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State Aid and Distortion of Competition, a Benchmark Model

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Abstract

This paper analyzes how State aid affects and distorts competition and trade within and across jurisdictions. We identify the circumstances in which state aid is likely to involve the largest distortions. In the context of the paper distortion of competition" is interpreted as the effect on rivals' profits. We consider three types of state intervention, namely subsidies which affect marginal cost, entry and quality and analyse whether particular market characteristics are robust indicators of the magnitude of the distortions. We obtain the following results: (i) it appears that concentration is a fairly robust indicator; (ii) A high degree of substitution across differentiated products is not a robust indicator of the magnitude of the distortions. Its effect depends on the type of state intervention; (iii) The substitution among domestic products may have opposite effects respectively on domestic and foreign …rms. In particular, when the market is not concentrated and state aid takes the form of a production subsidy, a stronger substitution among domestic products will reduce the distortions felt by the foreign firm (but increase that felt by domestic rivals); Finally, (iv) the paper demonstrates that the impact of selective State aid on market prices and competitors can depend on the particular characteristics of the market.

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1 Introduction

In June of this year, the EU Commission launched an action plan seeking to achieve less and better targeted aid in the next five years. One of the key features of this action plan is a new emphasis on the role of economic analysis in state aid control. In particular, the action plan suggests that the evaluation of state aids project involves the evaluation of a trade-off between the correction of market failures and the distortions of competiton that they imply. The objective of this paper is to contribute to the analysis of the distortions of competition induced by state aids.

According to the EU's approach, state aids distorts competition to the extent that it favours some undertakings (Commission Report, 2003a). In the absence of a more explicit definition by the Commission, the question arises how to interpret and operationalize this approach, which seems to focuse on the extent to which state aids shifts rents in favour of some firms and away from others. Presumably this approach can be rationalised in terms of a wish to expose all firms to the same market discipline. Accordingly, it may express a concern that, to the extent that state aids allocates rents, it may induce wasteful rent seeking or reduce firms' incentive to compete and improve efficiency. In what follows, we will adopt this approach and assume that in trying to avoid distortions of "competition", the Commission is really concerned about the rents accruing to the firms which do not receive the assistance. It is worth emphasizing however that, whatever its rationale, the Commission's approach is likely to prohibit state aid which would increase welfare. State aid which shift opportunities across firms and reduce the profits of competitors can nonetheless increase welfare, in particular because they enhance consumer surplus.

Tracing out the effect of state aid on competition (and competitors) requires both a theory of the firm to analyze how state aid will affect the recipient's decisions and a theory of competitive interactions to understand how changes in the strategy of the recipient(s) affects the outcome of competition. Regarding the recipient's decision making, we adopt a neo-classical view of the the firm and focus on the effect of state aids on its cost conditions as useful starting point [1]. This approach however abstracts from important considerations like the internal governance structure and the financial structure of the firm. In particular, we consider state aids which reduce the marginal cost of the recipient and state aid which affects costs that are fixed but not sunk (like subsidies to finance generic capital equipment or subsidized leasing rates for such capital) which affect the profitability of entry and exit decisions (or more generally the profitability of capacity expansions or reductions). We also consider the effect of state aids which affects the cost of increasing product quality. Regarding the competitive interactions, we focus on a workhorse model of price competition with product differentiation (à la Bowman).

The effect that a change in the cost condition of one firm can have on the rents of others has not been subject to much specific attention so far. Besley

[1] See Harbord and Yarrow (1999) for a detailed discussion.

and Seabright (1999) offer the conjecture that aid to firms that do not have significant market power is not likely to lead to rent-shifting on an important scale[2]. Furthermore, the authors note that this market power need not be confined to output markets: "a firm with little market power in output markets but with substantial market power in markets for a specialized input (say skilled labour) could still use state aid to gain rents in this way (e.g. by poaching research scientists from other countries)". Some insight can also be gained from the literature on cost pass through. The conclusion reached by Stenneck and Verboven, (2001) who survey the literature on this issue would imply that the equilibrium is more likely to reflect the subsidy the larger is the proportion of output accounted for by the recipient but that recipients with a large market share may enjoy market power and may have an incentive to pass on their cost reduction incompletely. As a result, they predict that the pass-through of a subsidy on equilibrium prices is likely to be largest for recipients with an intermediate market share. However, these authors do not focus either on the effect of a change in cost on competitors' profit.

The effect that state aid may have on product design has not been extensively considered. One exception is Mollgaard (2004) who analyses a model with endogenous sunk cost and vertical differentiation, in which firms decide on their investment in product improvement in the first stage and compete in price at the second stage. In this framework, a reduction in the cost of product improvement (for instance through a reduction in the cost of capital) will induce recipient to invest more, so that they will subsequently compete with higher quality products (which reduces the profits of their competitors).

The first section outlines the model. Section 2 derives the effect of subsidies which affect marginal cost. Section 3 focuses on subsidies which affect fixed costs and hence the number of firms and section 4 considers state aid which affect firms' investment decisions in product quality.

1.1 A benchmark model

Given that EU policy is in principle concerned about distortions which occur across countries, we consider a model with several geographic markets. Since the objective is to trace out the effect of state aid, we also want to specify a model which is sufficiently rich in terms of parameters so that a broad range of potential effects can be analyzed. The model that we outline will, in particular allow for variations in (i) the magnitude of the fixed cost of entry (which will determine the number of firms in equilibrium), (ii) the degree of horizontal product differentiation between domestic products, (iii) vertical differentiation among domestic products, (iv) the degree of substitution between domestic and foreign products and (v) the magnitude of trade costs.

Assume that there are two countries, a home country labelled as country one and a foreign country labelled as country two. Let us also consider an oligopolistic industry in country 1, with n firms, each of which produces a symmetrically

[2] Sleuwagen and Pennings, (2001a) and Besley and Seabright (1999) discuss the use of market share as criteria for identifying important distortions of competition.

differentiated product. Country 2 has a single imperfectly competitive firm producing also a differentiated good. For completeness, there is also a perfectly competitive industry in both countries producing a homogeneous good using constant returns to scale technology. This good is traded freely between the two countries and acts as the numeraire good.

Under multilateral free trade, the $n+1$ firms compete in a Bertrand oligopoly in both home and the foreign markets. We assume that markets are internationally segmented, so firms may choose prices in each national market separately. For simplicity, we assume that the home firms have constant marginal cost c whereas the foreign firm has constant marginal cost $\tilde{c}$. In addition, home firm i sets price p_{i1} in the home market and p_{i2} in the foreign market. The home firm i sells output q_{i1} in the home market and q_{i2} in the foreign market, where $i = 1, ..., n$. Furthermore, the foreign firm sets price $\tilde{p}_2$ in the foreign country and price $\tilde{p}_1$ in country 1. The foreign firm sells output $\tilde{q}_2$ in its home market and sells output $\tilde{q}_1$ in the country 1. Consumption of the home (foreign) firm's differentiated product in the home (foreign) market is equal to the sales of the home (foreign) firm in the home (foreign) market, q_{i1} ($\tilde{q}_2$); consumption of the foreign (home) firm's differentiated product in the home (foreign) market is equal to the sales of the foreign (home) firm in the home (foreign) market, $\tilde{q}_1$ (q_{i2}); and consumption of the numeraire good in each country is z. It is assumed that there is a representative consumer in each country k with quasi-linear preferences that can be represented by a quadratic utility function, as in Vives (1985):

$$U_k(y,z) = z + \left[\sum_{i=1}^{n} \alpha_{ik} q_{ik} + \tilde{\alpha}_k \tilde{q}_k\right] - \frac{1}{2}\left[\sum_{i=1}^{n} \beta_{ik} q_{ik}^2 + \tilde{\beta}_k \tilde{q}_k^2\right] - \left[\theta_k \sum_{i=1}^{n} \sum_{i \neq l}^{n} q_{ik} q_{lk} + \theta_k^0 \sum_{i=1}^{n} q_{ik} \tilde{q}_k\right], \ k = 1, 2, \tag{1}$$

where $\alpha_{ik}, \beta_{ik}, \tilde{\alpha}_k, \tilde{\beta}_k$ are positive; $0 < \frac{\theta_k^2}{\beta_{ik}\beta_{jk}} < 1$, $for\ i \neq j,\ i = 1, ..., n, k = 1, 2$; and $0 < \frac{\theta_k \theta_k^0}{\beta_{ik}\tilde{\beta}_k} < 1$, for $i = 1, ..., n,\ k = 1, 2$. It is assumed that $\alpha_{ik} > c$ and $\tilde{\alpha}_k > c_2$ otherwise the ith firm will not produce any output even if it has a monopoly.

Note that α_{ik} and $\tilde{\alpha}_k$ are the demand intercepts and if different from α_{jk}, is a measure of vertical product differentiation. If $\alpha_{ik} > \alpha_{jk}$, then firm i is perceived as providing a better quality than firm j. On the other hand, the relation between parameters θ_k and β_{ik}, measures the degree of horizontal product differentiation, i.e. customer preferences for a given brand independently of quality levels. When $\alpha_{ik} = \alpha_{lk}$, $\frac{\theta_k^2}{\beta_{ik}\beta_{lk}}, i \neq l,\ i = 1, ..., n,\ k = 1, 2$, is a measure of the degree of product substitutability among products i and l supplied by home firms in the country k ranging from zero when products are independent to one when the products are perfect substitutes. In the same manner, when $\alpha_{ik} = \tilde{\alpha}_k$,

$\frac{\theta_k \theta_k'}{\beta_{ik}\tilde{\beta}_k}$, $i = 1,...,n,\ k = 1,2$, is a measure of the degree of product substitutability among products j supplied by a local firm and the product supplied by the foreign firm in the country k ranging from zero when products are independent to one when the products are perfect substitutes. For simplicity we will also assume that $1 = \beta_{ik} = \beta_{lk} = \tilde{\beta}_k,\ i \neq l,\ i = 1,...,n,\ k = 1,2$.

