asset position of the home country, in line with the empirical evidence described in the introduction. Actually, matching these dynamics is possible with both assumptions, homogeneously pledgeable assets and heterogeneously pledgeable assets. And the corresponding movements in the trade balance are always due to worldwide equilibrium requirements in a two-country framework where there is a unique traded good. But the fact that, as just said, a model with homogeneously pledgeable assets does not help understand the behaviour of home and foreign collateral plays a role.

In the homogeneous pledgeability case, the net foreign assets of the home country decrease markedly after the shock (second panel of Figure 5.). This sizeable decrease is mainly due to the fact that lenders lose confidence in home investors as opposed to foreign investors. So, as shown by Figure 4. (panels 2, 4, and 5): the equity premium earned by home borrowers increases regardless of the assets composing their portfolios, while that of foreign investors remains unaffected. As a consequence, only home investors' borrowing falls, and the reaction of prices is very contained. It follows that the predicted fall in NFA_t is due to *first order* valuation effects which are specific to the comparison between home and foreign borrowers, not between home and foreign assets as well as the way these assets enter into borrowers' portfolios. This is at odds with observed flight-to-safety phenomena, the numbers reported in Table 1. and what generally happened during the last and similar crises.

FIGURE 5. International Accounts after Financial Shocks



In contrast, in the model with heterogeneously pledgeable asset, the shock reduces the suitability of home equities to be pledged as collateral. This creates a wedge between the rate of return on home versus foreign equities of an opposite sign (Figure 4., panel 2), with consequences in all countries because both home and foreign agents hold home equities. Valuation effects that cause NFA_t to fall, as in Figure 5., follows from this wedge. After the shock, domestic investors must earn a lower return on their foreign equity holdings than the return earned by foreign investors on their home equity holdings. That is, the shock to the riskiness of home collateral has higher effects on home country external claims than on its external liabilities. This matches the greater impact of the flight-to-safety on U.S. external claims than on its external liabilities underlying the dynamics in Figure 1. This result is suggestive of what kind of financial shock an international transmission framework should embed.

Finally, I check for the robustness of the result described in this section, changing the sensitivity e_Y of debt-to-asset ratios to the state of the economy. I find that, as e_Y decreases, the fall in NFA_t is more pronounced. Yet, even with purely exogenous debt-to-asset ratios the decline in NFA_t is half in size as the one predicted by a model with homogeneously pledgeable collateral (see Figure 2.A in appendix A. 4 and compare it with Figure 5.).

7 Conclusion

Although the demand for an asset is principally explained by the corresponding rate of return, this paper shows that other factors can be important as well. The additional factors considered here is the usefulness of an asset to serve as collateral. This becomes a determinant of the choice between alternative investments only if the leverage constraints are modeled in such a way that each collateral asset available to investors carries its own specific margin.

Embedding leverage constraints of this sort, the two-country model analyzed here is affcted by an endogenous portfolio choice not only through investors' budget constraints, but also through their leverage constraints. The solution of this portfolio problem is obtained taking advantage of the fact that the riskiness of collateral assets is state-dependent and assuming that it dies out with the effects of a given shock. So, in the steady state, the debt-to-asset ratios of all assets assume a symmetric value.

Applying this solution strategy, the model shows that the heterogeneous pledgeability between assets produce both steady state effects and dynamic effects. In fact, fluctuations in the relative riskiness of home versus foreign collateral assets must be compensanted by changes in the expected excess return. Nothwithstanding this compensation, investors show an interest in holding assets that work well as collateral.

First, this interest curbs the local bias in assets, as agents want to insure against the risk of being overly exposed to equities whose pledgeability is positively correlated with the domestic cycle. Second, here the international transmission channel created by binding leverage constraints works also through the dynamics of the equity premiums, which justifies the reaction of the expected excess return to current changes in collateral values. Finally, this type of transmission implies that the composition of agents' portfolios matters when one of the countries is subject to a financial shock. And the external adjustment such a shock gives rise to generates valuation effects that are in line with those observed in U.S. data, after the flight-to-safety that characterized the recent financial crisis.

References

[1] Aiyagari, R. and M. Gertler (1999) "Overreaction of Asset Prices in General Equilibrium", *Review of Economic Dynamics*, 2, 3-35.