The goods are substitutes, independent of complements according to whether $\theta_i \gtreqless 0$, and $\theta_i' \gtreqless 0$. Demand for good ij is always downward sloping in its own price and increases (decreases) with increases in the price of the competitor if the goods are substitutes (complements). It is straightforward to show that the utility function (1) yields the following inverse demand functions for the home firms in country k

$$p_{ik} = \alpha_{ik} - q_{ik} - \theta_k \sum_{l \neq i}^{n} q_{lk} - \theta_k' \tilde{q}_k, \quad i = 1,...,n,\ k = 1,2, \tag{2}$$

and the foreign firms in the country k

$$\tilde{p}_k = \tilde{\alpha}_k - \tilde{q}_k - \theta_k' \sum_{i=1}^{n} q_{ik}, \quad k = 1,2. \tag{3}$$

The inverse demand functions can be more conveniently written as:

$$p_{ik} = \alpha_{ik} - (1 - \theta_k)\, q_{ik} - \theta_k Q^k - \theta_k' \tilde{q}_k, \quad i = 1,...,n,\ k = 1,2, \text{ and} \tag{4}$$

$$\tilde{p}_k = \tilde{\alpha}_k - \left(1 - \tilde{\theta}_k\right) \tilde{q}_k - \theta_k' Q^k, \quad k = 1,2. \tag{5}$$

where $Q^k = \sum_{i=1}^{n} q_{lk}$ is the total output of the home firms in the country k.

1.2 Subsidies which affect marginal cost

This section assumes that the state can award a subsidy which reduces the marginal cost of some firm(s). We consider the effect of this subsidy by assuming that there is no vertical differentiation across products, and analyze how the degree of substitution across products (within and across countries) and the number of firms affect the magnitude of the distortions that such state intervention induces[3]. In a model with no differences in vertical differentiation equation (2) and equation (3) can then be rewritten as:

$$p_{ik} = \alpha_k - q_{ik} - \theta_k \sum_{l \neq i}^{n} q_{lk} - \theta_k' \tilde{q}_k, \quad i = 1,...,n,\ k = 1,2, \text{ and} \tag{6}$$

$$\tilde{p}_k = \tilde{\alpha}_k - \tilde{q}_k - \theta_k' \sum_{i=1}^{n} q_{ik}, \quad k = 1,2. \tag{7}$$

[3] In appendix C, we show that assuming different degrees of vertical differentiation across products does not affect the results.

Let us define the following parameters $R_k = (1-\theta_k)\left[1+(n-1)\theta_k - n\left(\theta'_k\right)^2\right]$, $a_k = \frac{(1-\theta_k)(1-\theta'_k)\alpha_k}{R_k}$, $a'_k = \frac{(1-\theta_k)(1+(n-1)\theta_k - n\theta'_k)\alpha_k}{R_k}$, $b_k = \frac{1+(n-2)\theta_k-(n-1)(\theta'_k)^2}{R_k}$, $b'_k = \frac{(1-\theta_k)(1+(n-1)\theta_k)}{R_k}$ $x_k = \frac{\theta_k-(\theta'_k)^2}{R_k}$, and $d_k = \frac{(1-\theta_k)\theta'_k}{R_k}$. We can now derive the direct demand functions in country k as follows:

$$q_{ik} = a_k - b_k p_{ik} + x_k \sum_{l \neq i}^{n} p_{lk} + d_k \tilde{p}_k, \quad i = 1, ..., n, \; k = 1, 2, \tag{8}$$

and direct demand function for the foreign firm in country k as

$$\tilde{q}_k = a'_k + d_k \sum_{i=1}^{n} p_{ik} - b'_k \tilde{p}_k, \quad k = 1, 2. \tag{9}$$

With segmented markets and constant marginal costs, the Bertrand oligopoly in the home market can be analyzed independently of the foreign market so the analysis will focus on the home market. Hereafter and for simplicity in the presentation we avoid the use of the subindex $k = 1$. Since the utility function is quadratic, these functions are linear in prices.

1.2.1 Base Case Scenario with no subsidies

We first present the equilibrium outputs of the market in a benchmark scenario in which no firm receives any kind of subsidies (state aid) from their national government. Following subsections will introduce alternative scenarios in which state aid are introduced by local authorities in order to trace the anticompetitive effect of this differential treatment.

The profit functions of the home firms and the foreign firm, respectively from sales in the home country market are written:

$$\pi_i = (p_i - c)q_i - F, \; i = 1, ..., n, \tag{10}$$

and

$$\tilde{\pi} = (\tilde{p} - \tilde{c} - t)\tilde{q} - \tilde{F}, \tag{11}$$

where t represents the transportation costs per production unit and F and $\tilde{F}$ denotes a measure of fixed costs of production in the home and foreign country respectively.

We will then solve for the Bertrand equilibrium and for the number of firms at the free entry equilibrium as function of the fixed cost of entry (in the domestic market). All domestic firms are symmetric and we focus on the symmetric equilibrium where all domestic firms charge the same price. The system of best response curves then reduces to a couple of equations, which jointly determine the price of domestic firms, p_i^* and foreign firm, $\tilde{p}^*$. For general values of the product differentiation parameters, the equation of the home firms and foreign firm's best response curve are, respectively:

$$p_i^* = \frac{1}{2b}\left(a + x\sum_{l\neq i}^{n} p_l^* + d\tilde{p} + bc\right), \; i = 1, ..., n, \tag{12}$$

and

$$\tilde{p}^* = \frac{1}{2b'}\left(a' + d\sum_{i=1}^{n} p_i^* + b'\tilde{c} + b't\right). \tag{13}$$

Solving this system for the equilibrium prices p_i^* and $\tilde{p}^*$ one obtains:

$$p_i^* = \frac{1}{W}\left\{2b'a + a'd + 2b'bc + b'd(\tilde{c} + t)\right\}, \text{ and} \tag{14}$$

$$\tilde{p}^* = \frac{1}{W}\left\{2a'b - (n-1)a'x + nda + ndbc + [b'(2b - (n-1)x)](\tilde{c} + t)\right\}, \tag{15}$$

where $W = 2b'[2b - (n-1)x] - nd^2$.

A number of interesting comparative statics results can then be derived :

$$\frac{\partial p_i^*}{\partial c} > 0, \frac{\partial p_i^*}{\partial \tilde{c}} > 0, \frac{\partial \tilde{p}^*}{\partial c} > 0, \frac{\partial \tilde{p}^*}{\partial \tilde{c}} > 0, \frac{\partial \pi_1^{**}}{\partial \tilde{c}} > 0, \frac{\partial \pi_i^{**}}{\partial \tilde{c}} > 0, \frac{\partial \tilde{\pi}^{**}}{\partial \tilde{c}} < 0. \tag{16}$$

Hence, it appears that the equilibrium price for the home producers, p_i^*, is an increasing function of its own marginal cost, c. The price of foreign firms also increases with the cost of the domestic firm but by less. As one would expect, an increase in the marginal cost of domestic firms reduces their profits but increases the profit of the foreign firm. The intuition is as follows, an increase in the marginal cost, c, forces equilibrium prices up. Since the increase in the equilibrium price for the domestic firm falls short of the increase in cost (that is, $\frac{\partial p_i^*}{\partial c} = \frac{1}{W}[b'2b] < 1$), the margin of domestic firms falls. Since the price of the foreign firm increases by less, the sale of domestic firms must also fall, and hence their profit falls. At the opposite, the margin of the foreign firm increases and their sales fall less than proportionally so that their profit increases.

In a similar way, the price set by the foreign producer in equilibrium is an increasing function of its own marginal cost, $\tilde{c}$. The increase in home producers' price as a consequence of an increase in the marginal cost of the foreign firms lower than the increase in the foreign price. The profit of domestic firms increases and that of the foreign firm falls. Of course, the effect of an increase in transportation costs is the same at the effect of a change in the cost of the foreign firms, namely :

$$\frac{\partial p_1^{**}}{\partial t} > 0, \frac{\partial p_i^{**}}{\partial t} > 0, \frac{\partial \tilde{p}^{**}}{\partial t} > 0, \text{ and } \frac{\partial \pi_1^{**}}{\partial t} > 0, \frac{\partial \pi_i^{**}}{\partial t} > 0, \frac{\partial \tilde{\pi}^{**}}{\partial t} < 0. \tag{17}$$

With respect to the degree of product differentiation and the number of firms, one can check that :

$$\frac{\partial p_i^*}{\partial \mu} < 0,\ \frac{\partial \tilde{p}^*}{\partial \mu} > 0,\ \frac{\partial p_i^*}{\partial \mu^0} < 0 \text{ and } \frac{\partial \tilde{p}^*}{\partial \mu^0} < 0. \tag{18}$$

$$\frac{\partial p_i^*}{\partial n} < 0\,, \frac{\partial \tilde{p}^*}{\partial n} < 0. \tag{19}$$

Hence, as intuition would suggest, the greater the degree of product differentiation among local producers (the lower is μ), the greater the equilibrium price-cost margin of the local producers. What may more surprising however is that the price charged by the foreign firm falls as the degree of product differentiation among domestic firms (a lower μ) increases. This arises because the foreign product becomes, in relative term, a closer substitute to local products. Furthermore, the greater the degree of product differentiation between local and foreign producer (the lower is μ^0), the larger the equilibrium price-cost margin for all the players in the market. Finally, the equilibrium price-cost margins fall as the number of home firms rises.

To illustrate these effects, Table 1 and Table 2 below present some simulations for a scenario with three home firms and the following parameter values $\alpha = 100$, $n = 3$, $c = \tilde{c} = 5$, $F = \tilde{F} = 25$ and $t = 2$. One observes in particular that the price and profit of domestic and foreign firms change in opposite directions as the degree of local product differentiation changes.

Table 1: Alternative scenarios in terms of local product differentiation, μ ($\mu^0 = 0.7$)

	$\mu = 0.8$	$\mu = 0.7$	$\mu = 0.65$	$\mu = 0.6$
p_i^*	12.39	15.78	17.37	18.86
$\tilde{p}^*$	18.12	16.65	15.78	14.77
π_i^*	173.09	275.00	327.80	382.85
$\tilde{\pi}^*$	259.44	215.55	188.46	157.12

Table 2: Alternative scenarios in terms of foreign product differentiation, μ^0 ($\mu = 0.7$)

	$\mu^0 = 0.8$	$\mu^0 = 0.7$	$\mu^0 = 0.65$	$\mu^0 = 0.6$
p_i^*	14.90	15.78	16.07	16.31
$\tilde{p}^*$	10.95	16.65	19.40	22.12
π_i^*	260.86	275.00	283.28	291.75
$\tilde{\pi}^*$	53.01	215.55	300.99	390.52

Before analyzing the effect of state aid in this context, it is important to note that the change in prices induced by a change in cost or a change in the degree of product differentiation (μ^0 and μ) provide a reliable guide to the effect of profits. This was shown earlier for a change in cost and is confirmed in the tables above for a change in product differentiation. This arises because, in the context of this model, products are strategic complements. As a consequence, the distortion of competition which is induced by a change in marginal cost (in terms of profit) can also be proxied by the change in price that it induces. In what follows, we analyze how the magnitude of the distortion is affected by particular parameters. These effects involve second derivatives of equilibrium prices and profits, and it is not clear that these second order effects will be necessarily the same for price and profits. It is very difficult to obtain analytical results for profits and in what follows, we will thus focus on price distortions. We develop some simulations in appendix A which confirm that important second order effects (in particular those involving concentration and the degree of substitution between firms) go in the same direction for prices and profits. These simulation provide some confidence that price distortions are a good proxy for profit distortions, even with respect to second order effects. This limitation should however be kept in mind.

Let us now turn to the effect of state aid which reduces the marginal cost of one domestic firm.

1.2.2 The effect of subsidies

State aid is modelled as a production subsidy that has the effect of reducing the recipient firm's marginal cost of production. Specifically, let us assume that the home state decides to grant a production subsidy, s_1, to firm 1 in the home country. The profit functions of the home firms and the foreign firm, respectively from sales in the home country market are the written:

$$\Pi_1 = (p_1 - c + s_1)q_1 - F, \tag{20}$$

$$\Pi_i = (p_i - c)q_i - F, \; i = 2, ..., n \tag{21}$$

and

$$\tilde{\Pi} = (\tilde{p} - \tilde{c} - t)\tilde{q} - \tilde{F}. \tag{22}$$

All domestic firms but firm 1 are now symmetric. The system of best response curves then involves three equations, which jointly determine the price of domestic firms, p_1^{**} and p_i^{**} and the price of the foreign firm, $\tilde{p}^{**}$. For general values of the product differentiation parameters, the equation of the home firms and foreign firm's best response curve are, respectively:

$$p_1^{**} = \frac{1}{2b}\left(a + x\sum_{i=2}^{n} p_l^{*} + d\tilde{p} + b(c - s_1)\right), \tag{23}$$

$$p_i^{**} = \frac{1}{2b}\left(a + x\sum_{l \neq i}^{n} p_l^{*} + d\tilde{p} + bc\right), \; i = 2, ..., n, \tag{24}$$

and

$$\tilde{p}^{**} = \frac{1}{2b'}\left(a' + d\sum_{i=1}^{n} p_i^{*} + b'(\tilde{c} + t)\right). \tag{25}$$

Solving this system for the equilibrium prices p_1^{**}, p_i^{**} and $\tilde{p}^{**}$ one obtains:

$$p_1^{**} = \frac{1}{W}\left\{2b'a + a'd + 2b'bc - \gamma s_1 + b'd(\tilde{c}+t)\right\}, \tag{26}$$

$$p_i^{**} = \frac{1}{W}\left\{2b'a + a'd + 2b'bc - \gamma' s_1 + b'd(\tilde{c}+t)\right\}, \text{ and} \tag{27}$$

$$\tilde{p}^{**} = \frac{1}{W}\left\{2a'b - (n-1)a'x + nda + ndbc - \gamma'' s_1 + [b'(2b-(n-1)x)](\tilde{c}+t)\right\}, \tag{28}$$

where $\gamma = \frac{1}{(2b+x)}\left(4b'b^2 - (n-1)bd^2 - 2(n-2)bb'x\right)$, $\gamma' = \frac{b(2b'x+d^2)}{(2b+x)}$ and $\gamma'' = db.$

The effect of the subsidy on equilibrium prices are then given by :

$$p_1^{**} - p_1^{*} = -\frac{1}{W}\gamma s_1 < 0, \tag{29}$$

$$p_i^{**} - p_i^{*} = -\frac{1}{W}\gamma' s_1 < 0,\ i = 2, ..., n, \text{ and} \tag{30}$$

$$\tilde{p}^{**} - \tilde{p}^{*} = -\frac{1}{W}\gamma'' s_1 < 0,\ i = 2, ..., n. \tag{31}$$

So that $\frac{\partial p_1^{**}}{\partial s_1} < 0$, $\frac{\partial p_i^{**}}{\partial s_1} < 0$, and $\frac{\partial \tilde{p}^{**}}{\partial s_1} < 0$.

Hence, when a subsidy is granted to a home firm, so that its marginal cost decreases, its price decreases. The price of domestic rivals and that of the foreign firm also decrease but by less. It is easy to check that the derivative of the equilibrium price for firm 1, p_1^{**}, with respect to the subsidy, s_1, is, in absolute value, lower than one (that is, $\left|\frac{\partial p_1^{**}}{\partial s_1}\right| = \left|-\frac{1}{W}\gamma\right| < 1$). It means that the decrease in the equilibrium price induced by the subsidy is always lower than magnitude of the subsidy itself (less than complete pass-through). Domestic as well as foreign competitors increase their own price, but by less. As noted above, profits are affected in the same way as prices, so that the profit of the domestic firm increases and that of the other firms decreases, i.e.:

$$\frac{\partial \pi_1^{**}}{\partial s_1} > 0,\ \frac{\partial \pi_i^{**}}{\partial s_1} < 0, \text{ and } \frac{\partial \tilde{\pi}^{**}}{\partial s_1} < 0. \tag{32}$$

Let us now evaluate the magnitude of the distortion induced by the subsidy, in terms of the degree of product differentiation and the number of firms. In general, the effect of the subsidy is not a monotonic function of the relevant parameters, so that :

$$\frac{\partial^2 p_1^{**}}{\partial s_1 \partial n} \gtrless 0,\ \frac{\partial^2 p_1^{**}}{\partial s_1 \partial \theta} \gtrless 0 \text{ and } \frac{\partial^2 p_1^{**}}{\partial s_1 \partial \theta'} \gtrless 0. \tag{33}$$

The decrease in the price of the firm enjoying the subsidy is not a monotonic function neither of the number of firms in the domestic products, n, nor of the degree of substitution between home products, θ, and home and foreign products, μ'. Some natural restriction on parameter values can however be considered. In particular, it is natural to assume, in the context of this model, that the degree of product differentiation between local and foreign products exceeds the degree of product differentiation among local products. We first focus on the recipient.

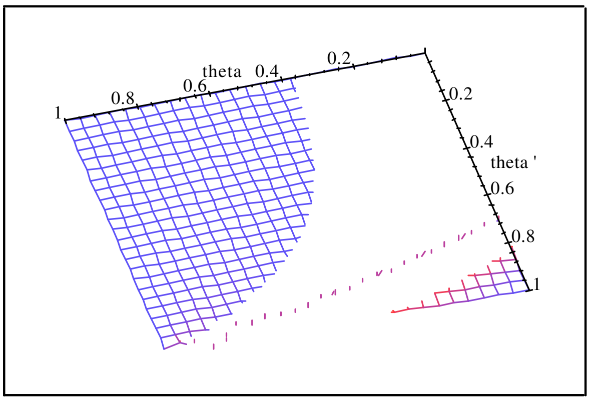
Effect on the recipient With the restriction that degree of product differentiation between local and foreign firms exceeds that among local firms, we obtain that:

$$If\ \mu > \mu' \Rightarrow \frac{\partial^2 p_1^{**}}{\partial s_1 \partial \mu} < 0.$$

Hence, the pass-through by the recipient firm is greater, the larger is the substitution among domestic products. In addition, the larger is the number of firms, the broader is the range of values of μ and μ' for which this effect obtains ($\frac{\partial^2 p_1^{**}}{\partial s_1 \partial \mu} < 0$).

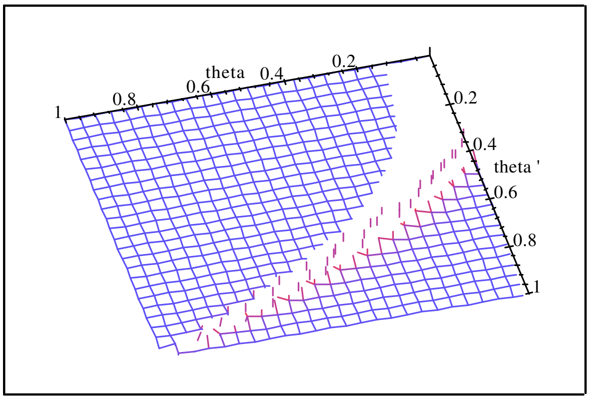
The effect of the substitution between domestic and foreign product is more intricate. When the number of firms is large, the pass-through of the recipient firms is greater, the lower is the substitution between domestic and foreign product (i.e., $\frac{\partial^2 p_1^{**}}{\partial s_1 \partial \mu'} > 0$). When there are only few domestic competitors, an additional condition needs to be satisfied, such that the substitution between domestic products is large enough (μ is large enough, with μ larger than μ')[4]. Figure 1 and 2 illustrate, showing the range of parameters (the shaded area) for which the pass-through falls with an increase in the substitution between domestic and foreign products.

Figure 1: Range of μ and μ' values for which $\frac{\partial^2 p_1^{**}}{\partial s_1 \partial \mu'} > 0$ ($n = 3$)



[4] Alternatively, if μ' is larger than μ, then $(\mu' - \mu)$ needs to be large enough.