[2] Baxter, M. and U. J. Jermann (1997) "The International Diversification Puzzle is Worse Than You Think", *American Economic Review*, 87(1), 170-80.

[3] Bertaut, C. C. and L. Pounder (2009) "The Financial Crisis and U.S. Cross-Border Financial Flows", *Federal Reserve Bulletin*, November, A147-A167.

[4] Blengini, I. (2011) "Portfolio Choice under Uncertainty", University of Lausanne, *mimeo.*

[5] Broner F., T. Didier, A. Ercer and S. L. Schmukler (2011) "Gross Capital Flows: Dynamics and Crises", *CEPR Discussion Paper* 8591.

[6] Caballero, R. J., E. Farhi and P.-O. Gourinchas (2008) "An Equilibrium Model of "Global Imbalances" and Low Interest Rates", *American Economic Review*, 98(1), 358-93.

[7] Cao, D. and P. Gete (2011) "Demand for Collateral and Foreign Holdings of U.S. Assets", *Mortara Working Paper* 2011-16.

[8] Curcuru, S. E., T. Dvorak and F. E.. Warnock (2008) "Cross-Border Returns Differentials", *Quarterly Journal of Economics*, 123(4), 1495-1530.

[9] Curcuru, S. E., C. P. Thomas and F. E. Warnock (2012) "On Return Differentials", Darden, University of Virginia, *mimeo.*

[10] Devereux, M. B. and J. Yetman (2010) "Leverage Constraints and the International Transmission of Shocks", *Journal of Money, Credit and Banking*, 42(6).

[11] Devereux, M. B. and M. Saito (2006) "A Portfolio Theory of International Capital Flows", *CEPR Discussion Paper* 5746.

[12] Devereux, M. B. and A. Sutherland (2010) "Country Portfolio Dynamics", *Journal of Economic Dynamics and Control*, 34(7), 1325-1342.

[13] Devereux, M. B. and A. Sutherland (2011a) "Country Portfolios in Open Economy Macro Models", *Journal of European Economic Association*, 9(2), 337-369.

[14] Devereux, M. B. and A. Sutherland (2011b) "Evaluating International Financial Integration under Leverage Constraints", *European Economic Review*, 55(3), 427-442.

[15] Evans, M. D. D. and V. Hnatkovska (2011) "A Method for Solving General Equilibrium Models with Incomplete Markets and Many Financial Assets", Georgetown University and University of British Columbia, *mimeo.*

[16] Garleanu, N. and L. H. Pedersen (2011) "Margin-Based Asset Pricing and Deviations from the Law of One Price", *Review of Financial Studies*, 24(6), 1980-2022.

[17] Gohrband, C. A. and K. L. Howell (2010) "U.S. International Financial Flows and the U.S. Net Investment Position: New Perspectives Arising from New International Standards", in *Wealth, Financial Intermediation and the Real Economy*, NBER.

[18] Gorton, G. (2009) "Slapped in the Face by the Invisible Hand: Banking and the Panic of 2007", *Federal Reserve Bank of Atlanta*, Conference Paper.

[19] Gourinchas, P.-O. and H. Rey (2007) "From World Banker to World Venture Capitalist: The U.S. External Adjustment and the Exorbitant Privilege" in R. Clarida (ed.) *G7 Current Account Imbalances: Sustainability and Adjustment,* University of Chicago Press, 11-55.

[20] Jermann, U. and V. Quadrini (2012) "Macroeconomic Effects of Financial Shocks", *American Economic Review*, 102(1), 238-271.

[21] Judd, K. (1998) *Numerical Methods in Economics*, MIT Press.

[22] Kiyotaki, N. and J. Moore (1997) "Credit Cycles", *The Journal of Political Economy*, 105(2), 211-248.

[23] King, R. J. and M. Watson (1998) "The Solution of Singular Linear Difference Systems under Rational Expectations", *International Economic Review*, 39(4), 1015-1026.

[24] Klein, P. (2000) "Using the Generalized Schur Form to Solve a Multivariate Linear Rational Expectations Model", *Journal of Economic Dynamics and Control*, 24(10), 1405-1423.

[25] Lane, Philip R., and G. M. Milesi-Ferretti (2005) "A Global Perspective on External Positions" *NBER Working Paper* 11589.

[26] Mendoza, E. G., V. Quadrini and J.-V. Rios-Rull (2007) "Financial Integration, Financial Deepness and Global Imbalances", *NBER Working Paper* 12909.