Figure 2: Range of θ and θ' values for which $\frac{\partial^2 p_1^{**}}{\partial s_1 \partial \theta'} > 0$ ($n = 10$)



Considering the effect of an increase in the number of firms, we observe that the pass-through falls with an increase in the number of firms ($\frac{\partial^2 p_1^{**}}{\partial s_1 \partial n} > 0$), at least if the degree of substitution between domestic and foreign products and across domestic products is high enough. When substitution is low, there is a threshold number of firms such that for highly concentrated market, the pass-through initially increases with the number of firms (that is, $\frac{\partial^2 p_1^{**}}{\partial s_1 \partial n} < 0$) up to the threshold and subsequently falls. In other words, the pass-through will then to low when rivalry[5] is limited and the market is concentrated and when the market is atomistic. If rivalry is low, a fall in concentration will initially increase the pass-through up to a point and subsequently fall. This arises because in highly concentrated market, the recipient will not pass on the benefit of the subsidy to consumers, at least if rivalry does not induce them to do so. As concentration increases, competition is enhanced and the recipient will pass on a greater proportion of the subsidy. However, as concentration falls further, the recipient firms becomes small relative to the markets and pass-through is less. By contrast if rivalry is intense enough, the recipient will pass on large proportion of the subsidy even if concentration is high. And the pass-through will decrease monotonically as the number of firms increases.

Let's now analyze the effect of the subsidy received by firm 1 on the equilibrium prices of rival firms in the market.

Effect on rivals We first consider domestic rivals. The extent of the distortion induced by the subsidy on their equilibrium prices can be described as follows:

$$\frac{\partial^2 p_i^{**}}{\partial s_1 \partial n} > 0, \ \frac{\partial^2 p_i^{**}}{\partial s_1 \partial \theta} < 0, \ \frac{\partial^2 p_i^{**}}{\partial s_1 \partial \theta'} \gtrless 0, \ i = 2, ..., n,$$

[5] In what follows, a high value of θ is interpreted as involving either a low degree of product differentiation, a high degree of substitution across products or a high degree of rivalry.

Hence, the distortion (proxied by the decrease in the price for a local firm) induced by the subsidy received by the rival firm 1 is larger, when the number of domestic firms is smaller and the degree of substitution across local products is larger (product differentiation among home products is smaller). In other words, domestic competitors are more affected in concentrated markets and in markets where rivalry is greater. Intuitively, concentration matters because with a small number of firms, the recipients accounts for a relatively large share of output and hence has more of an effect on competitors. Rivalry matters because competitors are induced to respond more sharply to the price reduction of the recipient.

The effect of the substitution between domestic and foreign products is however often the opposite of the substitution across domestic products. Assuming as before that substitution across domestic products is greater than substitution between domestic and foreign products ($\mu > \mu'$) one observes that the distortion in the price for the local firms as a consequence of the subsidy received by the rival firm 1 is always larger, when the degree of substitution between home and foreign products is smaller, (i.e., $\frac{\partial^2 p_i^{**}}{\partial s_1 \partial \mu'} > 0$). That is,

$$If\ \mu > \mu' \Rightarrow \frac{\partial^2 p_i^{**}}{\partial s_1 \partial \mu'} > 0.$$

Hence, a lower rivalry with the foreign product will actually increase the distortion. In other words, more segmented markets will lead a greater domestic distortion. The intuition behind this result can be described as follows; when the foreign product operates in a niche, much of the burden of the adjustment to the more aggressive pricing of the recipient is supported by domestic firms. When it becomes more like domestic products, the burden of the adjustment will be shared more evenly.

This result can also be reinterpreted in terms of the effect that asymmetry has on the distortion of competition. An increase in the degree of substitution between domestic and foreign will tend to induce a more symmetric pattern of prices and market shares. It would appear that, for a given number of firms, symmetry will reduce the distortions imposed on firms that are alike.

The effect on the foreign rival can be described as follows :

$$\frac{\partial^2 \mathbf{p}^{**}}{\partial s_1 \partial n} > 0,\ \frac{\partial^2 \mathbf{p}^{**}}{\partial s_1 \partial \mu} \gtrless 0,\ \frac{\partial^2 \mathbf{p}^{**}}{\partial s_1 \partial \mu'} \gtrless 0. \tag{34}$$

Hence, the distortion imposed on the foreign firm as a consequence of the subsidy received by the rival firm 1 is larger, when the number of domestic firms is smaller. This effect is the same as that found for domestic rivalry. Furthermore, if $\mu > \mu'$ the distortion imposed on the foreign firm as a consequence of the subsidy received by the rival firm 1 is always larger, when the degree of substitution across foreign and domestic products is larger (μ' is larger). That is

$$If\ \mu > \mu' \Rightarrow \frac{\partial^2 \mathbf{p}^{**}}{\partial s_1 \partial \mu'} < 0.$$

In other words, foreign rivals will be less affected in segmented markets. As one would expect, as the foreign product becomes a closer substitute to domestic alternatives, it will more affected by a subsidy granted to a domestic rival. That is also to say that an increase in symmetry across firms will also tend to increase the distortion imposed on the firm that is unlike its competitors.

The effect of the substitution across domestic products is more intricate. The distortion is not a monotonic function of the degree of substitution across local products. When the number of firms in the market is high enough and $\mu > \mu^0$, $\frac{\partial^2 \widetilde{p}^{**}}{\partial s_1 \partial \mu}$ remains always positive, that is, the distortion in the price of the foreign firm is larger, when the degree of product differentiation across local products increases (μ is lower).

$$If\ \mu > \mu^0 \text{ and } n \text{ is large enough } \Rightarrow \frac{\partial^2 \widetilde{p}^{**}}{\partial s_1 \partial \mu} > 0.$$

This effect is the opposite of that found for domestic rivals. It can be explained as follows: an increase in the degree of product differentiation among domestic firms will actually make foreign and domestic firms more alike (as μ falls, it becomes closer to μ^0) . Hence, the foreign firm will be more affected by the subsidy granted to a domestic rival. However, there is a second effect at work. An increase in the degree of product differentiation will reduce the rivalry among domestic firms. Their prices will fall by less in response to a subsidy and as a result, the foreign firm will also be less affected. This second effect will be particularly strong when the number of domestic firms is small and it may actually dominate the first effect so that the distortion imposed on the foreign firm may actually fall when the degree of product differentiation increases (i.e., $\frac{\partial^2 \widetilde{p}^{**}}{\partial s_1 \partial \mu} < 0$) if concentration is high enough[6].

State aid affecting marginal cost : summary A number of conclusions emerge. First, an increase in concentration (associated with a reduced number of firms) will always increase the distortion incurred by all competitors, both domestic and foreign. This arises mostly because a reduction in the number of firms increases the proportion of output which is subsidized.

Second, a more intense rivalry among domestic firms (associated with less product differentiation), will increase the distortion on both domestic and foreign firms when concentration is high enough. When concentration is low, a more intense rivalry will still increase the distortion on domestic firms but will reduce the distortion on foreign firms.

[6] For instance, it can be shown that:

$$\begin{aligned} \text{If } n &= 2, \text{ then } \frac{\partial^2 \widetilde{p}^{**}}{\partial s_1 \partial \mu} < 0 \Leftrightarrow \mu > \frac{1}{2}, \text{ and} \\ \text{If } n &= 3, \text{ then } \frac{\partial^2 \widetilde{p}^{**}}{\partial s_1 \partial \mu} < 0 \Leftrightarrow \mu^0 > \frac{1}{5}\sqrt{10}. \end{aligned}$$

Third, more segmented markets will have opposite e¤ects on domestic and foreign ..rms. The distortion on domestic ..rms will increase and that imposed on foreign ..rms will decrease.

Fourth, a reduction in the asymmetry across ..rms (for a given number) will reallocate the distortion more evenly. It is not clear whether a more symmetric pattern (for instance, a lower Her..ndahl index) a¤ects the overall magnitude of the distortion.

Hence, it would appear that concentration and the degree of segmentation across markets are unambiguously related to the magnitude of the distortions induced by state aid. Domestic rivalry is also reliable indicator of the distortion imposed on domestic ..rms but, interestingly, the importance of the spillover across countries is a¤ected by concentration in the domestic industry. Industries which feature both a high degree of rivalry and high concentration will involve higher spillovers to foreign ..rms.

1.3 Subsidies which a¤ect entry

In this section, we analyze the e¤ect of a subsidy which prevents exits or induces entry. Hence, we seek to identify the circumstances in which the prices and pro..ts of existing competitors are particularly a¤ected by the presence of a subsidized ..rm (which would otherwise be absent from the industry). We use the same underlying model as that presented in the previous section (in which there is no vertical di¤erentiation across products). As one would expect, the extent to which competitors' prices are a¤ected depends on the degree of product di¤erentiation among local ..rms (μ) and among local and foreign ..rms (μ^0). However, the e¤ect on prices of a change in the number of ..rms is not a monotonic function of the degree of product di¤erentiation among the products in the market. Analytical results are also di¢cult to derive. Some conclusions can be still be obtained from simulations. The base parameter used for the simulations are c = ¢ = 5, t = 2, ® = 100 and s_1 = 1. We report results for the entire range of admissible values for μ and μ^0.

1.3.1 E¤ects on domestic rivals

The distortion on local rivals which is induced by an additional competitor will decrease as the degree of substitution among local ..rms increases (i.e., $\frac{\partial^2 p_i^{¤¤}}{\partial n \partial \mu} > 0$), at least if the degree of substitution among local ..rms is large enough (and $\mu > \mu^0$). In addition, the larger is the initial number of ..rms in the market the lower is the minimum level of μ necessary to ensure the direction of this e¤ect. This can be interpreted as follows : in principle. one would expect that when product di¤erentiation is strong (so that products operate in niches), the e¤ect of an additional competitors will be felt more strongly as the degree of product di¤erentiation increases. At the same time, when product di¤erentiation is already very high, margins will be low and the reduction in price induced by an additional ..rm will hardly be a¤ected any longer by a reduction in product di¤erentiation (for instance, when products are almost perfect substitutes). Hence, it is natural to expect that the effect of entry will first increase and then fall with the degree of substitution. This is what is found here. In addition, we observe that the range of substitution parameter for which the effect of entry increase with substitution is smaller when there is a large number of incumbent firms. This arises because the rivalry has less of an impact on margins when the number of firms is large (in those circumstances, margins are largely determined by the number of competitors). These effects are illustrated in Table 3 and figures 3.1. to 3.3. below:

Table 3: Values of $\frac{\partial^2 p_i^{**}}{\partial n \partial \mu}$ as a function of μ, μ' and n.

	$n = 2$	$n = 3$	$n = 10$
$\mu = \mu' = .8$	-7.57	1.29	1.30
$\mu = .8,\ \mu' = .5$	10.98	10.15	1.27
$\mu = .8,\ \mu' = .3$	17.26	12.18	1.32
$\mu = \mu' = .5$	-14.35	4.09	1.56
$\mu = .5,\ \mu' = .3$	-13.63	-1.25	1.78
$\mu = \mu' = .2$	-20.56	-13.60	-0.04

Figure 3.1.: Values of μ and μ' such that $\frac{\partial^2 p_i^{**}}{\partial n \partial \mu} > 0$, $(n = 2)$.

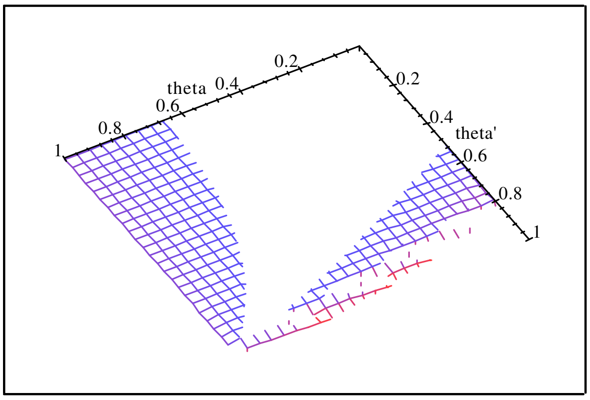


Figure 3.2.: Values of μ and μ' such that $\frac{\partial^2 p_i^{**}}{\partial n \partial \mu} > 0$, $(n = 3)$.

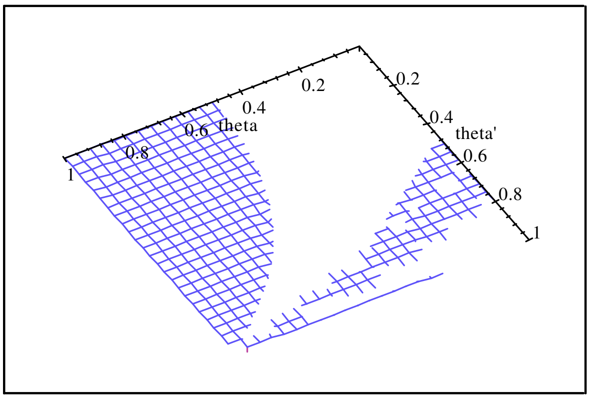
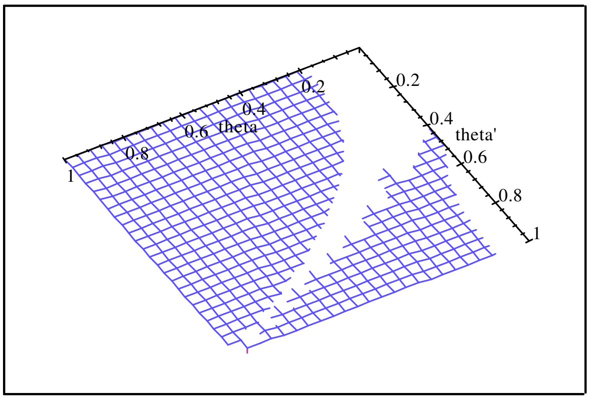


Figure 3.3.: Values of µ and µ' such that $\frac{\partial^2 p_i^{**}}{\partial n \partial \mu} > 0$, $(n = 10)$.



If $\mu > \mu^0$, the decrease on local rivals' price as a consequence of an increase in the number of ..rms is larger when the degree of substitution among local and foreign ..rms decreases (i.e., $\frac{\partial^2 p_i^{**}}{\partial n \partial \mu^0} > 0$). See Table 4 and ..gures 4.1. to 4.3. below. This result can be interpreted as follows; as the substitution between domestic and foreign products become closer to the substitution across domestic products, a greater share of the adjustment associated with entry will be felt by the foreign ..rm. The distortion imposed on local ..rms will then to be less. Note however (see table 4 and ..gures 4.1 to 4.3) that when the number of domestic ..rms is large, this e¤ect hardly matters. This arises simply because the (sole) foreign ..rm becomes relatively unimportant.

Table 4: Values of $\frac{\partial^2 p_i^{**}}{\partial n \partial \mu^0}$ as a function of μ, μ^0 and n.

	$\mu = .8, \mu^0 = .8$	$\mu = .8, \mu^0 = .5$	$\mu = .8, \mu^0 = .3$
$n = 2$	21.71	7.57	5.56
$n = 3$	7.31	1.76	1.50
$n = 10$	¡ 0.01	0.04	.068

	$\mu = .5$, $\mu^0 = .5$	$\mu = .5$, $\mu^0 = .3$	$\mu = .2$, $\mu^0 = .2$
$n = 2$	14.27	9.71	6.27
$n = 3$	7.08	4.20	4.65
$n = 10$	.19	.20	.85

Figure 4.1.: Values of µ and µ' such that $\frac{\partial^2 p_i^{**}}{\partial n \partial \mu^0} > 0$, $(n = 2)$.

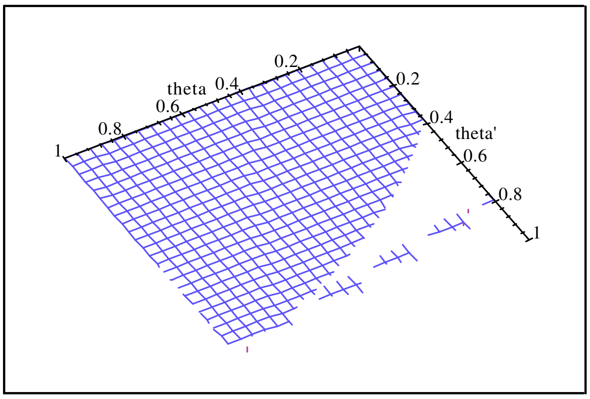


Figure 4.2.: Values of µ and µ' such that $\frac{\partial^2 p_i^{**}}{\partial n \partial \mu^0} > 0$, $(n = 3)$.

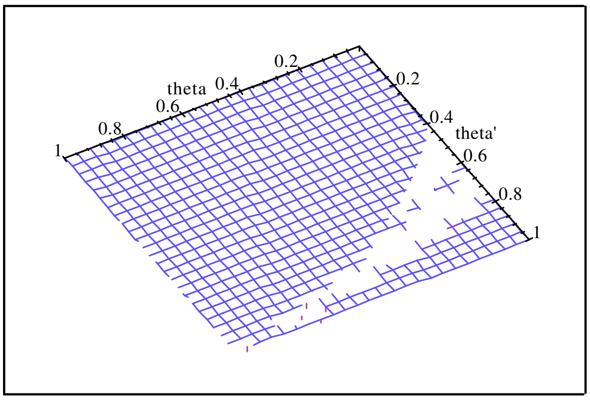
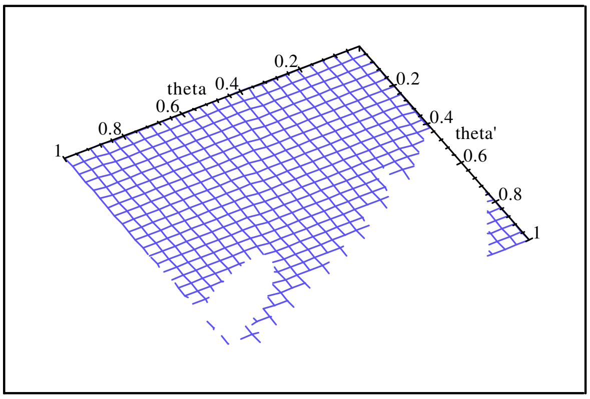


Figure 4.3.: Values of µ and µ' such that $\frac{\partial^2 p_i^{**}}{\partial n \partial \mu^0} > 0$, $(n = 10)$.



1.3.2 Effect on the foreign rival

If $\mu > \mu'$, then the distortion imposed on the foreign firm as a consequence of an increase in the number of firms is larger when the degree of substitution among local firms decreases (i.e., $\frac{\partial^2 \tilde{p}^{**}}{\partial n \partial \mu} > 0$). Hence, unlike what happens with domestic firms, the effect of entry falls monotonically with the degree of substitution among domestic products. This arise presumably because the effect of entry on the foreign firm is largely determined by the degree of substitution between domestic and foreign firm - which is by assumption less than the degree of substitution across domestic products. Hence, when the degree of substitution across domestic products is low (a parameter range for which the distortion on domestic firms increases with the substitution among domestic products), the rivalry between domestic and foreign firms is even less. See Table 5 and figures 5.1. to 5.3. below:

Table 5: Values of $\frac{\partial^2 \tilde{p}^{**}}{\partial n \partial \mu}$ as a function of μ, μ' and n.

	$\mu = .8, \mu' = .8$	$\mu = .8, \mu' = .5$	$\mu = .8, \mu' = .3$
$n = 2$	9.61	11.20	7.76
$n = 3$	7.70	7.50	4.87
$n = 10$	1.52	0.95	0.58

	$\mu = .5, \mu' = .5$	$\mu = .5, \mu' = .3$	$\mu = .2, \mu' = .2$
$n = 2$	8.16	4.73	4.90
$n = 3$	8.25	5.60	6.44
$n = 10$	2.98	1.87	6.38

Figure 5.1.: Values of μ and μ' such that $\frac{\partial^2 \tilde{p}^{**}}{\partial n \partial \mu} > 0$, $(n = 2)$.