[27] Pavlova, A. and R. Rigobon (2008) "An Asset-Pricing View of External Adjustment", *Journal of International Economics*, 80(1), 144-156.

[28] Tille, C. and E. Van Wincoop (2007) "International Capital Flows", *NBER Working Paper* No. 12856.

[29] Tille, C. and E. Van Wincoop (2010) "International Capital Flows", *Journal of International Economics*, 80(2), 157-175.

[30] Trani, T. (2012) "Funding under Borrowing Limits in International Portfolios", The Graduate Institute | Geneva, *mimeo.*

A Appendix

A.1 Equilibrium Conditions: Home Country

- **Investors**

  Given $\lambda_t^I = \left(c_t^I\right)^{-\sigma}$, investors' demand functions for loans, home and foreign equities are from

$$
\begin{aligned}
\left(\lambda_t^I - \mu_t\right) q_t^b &= \beta\left(c_t^I\right) E_t \lambda_{t+1}^I \\
\left(\lambda_t^I - \mu_t \kappa_{Ht}\right) q_{Ht}^e &= \beta\left(c_t^I\right) E_t \lambda_{t+1}^I \left(q_{Ht+1}^e + d_{Ht+1}\right) \qquad \text{(A.1)} \\
\left(\lambda_t^I - \mu_t \kappa_{Ft}\right) q_{Ft}^e &= \beta\left(c_t^I\right) E_t \lambda_{t+1}^I \left(q_{Ft+1}^e + d_{Ft+1}\right) \qquad \text{(A.2)}
\end{aligned}
$$

  Equations (1)-(2) complete the set of conditions.

- **Savers**

  Given $\lambda_t^S = \left(c_t^S\right)^{-\sigma}$, savers' demand functions for bonds and capital are from

$$
\begin{aligned}
\lambda_t^S q_t^b &= \beta\left(c_t^S\right) E_t \lambda_{t+1}^S \qquad \text{(A.3)} \\
\lambda_t^S q_{Ht}^e &= \beta\left(c_t^S\right) E_t \lambda_{t+1}^S \left[q_{Ht+1}^e + \nu z \left(k_{Ht}^S\right)^{\nu-1}\right]
\end{aligned}
$$

  Equation (4) is the additional condition.

- **Firms**

  The demand of firms for capital and labour must respectively satisfy

$$
\begin{aligned}
d_{Ht} &= \alpha \frac{Y_{Ht}}{K_{Ht-1}} \\
w_t &= (1-\alpha) Y_{Ht}
\end{aligned}
$$

- **Market clearing conditions**

  As shows in the text, markets clear when equations (5)-(7) are satisfied.

A.2 Second Order Terms of Portfolio Equations

The second-order terms of the expected excess return corresponding to the equilibrium portfolios are:

- in equation (21)

$$
T(2) = -E_t \left[r_{xt+1}(2) + \frac{1}{2} r_{xt}(1)^2 + \frac{1}{2}\left(\lambda_{t+1}^I(1) + \lambda_{t+1}^{*I}(1)\right) r_{xt+1}(1) \right]
$$

- in equation (25)

$$
\begin{aligned}
T(2) &= -r_x(0)\, E_t\left[\left(\lambda^I_{t+1}(2)+\lambda^{*I}_{t+1}(2)\right)+\frac{1}{2}\left(\lambda^I_{t+1}(1)^2+\lambda^{*I}_{t+1}(1)^2\right)\right] \\
&\quad -\frac{1}{2}E_t\left(\lambda^I_{t+1}(1)+\lambda^{*I}_{t+1}(1)\right)\left(r_H(0)\, r_{Ht}(1)-r_F(0)\, r_{Ft}(1)\right) \\
&\quad -\frac{1}{2}GP(0)\left[\kappa_H(0)\left(\kappa_{Ht}(2)+\frac{1}{2}\kappa_{Ht}(1)^2\right)-\kappa_F(0)\left(\kappa_{Ft}(2)+\frac{1}{2}\kappa_{Ft}(1)^2\right)\right] \\
&\quad -\frac{1}{2}GP(0)\,\kappa_x(0)\left[M_t(2)+M^*_t(2)+\frac{1}{2}\left(M_t(1)^2+M^*_t(1)^2\right)\right] \\
&\quad -\frac{1}{2}GP(0)\left(M_t(1)+M^*_t(1)\right)\left(\kappa_H(0)\,\kappa_{Ht}(1)-\kappa_F(0)\,\kappa_{Ft}(1)\right)
\end{aligned}
$$