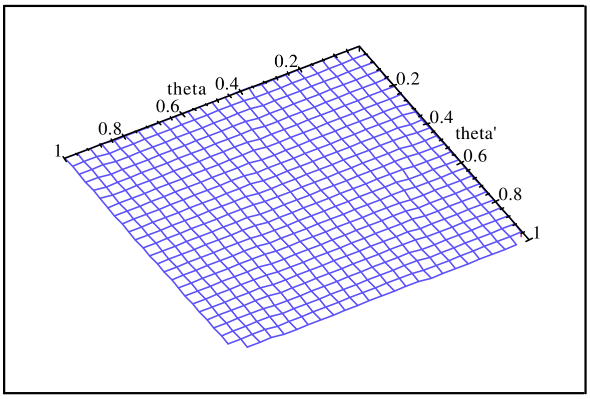


Figure 5.2.: Values of μ and μ' such that $\frac{\partial^2 \tilde{p}^{**}}{\partial n \partial \mu} > 0$, $(n = 3)$.

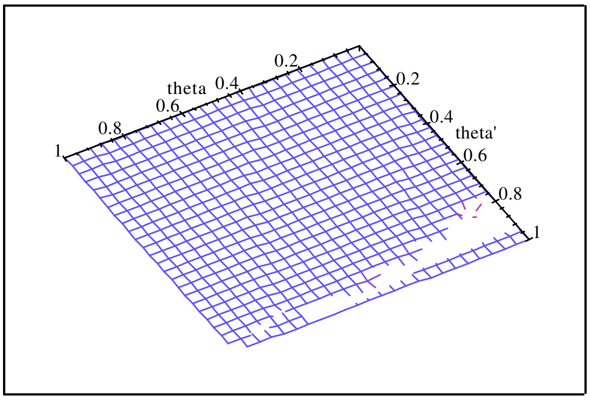
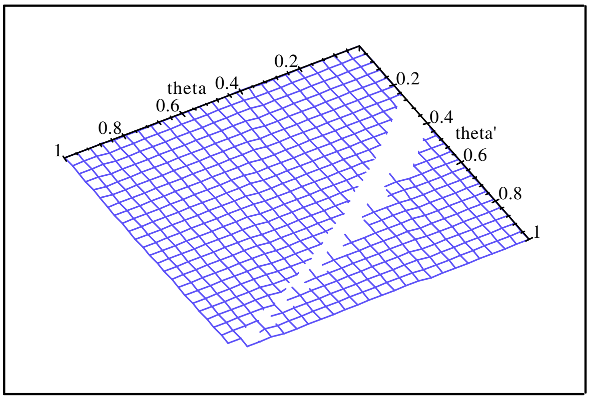


Figure 5.3.: Values of μ and μ' such that $\frac{\partial^2 \tilde{p}^{**}}{\partial n \partial \mu} > 0$, $(n = 10)$.



If $\mu > \mu^0$, the distortion imposed on the foreign price as a consequence of an increase in the number of firms is larger when the degree of substitution among local and foreign products increases (i.e., $\frac{\partial^2 \widetilde{p}^{**}}{\partial n \partial \mu^0} < 0$). This effect is the mirror image of the effect discussed for domestic rivals. As the substitution between domestic and foreign firm increases, the foreign firm becomes more similar to the domestic firms and accordingly is more affected by the entry of a domestic competitor. See Table 6 and figures 6.1. to 6.3. below:

Table 6: Values of $\frac{\partial^2 \widetilde{p}^{**}}{\partial n \partial \mu^0}$ as a function of μ, μ^0 and n.

	$\mu = .8, \mu^0 = .8$	$\mu = .8, \mu^0 = .5$	$\mu = .8, \mu^0 = .3$
$n = 2$	4.59	$-$4.74	$-$6.55
$n = 3$	0.93	$-$2.48	$-$2.90
$n = 10$	$-$0.30	$-$0.26	$-$0.26

	$\mu = .5, \mu^0 = .5$	$\mu = .5, \mu^0 = .3$	$\mu = .2, \mu^0 = .2$
$n = 2$	$-$8.14	$-$11.60	$-$19.10
$n = 3$	$-$5.20	$-$7.50	$-$15.33
$n = 10$	$-$1.22	$-$1.30	$-$5.66

Figure 6.1.: Values of μ and μ' such that $\frac{\partial^2 \widetilde{p}^{**}}{\partial n \partial \mu^0} < 0$, $(n = 2)$.

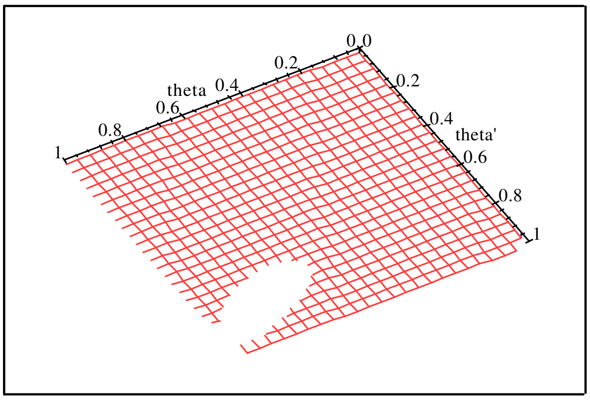


Figure 6.2.: Values of μ and μ' such that $\frac{\partial^2 \widetilde{p}^{**}}{\partial n \partial \mu^0} < 0$, $(n = 3)$.

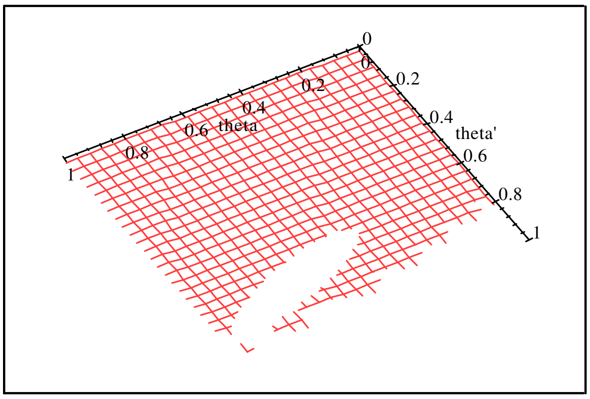
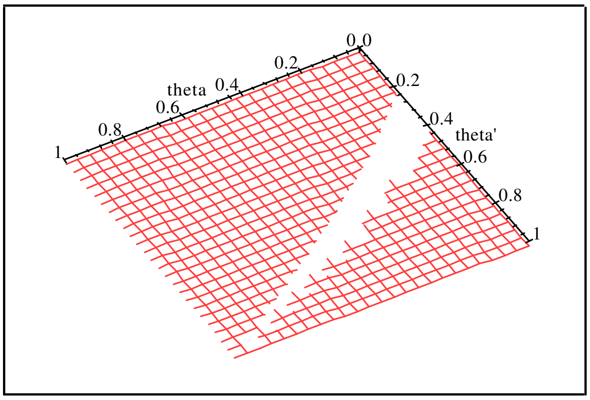


Figure 6.3.: Values of μ and μ' such that $\frac{\partial^2 \tilde{p}^{**}}{\partial n \partial \mu'} < 0$, $(n = 10)$.



1.3.3 Subsidies which affect entry - summary

The following results emerge. First, it appears the distortion induced by the entry (or lack of exit) of a subsidized firm is likely to be limited when there is intense rivalry between domestic firms. This holds both for domestic and foreign firms.

Second, the distortion induced by entry is likely to be most pronounced when rivalry between domestic firms is moderate and when concentration is relatively high.

Third, a reduction in concentration will reduce the importance of the distortion, both because entry is less significant at the margin with a higher number of firms but also because a reduction in concentration will enlarge the set of substitution parameters for which the distortion will be relatively small. In other words, there is a complementarity between rivalry and low concentration to reduce the distortion.

Fourth, increased market segmentation will increase the distortion on domestic firms but reduce the distortion which is incurred by the foreign competitor.

1.4 State aid which affects vertical differentiation

This section studies subsidies that do not affect the cost structure of the firm but the degree of vertical product differentiation in the market. The state aid is assumed to reduce the cost of raising quality for the recipient (for instance, it may be a government intervention which reduces the cost of research and development). As a result, the recipient will end up selling a product of higher quality. Hence, we analyze in what circumstances rivals will be most affected by an increase in the quality sold by the recipient firm.

For simplicity and tractability, we assume that there are two local firms in the market ($n = 2$) and one foreign firm. In a model of vertical differentiation, inverse demand curves for the local and foreign firm can then be written as:

$$p_1 = \alpha_1 - q_1 - \theta q_2 - \theta' \tilde{q}, \tag{35}$$

$$p_2 = \alpha_2 - q_2 - \theta q_1 - \theta' \tilde{q}, \text{ and} \tag{36}$$

$$\tilde{p} = \tilde{\alpha} - \tilde{q} - \theta' (q_1 + q_2). \tag{37}$$

If $\alpha_i > \alpha_j$, then firm j is perceived as providing a better quality than firm i.

We can now derive the direct demand functions as follows:

$$q_1 = -b(p_1 - \alpha_1) + x(p_2 - \alpha_2) + d(\tilde{p} - \tilde{\alpha}), \tag{38}$$

$$q_2 = -b(p_2 - \alpha_2) + x(p_1 - \alpha_1) + d(\tilde{p} - \tilde{\alpha}), \text{ and} \tag{39}$$

$$\tilde{q} = -b'(\tilde{p} - \tilde{\alpha}) + d\sum_{i=1}^{2}(p_i - \alpha_i), \tag{40}$$

where $W = (1-\theta)\left(1+\theta-2\theta'^2\right)$, $b = \frac{1}{W}\left[1-\left(\theta'\right)^2\right]$, $b' = \frac{1}{W}\left(1-\theta^2\right)$, $x = \frac{1}{W}\left[\theta-\left(\theta'\right)^2\right]$ and $d = \frac{1}{W}\left[\theta'(1-\theta)\right]$.

Let us assume that marginal cost of production for local firms is c, whereas marginal cost of foreign firm is $\tilde{c}$. When the strategic variables are prices and there are no subsidies in the market, it can be shown that equilibrium prices in the market are defined by:

$$\tilde{p}_1 = \frac{1}{W'}\Big(\left(3d^2 b + 2b'x^2 - 4b'b^2 + 2d^2 x\right)\alpha_1 + b\left(d^2 + 2b'x\right)\alpha_2 \tag{41}$$
$$- 2b'b(2b+x)c + b'd(x+2b)(\tilde{\alpha} - \tilde{c} - t)\Big),$$

$$p_2^* = \frac{1}{W'}\left(\left(3d^2b + 2b'x^2 - \left(4b'b^2 + 2d^2x\right)\right)\alpha_2 + b\left(d^2 + 2b'x\right)\alpha_1 \right. \tag{42}$$
$$\left. - 2b'b\,(2b + x)\,c + b'd\,(x + 2b)\,(\bar{\alpha} - \bar{c} - t)\right),$$

and

$$\bar{p}^* = \frac{(2b + x)}{W'}\left(\left(b'x - \left(2b'b + 2d^2\right)\right)\bar{\alpha} + db\,(\alpha_1 + \alpha_2) + \right. \tag{43}$$
$$\left. (b'x - 2b'b)\,(\bar{c} + t) - 2dbc\right).$$

where $W' = 2\left(d^2 - 2b'b + b'x\right)(2b + x)$.

The main comparative statics results can then be summarized as follows :

$$\frac{\partial p_i^*}{\partial \alpha_i} > 0, \frac{\partial p_i^*}{\partial \alpha_j} < 0, \frac{\partial p_i^*}{\partial \bar{\alpha}} < 0, \frac{\partial p_i^*}{\partial c} > 0, \frac{\partial p_i^*}{\partial \bar{c}} > 0, \frac{\partial p_i^*}{\partial t} > 0, \tag{44}$$

$$\frac{\partial \bar{p}^*}{\partial \bar{\alpha}} > 0, \frac{\partial \bar{p}^*}{\partial \alpha_i} < 0, \frac{\partial \bar{p}^*}{\partial c} > 0, \frac{\partial \bar{p}^*}{\partial \bar{c}} > 0, \frac{\partial \bar{p}^*}{\partial t} > 0. \tag{45}$$

That is, an increase in the quality of a rival will reduce the equilibrium price of both local and foreign firms. An increase in own quality will lead to a higher price. In other words, the rivals of the recipient firms will be induced to reduce their prices by a state aid which reduces the cost of quality. It is important to note however that the profits of rivals will not necessarily fall. Unlike what happens with subsidies with affect marginal cost, the price of the recipient firm increases and that of the rivals fall. Hence, the link between the direction of changes in prices and profits that was observed for a subsidy to marginal cost (such that they moved in the same direction) may no longer hold. The increase in the price of the recipient may actually shift enough demand to the rivals so that their profit will increase (despite the fact that their equilibrium price falls). In what follows, we will first investigate the circumstances which affect the magnitude of this price distortion, given that it is difficult to derive analytical results with respect to profits. We subsequently undertake some simulations to investigate the effect of profits.

1.4.1 Effect on rivals

The circumstances which affect the price distortion for domestic rivals can be summarized as follows:

$$\frac{\partial^2 p_i^*}{\partial \alpha_j \partial \theta} < 0, \frac{\partial^2 p_i^*}{\partial \alpha_j \partial \theta'} > 0, \tag{46}$$

To fix ideas, assume that firm j is the recipient of the subsidy, such that its quality increases. The effect on the domestic rival is larger when the substitution across domestic products is large. This arises as before because competitors are induced to react more sharply when substitution is large. The distortion falls however when the degree of substitution between domestic and foreign products is higher. In other words, more segmented market will lead to a greater distortion.

Turning to the foreign firm, there is no monotonic relation between the distortion and the degree of differentiation between local firms.

$$\frac{\partial^2 \tilde{p}^{*}}{\partial \alpha_i \partial \theta} = \frac{1}{2} \frac{\left(1 - \beta^2\right)\beta}{\left(\theta^2 - \theta + 2\beta^2 - 2\right)^2} (1 - 2\theta) \gtrless 0, \; i = 1, 2. \tag{47}$$

From this equation, it appears that the distortion increases with the degree of substitution among local firms as long as the degree of substitution among local products is sufficiently high (θ values larger than .5). That is, for values of θ high enough, $\frac{\partial^2 \tilde{p}^{*}}{\partial \alpha_i \partial \theta} < 0$.

Furthermore when $\theta > \theta'$(that is the degree of product substitution is lower between foreign and local firms than among local firms), the distortion incurred by the foreign product increases with the degree of substitution between domestic and foreign products, i.e. :

$$\frac{\partial^2 \tilde{p}^{*}}{\partial \alpha_i \partial \theta'} < 0. \tag{48}$$

With respect to the recipient, we further observe that :

$$\frac{\partial^2 p_i^{*}}{\partial \alpha_i \partial \theta'} \lessgtr 0, \text{ and if } \theta > \theta' \frac{\partial^2 p_i^{*}}{\partial \alpha_i \partial \theta} < 0. \tag{49}$$

That is, the increase in price as a consequence of an increase in own product quality is larger when the degree of product substitution among local product is lower (θ is lower), provided that the degree of product substitution is lower between foreign and local firms than among local firms. However, there is no a monotonic relation between the extent of the distortion and the the degree of differentiation between foreign and local firms $\left(\frac{\partial^2 p_i^{*}}{\partial \alpha_i \partial \theta'}\right)$. It can be shown that when the degree of substitution in the local market is sufficiently high (high θ values), the increase in price motivated by the increase in quality is an increasing function of the degree of substitution between local and foreign products (for values of θ high enough, $\frac{\partial^2 p_i^{*}}{\partial \alpha_i \partial \theta'} > 0$).

1.4.2 Subsidies that affect the degree of vertical product differentiation - summary

A number of results emerge. First, it appears that a high degree of rivalry will increase the distortion on domestic rivals but also on foreign rivals, as long as domestic rivalry is strong enough. Second, market segmentation will have opposite effects on domestic an foreign firms. The distortion on domestic firms will increase and that imposed on foreign firms will decrease.

In the context of the model presented here, the number of firms has been fixed. The effect of concentration is investigated through a limited simulation presented in appendix B. The results presented in this appendix confirm that an increase in concentration will tend to enhance the distortions on both domestic and foreign firms.

Overall, the circumstances in which price distortions induced by a subsidy which affects the quality of products are greatest appear to be similar to those found above in the case of subsidies which affect marginal cost. In particular, concentration and the degree of rivalry among domestic firms have the same effect of the distortions imposed on domestic firms.

However, as mentioned above, the effect of the subsidy on the price of rivals may differ from its effect on profits. The simulations presented in Appendix B suggest that profits of rivals are more likely to increase when the degree of substitution among domestic products is large (and concentration is high).

1.5 Factors which affect the distortions of competition

We briefly collect the results that we have derived with respect to three types of state intervention (marginal cost, entry and quality) and analyze whether particular market characteristics are robust indicators of the magnitude of the distortions. First, it appears that concentration is a fairly robust indicator. In all three cases, an increase in concentration tends to increase the price distortions that is incurred by domestic and foreign firms. The presumption that state aid is more likely to induce distortions in concentrated market thus receives some support. On should however be cautions in the case of subsidies which affect quality; if high concentration induces large price distortions, it may however not necessarily lead to a reduction in the profit of rivals.

Second, intense domestic rivalry (which could be proxied by low margins or low product differentiation) is not a robust indicator of the magnitude of the distortions. Its effect depend on the type of state intervention. When state aid takes the form of reductions in marginal cost, it will be a good indicator of the magnitude of the distortions, for domestic firms. With respect to state intervention which induces entry (or prevents exit), it is an intermediate degree of rivalry which will induce the greatest distortions. Finally, with respect to state intervention which affects quality, rivalry will tend to increase the price distortions but it will might also increase the likelihood that rivals will benefit in terms of profit. This would suggest that the degree of rivalry should be considered carefully as an indicator of the magnitude of the distortion. Its effect will depend on the whether the subsidy affects marginal cost or the quality of the product sold by the recipient.

Third, domestic rivalry may have opposite effects respectively on domestic and foreign firms. In particular, when the market is not concentrated and state aid takes the form of a production subsidy, domestic rivalry will reduce the distortions felt by the foreign firm (but increase that felt by domestic rivals). That is also to say that the importance of the spillover across countries is not only a function of the extent of market segmentation but also a function of the conditions of competition in the domestic market.

Fourth, the effect of market segmentation is without surprise; in all three cases, a greater segmentation will insulate the foreign firm from state intervention and increase the distortion which is felt by domestic firms.

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A A numerical example - horizontal product differentiation

This appendix provides a numerical example for the effect of production subsidies with horizontal product differentiation. Table 7 and Table 8 below present some simulations in terms of equilibrium prices and profits for two alternative scenarios with two and three home firms in the local market and the following parameter values $\alpha = 100$, $c = \tilde{c} = 5$, and $t = 2$. These simulations illustrate the various effects discussed in the text. They also confirm that second order effects for profits are the same as those developed in the text for prices. For instance, comparing the first and second panels of table 7, one observes that the profit distortion imposed on domestic (foreign) firms increases (decreases) as the substitution between domestic and foreign products increases. A comparison between the second and third panel confirms that the profit distortion imposed on domestic rivals increases with the degree of substitution among domestic firms. The profit distortion imposed on the foreign firm also increases, in line with what is found in the text with respect to prices when concentration is high. Finally, a comparison between tables 8 and 9 confirms that the profit distortion imposed on domestic and foreign firms increases with concentration.