- in equation (28)

$$
\begin{aligned}
T(2) &= -\left[E_t\left(r_{xt+1}(2)+\frac{1}{2}r_{xt+1}(1)^2\right)+\frac{GP(0)\,\kappa(0)}{r(0)}\left(\kappa_{xt}(2)+\frac{1}{2}\kappa_{xt}(1)^2\right)\right] \\
&\quad -\frac{1}{2}\left[E_t\left(\lambda^I_{t+1}(1)+\lambda^{*I}_{t+1}(1)\right)r_{xt+1}(1)+\frac{GP(0)\,\kappa(0)}{r(0)}\left(M_t(1)+M^*_t(1)\right)\kappa_{xt}(1)\right]
\end{aligned}
$$

A.3 Zero Order Portfolios: Practical Computation

The solution for the first order component of model variables is

$$
\begin{aligned}
\mathbf{s}_{t+1}(1) &= \mathbf{\Pi}_1\mathbf{x}_t(1)+\mathbf{\Pi}_2\mathbf{s}_t(1) && \text{(A.4)}\\
\mathbf{c}_t(1) &= \mathbf{\Phi}_1\mathbf{x}_t(1)+\mathbf{\Phi}_2\mathbf{s}_t(1) && \text{(A.5)}
\end{aligned}
$$

where $\mathbf{s}_{t+1}$ are the endogenous states, $\mathbf{c}_t$ are the jump variables (controls), $\mathbf{x}_t$ are the exogenous states and, finally, $\mathbf{\Pi}_1,\mathbf{\Pi}_2,\mathbf{\Phi}_1,\mathbf{\Phi}_2$ are matrices of numbers.

In order to find the portfolio share satisfying (27), the rows to extract from the solution (A.4)-(A.5) are as follows:

Pay-off	Corresponding Multipliers
$\Upsilon_{t+1}(1)=\mathbf{U}\boldsymbol{\varepsilon}_{t+1}(1)$	$\lambda^I_{t+1}(1)-\lambda^{*I}_{t+1}(1)=\mathbf{L}_1\boldsymbol{\varepsilon}_{t+1}(1)+\mathbf{L}_2\begin{pmatrix}\mathbf{x}_t(1)\\ \mathbf{s}_{t+1}(1)\end{pmatrix}$
$\kappa_{xt+1}(1)=\mathbb{k}_1\boldsymbol{\varepsilon}_{t+1}(1)+\mathbb{k}_2\begin{pmatrix}\mathbf{x}_t(1)\\ \mathbf{s}_{t+1}(1)\end{pmatrix}$	$M_{t+1}(1)-M^*_{t+1}(1)=\mathbf{M}_1\boldsymbol{\varepsilon}_{t+1}(1)+\mathbf{M}_2\begin{pmatrix}\mathbf{x}_t(1)\\ \mathbf{s}_{t+1}(1)\end{pmatrix}$

where $\boldsymbol{\varepsilon}_{t+1}(1)=\begin{pmatrix}\varepsilon_{At+1}(1) & \varepsilon_{A^*t+1}(1) & \varepsilon^H_{\kappa t+1}(1) & \varepsilon^F_{\kappa t+1}(1)\end{pmatrix}'$ is the 4×1 vector of shocks. As a result, condition (27) is an implicit function of $\omega_{he}(0)$, the portfolio share when collateral assets are heterogeneously pledgeable:

$$g\left(\omega_{he}\left(0\right)\right) \equiv 2\tau + \mathbf{U}E_t\boldsymbol{\varepsilon}_{t+1}\left(1\right)\boldsymbol{\varepsilon}_{t+1}\left(1\right)'\mathbf{L}_1' + \frac{GP\left(0\right)\kappa\left(0\right)}{r\left(0\right)}\mathbb{k}_1 E_t\boldsymbol{\varepsilon}_{t+1}\left(1\right)\boldsymbol{\varepsilon}_{t+1}\left(1\right)'\left(\mathbf{M}_1 - \mathbf{L}_1\right)' \tag{A.6}$$

One can thus proceed in two ways. First, since $\omega_t < 0$ only if there is home bias (otherwise, it is positive), the equilibrium portfolio share is $\omega_{he}^E\left(0\right)$, which satisfies (A.4) in a neighborhood around zero:

$$\begin{aligned} S^1 &= \left\{\omega_{he}\left(0\right):\ \omega_{he}\left(0\right) \in \left[-\delta^1\ ;\ \delta^1\right]\right\} \\ \omega_{he}^E\left(0\right) &= \underset{\omega_{he}(0)\in S^1}{\arg\min}\left|\mathbf{g}\left(\omega_{he}\left(0\right)\right) - 0\right| \end{aligned} \tag{A.7}$$

Second, if the equilibrium portfolio for the homogeneously pledgeable assets, $\omega_{ho}^E\left(0\right)$, is already available, $\omega_{he}^E\left(0\right)$ can be computed satisfying (A.4) in the neighbourhood around $\omega_{ho}^E\left(0\right)$:

$$\begin{aligned} S^2 &= \left\{\omega_{he}\left(0\right):\ \omega_{he}\left(0\right) \in \left[\omega_{ho}^E\left(0\right) - \delta^2\ ;\ \omega_{ho}^E\left(0\right) + \delta^2\right]\right\} \\ \omega_{he}^E\left(0\right) &= \underset{\omega_{he}(0)\in S^2}{\arg\min}\left|\mathbf{g}\left(\omega_{he}\left(0\right)\right) - 0\right| \end{aligned} \tag{A.8}$$

In either case, the solution procedure is as follows: (a) choose a starting value for the portfolio share ($\omega_{he}\left(0\right) \in S^1$ or $\omega_{he}\left(0\right) \in S^2$); (b) solve for the first order component of model variables; (c) extract the appropriate rows to compute (A.4) and apply it; (d) iterate until either (A.7) or (A.8) is satisfied with a certain degree of approximation.

A.4 Effects of Shocks: Robustness

Figure 1-A. H Productivity Shock, Less Sensitive Debt-to-Assets ($e_Y = 0.3$)

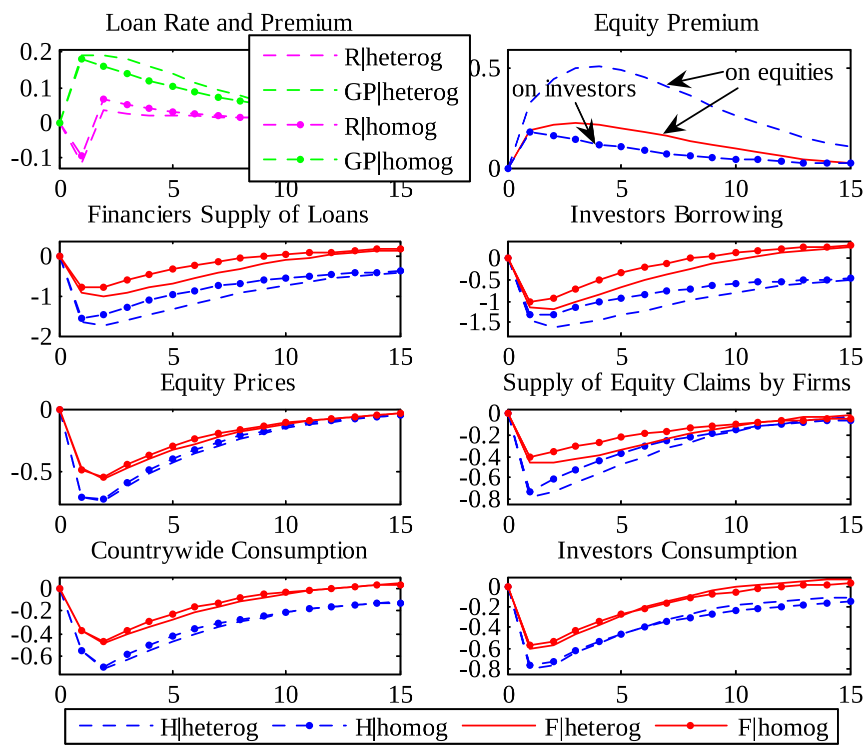


Figure 2-A. NFA: H Financial Shocks, Less Sensitive Debt-to-Assets