Table 8: Competitive effects of state aid ($n = 2$)

	$\theta = 0.8$ and $\theta' = 0.7$
p_1^{**}	$16.16 - .58 s_1$
p_i^{**}	$16.16 - .20 s_1$
$\tilde{p}^{**}$	$20.90 - .15 s_1$
Π_1^{**}	$387.50 + .547 s_1 (53.25 + s_1) - F$
Π_i^{**}	$387.50 - .121 s_1 (113.19 - s_1) - F$
$\tilde{\Pi}^{**}$	$423.92 - 0.050 s_1 (183.74 - s_1) - \tilde{F}$
	$\theta = 0.8$ and $\theta' = 0.1$
p_1^{**}	$20.11 - .59 s_1$
p_i^{**}	$20.11 - .24 s_1$
$\tilde{p}^{**}$	$49.06 - .02 s_1$
Π_1^{**}	$634.93 + .456 s_1 (74.63 + s_1) - F$
Π_i^{**}	$634.93 - .157 s_1 (127.16 - s_1) - F$
$\tilde{\Pi}^{**}$	$1789.1 - 0.0005 s_1 (3636.80 - s_1) - \tilde{F}$
	$\theta = 0.2$ and $\theta' = 0.1$
p_1^{**}	$45.18 - .51 s_1$
p_i^{**}	$45.18 - .049 s_1$
$\tilde{p}^{**}$	$48.93 - .023 s_1$
Π_1^{**}	$1692.70 + .256 s_1 (162.55 + s_1) - F$
Π_i^{**}	$1692.70 - 0.002 s_1 (1624.6 - s_1) - F$
$\tilde{\Pi}^{**}$	$1788.0 - 0.0005 s_1 (3625.6 - s_1) - \tilde{F}$

Table 9: Competitive effects of state aid ($n = 3$)

	$\mu = 0.8$ and $\mu' = 0.7$
p_1^{**}	$12.39 - .56 s_1$
p_i^{**}	$12.39 - .14 s_1$
$\bar{p}^{**}$	$18.12 - .11 s_1$
π_1^{**}	$198.09 + .689 s_1 (33.89 + s_1) - F$
π_i^{**}	$198.09 - .075 s_1 (102.96 - s_1) - F$
$\bar{\pi}^{**}$	$284.44 - .030 s_1 (194.09 - s_1) - \bar{F}$

	$\mu = 0.8$ and $\mu' = 0.1$
p_1^{**}	$14.08 - .57 s_1$
p_i^{**}	$14.08 - .16 s_1$
$\bar{p}^{**}$	$48.55 - .02 s_1$
π_1^{**}	$285.27 + .632 s_1 (42.475 + s_1) - F$
π_i^{**}	$285.27 - .093 s_1 (111.06 - s_1) - F$
$\bar{\pi}^{**}$	$1746.00 - .0003 s_1 (4803.10 - s_1) - \bar{F}$
	$\mu = 0.2$ and $\mu' = 0.1$
p_1^{**}	$41.25 - .51 s_1$
p_i^{**}	$41.25 - .05 s_1$
$\bar{p}^{**}$	$47.20 - .02 s_1$
π_1^{**}	$1414.80 + .260 s_1 (147.37 + s_1) - F$
π_i^{**}	$1414.80 - .002 s_1 (1601.60 - s_1) - F$
$\bar{\pi}^{**}$	$1651.90 - .0005 s_1 (3761.70 - s_1) - \bar{F}$

B A numerical example - vertical product differentiation

This appendix develops a numerical example to illustrate the effects of an increase in the quality of a product (product 1). Let us assume that marginal costs in the industry are such that $c = \bar{c} = 5$ and the level of transportation costs is $t = 2$. Substituting these values into equation (41), equation (42) and equation (43) we obtain the following results depending on the degree of vertical and horizontal product differentiation. It is worth noting in particular that the profit of the domestic rival increases as a result of a higher quality, when there are two firms in the market and when substitution is high (see Table 11).

Table 10a: Bertrand Competition with Vertical Product Differentiation ($n = 2$)

	$\mu = 0.8$ and $\mu' = 0.7$	$\mu = 0.8$ and $\mu' = 0.1$
p_1^{*}	$4.64 + .42\alpha_1 - .20\alpha_2 - .11\bar{\alpha}$	$4.22 + .40\alpha_1 - .24\alpha_2 - 0.008\bar{\alpha}$
p_2^{*}	$4.64 - .20\alpha_1 + .42\alpha_2 - .11\bar{\alpha}$	$4.22 - .24\alpha_1 + .40\alpha_2 - 0.008\bar{\alpha}$
$\bar{p}^{*}$	$5.30 - .15\alpha_1 - .15\alpha_2 + .46\bar{\alpha}$	$3.73 - 0.02\alpha_1 - 0.02\alpha_2 + .499\bar{\alpha}$

	$\mu = 0.2$ and $\mu' = 0.1$
$\tilde{p}_1$	$2.93 + .49\alpha_1 - .05\alpha_2 - .022\alpha$
$\tilde{p}_2$	$2.93 - .05\alpha_1 + .49\alpha_2 - .022\alpha$
$\tilde{p}$	$3.74 - 0.02\alpha_1 - 0.02\alpha_2 + .498\alpha$

Table 10b: Bertrand Competition with Vertical Product Differentiation ($n = 3$)

	$\mu = 0.8$ and $\mu' = 0.7$	$\mu = 0.8$ and $\mu' = 0.1$
$\tilde{p}_1$	$4.76 + .44\alpha_1 - .14(\alpha_2 + \alpha_3) - 0.07\alpha_4$	$4.53 + .43\alpha_1 - .16(\alpha_2 + \alpha_3) - 0.005\alpha_4$
$\tilde{p}_2$	$4.76 + .44\alpha_2 - .14(\alpha_1 + \alpha_3) - 0.07\alpha_4$	$4.53 + .43\alpha_2 - .16(\alpha_1 + \alpha_3) - 0.005\alpha_4$
$\tilde{p}_3$	$4.76 + .44\alpha_3 - .14(\alpha_1 + \alpha_2) - 0.07\alpha_4$	$4.53 + .43\alpha_3 - .16(\alpha_1 + \alpha_2) - 0.005\alpha_4$
$\tilde{p}$	$5.42 - .11(\alpha_1 + \alpha_2 + \alpha_3) + .470\alpha_4$	$3.76 - 0.018(\alpha_1 + \alpha_2 + \alpha_3) + .499\alpha_4$

	$\mu = 0.2$ and $\mu' = 0.1$
$\tilde{p}_1$	$3.13 + .49\alpha_1 - 0.045(\alpha_2 + \alpha_3) - .020\alpha_4$
$\tilde{p}_2$	$3.13 + .49\alpha_2 - 0.045(\alpha_1 + \alpha_3) - .020\alpha_4$
$\tilde{p}_3$	$3.13 + .49\alpha_3 - 0.045(\alpha_1 + \alpha_2) - .020\alpha_4$
$\tilde{p}$	$3.84 - 0.02(\alpha_1 + \alpha_2 + \alpha_3) + .497\alpha_4$

Table 10a to Table 12b below simulate the changes in profits for particular values of the substitution parameters.

Table 11a: Equilibrium Profits and Vertical Product Differentiation ($n = 2$, $\mu = 0.8$ and $\mu' = 0.7$)

	$\alpha_1 = \alpha_2 = \alpha = 100$	$\alpha_1 = 125$ and $\alpha_2 = \alpha = 100$
Π_1^*	$388 - F$	$1457 - F$
Π_2^*	$388 - F$	$1227 - F$
Π^*	$424 - F$	$225 - F$

Table 11b: Equilibrium Profits and Vertical Product Differentiation ($n = 3$, $\mu = 0.8$ and $\mu' = 0.7$)

	$\alpha_1 = \alpha_2 = \alpha = 100$	$\alpha_1 = 125$ and $\alpha_2 = \alpha = 100$
Π_1^*	$198 - F$	$1214 - F$
$\Pi_2^* = \Pi_3^*$	$198 - F$	$52 - F$
Π^*	$284 - F$	$157 - F$

Table 12a: Equilibrium Profits and Vertical Product Differentiation ($n = 2$, $\mu = 0.8$ and $\mu' = 0.1$)

	$\alpha_1 = \alpha_2 = \alpha = 100$	$\alpha_1 = 125$ and $\alpha_2 = \alpha = 100$
Π_1^*	$634.9 - F$	$1770.7 - F$
Π_2^*	$634.9 - F$	$233.8 - F$
Π^*	$1789.1 - F$	$1740.2 - F$

Table 12b: Equilibrium Profits and Vertical Product Differentiation ($n = 3$, $\mu = 0.8$ and $\mu' = 0.1$)

	$\alpha_1 = \alpha_2 = \tilde{\alpha} = 100$	$\alpha_1 = 125$ and $\alpha_2 = \tilde{\alpha} = 100$
Π_1^*	$285 - F$	$1352 - F$
$\Pi_2^* = \Pi_3^*$	$285 - F$	$86.7 - F$
$\widetilde{\Pi}^*$	$1746 - \tilde{F}$	$1710 - \tilde{F}$

Table 13a: Equilibrium Profits and Vertical Product Differentiation ($n = 2$, $\theta = 0.2$ and $\theta^0 = 0.1$)

	$\alpha_1 = \alpha_2 = \tilde{\alpha} = 100$	$\alpha_1 = 125$ and $\alpha_2 = \tilde{\alpha} = 100$
Π_1^*	$1693 - F$	$2894 - F$
Π_2^*	$1693 - F$	$1590 - F$
$\widetilde{\Pi}^*$	$1788 - \tilde{F}$	$1739 - \tilde{F}$

Table 13b: Equilibrium Profits and Vertical Product Differentiation ($n = 3$, $\theta = 0.2$ and $\theta^0 = 0.1$)

	$\alpha_1 = \alpha_2 = \tilde{\alpha} = 100$	$\alpha_1 = 125$ and $\alpha_2 = \tilde{\alpha} = 100$
Π_1^*	$1415 - F$	$2538 - F$
Π_2^*	$1415 - F$	$1328 - F$
$\widetilde{\Pi}^*$	$1652 - \tilde{F}$	$1608 - \tilde{F}$

C The effect of a production subsidy with vertical product differentiation

In this appendix, we show that the effect of production subsidies on prices is unaffected by the degree of vertical differentiation.

Using the model of section 4.4 (Bertrand competition with both vertical and horizontal product differentiation), it is easy to prove that when we introduce a state aid that reduces the marginal cost of firm 1, the decline on rival's equilibrium prices is not a function of the degree of vertical product differentiation among firms:

$$\frac{\partial^2 p_i^*}{\partial s_1 \partial \alpha_j} = 0, \text{ and } \frac{\partial^2 \tilde{p}^*}{\partial s_1 \partial \alpha_j} = 0, \ \forall i, j = 1, ..., n. \tag{50}$$

This result may be due to the linearity of the demand specification that we use.